DISCUSSION: VARIETY, ANALOGY, AND PERIODICITY IN INDUCTIVE LOGIC.*

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l. The Variety of Instances. Peter Achinstein gives in his papers [1] and [2] interesting analyses of some problems of inductive logic and of some approaches I have proposed. I shall discuss here some of these problems in order to clarify my present position. My comments will mainly concern the variety of instances, and only briefly the analogy influence, and the inductive methods for a coordinate language.

In his paper [2] (the ms. of which he kindly made available to me) Achinstein derives some formulas concerning my confirmation function c^* for certain examples. He infers from the results that c^* does not satisfy the requirement of the variety of instances (RVI). I had asserted in my paper ([3], §15] and again in my book ([5], §110 I] that c^* satisfies RVI. He further generalizes his results by arguments intended to show that none of the confirmation functions in the lambda-system, explained in my monograph [6], satisfies RVI.

Many years ago, when I examined for the first time the question whether c^* satisfies RVI, I used examples similar to those of Achinstein and found the same result: that the value of c^* for the hypothesis was the same on each of two evidences, one with a variety of instances and the other with instances of one kind only. This led me too to the belief that the requirement was not fulfilled. However, on closer analysis I saw that the simple forms of evidence which I had used (and similarly now Achinstein) are not suitable for an examination with respect to RVI. The examination must rather be made in the following way. The observers X and Y start out with a common prior evidence e, which states the following about a class K of N objects: (a) N_1 specified objects of class K have the property P_1, N'_1 have not; (b) N_{13} specified objects of those with P_1 have also the property P_3, $N_{13'}$ others have not. Both observers intend to test the law l: "All objects with P_1 have also P_2". They make their tests separately, but they agree that each of them will test a sample of s specimens from the N_1 objects which were found to have P_1. Now X takes his s specimens all from the N_{13} with P_3, while Y, mindful of the requirement RVI, takes s_{13} specimens from those N_{13} and $s - s_{13}$ others from the $N_{13'}$ without P_3. X's report e_1 on his tests states that his s specimens have P_2; and Y's report says the same of his s specimens. Thus either observer has found s confirming instances and no disconfirming ones. Now let h be the hypothesis saying that the next object (or the next n objects) found to have P_1 will also have P_2 and thus satisfy the law l. Then RVI demands that the confirmation function c be such that $c(h, e.e_2) > c(h, e.e_1)$. This is the correct form of the requirement. And I shall show that this is fulfilled by my function c^*.

The essential point here is that the two observers have the same prior information about the observed frequencies of the various kinds of objects which come into consideration for a test of the law. This is necessary for a fair comparison. In Achinstein's example, the evidence for each observer is restricted to those objects which he tests for P_2. Thus the prior information of the first observer is restricted to those s ravens

* Received March, 1963.

[1] The author is indebted to the National Science Foundation for the support of research in inductive probability.

which he has found to be young. This is then his total knowledge about ravens, since we must assume that he has fulfilled the requirement of total evidence ([5], §45 B). For all he knows there may be no old ravens. Under these conditions we cannot say that he violated a methodological principle by not choosing any old ravens for his test of the law.

2. On the, function c*. We consider the law $\boldsymbol{l}$: '$(x)\ (Mx \supset Mx)$', interpreted, e.g., as "All metal bodies are good conductors of electricity". I shall use the concepts and notations of ([5], §§31, 32, 38). We take a finite language L with N individuals and κ Q-predicates. We define 'M_l' by '$M. \sim\ M''$', and 'M_2' by 'M.M''. The law $\boldsymbol{l}$ says that M_l is empty. Let the p predicates 'K_1', ..., 'K_p,' be pairwise L-exclusive, and their disjunction be L-equivalent to 'M'. Thus they designate p different, non-overlapping kinds of metal (e.g., iron, copper, etc.). For each q, let $K_{1,\,q}$, be the conjunction of M_1 and K_q, and $K_{2,\,q}$ that of M_2 and K_q. Let $w_{l,\,q}$ be the logical width of K_l, q, and w_2,q that of $K_{2,\,q}$. Each of the predicates mentioned is supposed to be a factual, molecular predicate in L, hence L-equivalent to a disjunction of (one or more) Q-predicates.

We assume that X's prior information is expressed in three sentences k, k^*, and j, as follows. Each of them is a conjunction of full sentences of certain predicates; all these full sentences contain distinct individual constants. k contains s' full sentences, to wit, for every q, s'_q full sentences of $K_{2,\,q}$. Thus k is a report about earlier tests of the law $\boldsymbol{l}$, with s' specimens of M, among them s'_q of the kind K_q, all of which were found to have M_2, and thus to confirm the law. k^* contains s^* full sentences of different predicates, each predicate L-implying non-M. This is a report on observations of s^* objects which were all found to be non-metals. j contains s_M full sentences with K-predicates, among them $s_{M,\,q}$, with K_q. j is a description of s_M metal specimens, each of them specified as belonging to one of the p subkinds K_1, ..., K_p.

Now X makes new tests for the law $\boldsymbol{l}$. He chooses s of the specimens described in j, among them, for every q (from 1 to p), s_q specimens from those described in j as belonging to the kind K_q. (Hence $s_q \leq s_{M,\,q}$; some of the numbers s_q may be 0.) All these tests have positive results, i.e., these s specimens are found to be M_2 and thus M', and hence to confirm the law. Let i be the report of these results. Thus i is a conjunction of s full sentences of M_2 with distinct individual constants; among these constants there are, for every q, s_q which occur in j with K_q. The posterior information consists of the prior one together with i.

I shall now give a theorem about c^* involving the sentences just described. (I referred to this theorem in [3, §15] without stating it explicitly; its proof is too complicated to be given here.)

$$(1) \qquad c^*(i, k \cdot k^* \cdot j) = \prod_{q=1}^{p} \frac{(s'_q + s_q + w_{2,\,q} - 1)!\,(s'_q + w_{1,\,q} + w_{2,\,q} - 1)!}{(s'_q + s_q + w_{1,\,q} + w_{2,\,q} - 1)!\,(s'_q + w_{2,\,q} - 1)!}$$

I have chosen here a language L with a finite number N of individuals, so that the law $\boldsymbol{l}$ is a finite conjunction of instances and has positive c^*-values. (For an infinite law, c^* is 0; see ([5], §110 F, (12)).

We define R as the ratio of the posterior confirmation of $\boldsymbol{l}$ to its prior confirmation:

$$(2) \qquad R =_{\mathrm{Df}} c^*(\boldsymbol{l}, k\,.k^*.j.i)/c^*(\boldsymbol{l}, k.k^*.j).$$

We see easily that i is L-implied by $j.\boldsymbol{l}$, and hence also by $k.k^*.\ j.\boldsymbol{l}$. Therefore, by ([5], T61-3c):

(3) $$R = 1/c^*(i,\ k.k^*.j).$$

We see from (3) and (1) that R is independent of the values of s^*, s_M, and $s_{M,\,q}$ (i.e., $s_{M,\,1}$, ..., $s_{M,p}$), although the two c^*-values in (2) are dependent on them.

3. Application to Nagel's Numerical Examples. Nagel has given some numerical examples ([9], pp. 68-71) in order to illustrate the great difficulties for the construction of a c-function that would be in accord with RVI. He makes use of two subkinds K_1 and K_2. Hence we have $p = 2$. In order to make our numerical calculations easier, we make the following simplifying assumptions: (A) we omit k; hence we have $s'_q = 0$. This means that the prior information does not contain any results of tests of metal specimens with respect to conductivity; thus the only results of this kind are those described in i. (B) We assume that each of the widths w_{11}, w_{21}, w_{12}+ w_{22} is 1; thus the width of M is 4. (We might take the width of non-M as 12; then $\kappa = 16$; but these numbers are irrelevant for R, though relevant for the c^*-values.) For the intended examination of c^* with respect to RVI, it is fortunately not necessary to calculate the posterior values of c^* for the various cases. It is sufficient to determine R. And for this we obtain, under the simplifying conditions mentioned, an extremely simple formula (from (3) and (1)):

(4) $$R = (s_1 + 1)\ (s_2 + 1).$$

Nagel discusses nine different possible cases P_1, ..., P_9 with respect to the total number s of specimens tested, and the numbers s_1 and s_2 of the two kinds, as given in the subsequent table. Let us imagine nine observers X_1, ..., X_9, who have the same prior information $k^*.\ j,$ but then make different decisions with respect to s, s_1, and s_2, as specified in Nagel's nine cases. (In order to make possible the values of s_1 and s_2 in all nine cases, we must choose the numbers in j sufficiently high so that, in each case, $s_{M,q} \geq s_q$; hence we must take $s_{M,1} \geq 200$ and $s_{M,2} \geq 100$; but here we need not bother about these numbers, since they do not affect R.) The nine observers obtain different results i, say i_1, ..., i_9, and hence different values of the posterior confirmation of the law. Now we have from (2):

(5) $$\text{For } n = 1, ..., 9,\ \ c^*(\boldsymbol{l},\ k^*.\ j.i_n) = R_n.c^*(\boldsymbol{l},\ k^*.j).$$

I have added to Nagel's table the last line, giving the values of R_1, ..., R_9, determined by (4). Since the prior confirmation is the same in all nine cases, the posterior confirmation is, according to (5), proportional to R_n.

	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9
s_1	50	50	100	101	99	100	200	100	198
s_2	0	50	0	49	52	90	0	100	2
s	50	100	100	150	151	190	200	200	200
R_n	51	2601	101	5100	5300	9191	201	10201	597

The values of R_n clearly bear out what I said about the nine cases in ([3], §15). In particular, these values show that the posterior confirmation is increased not only if the total number s of confirming instances is increased (and none of the numbers s_1 and s_2 is decreased), but also, and indeed much more, if with the same s the distribution is improved, i.e., the smaller of the numbers s_1 and s_2 is increased from 0 to a positive value, or from a positive value to another one nearer to $s/2$. Thus the function c^* is in good accordance with RVI.

4. The Analogy Influence. The principle of analogy says roughly this: the probability that an object b has a certain property, is increased by the information that one or more other objects, which are similar to b in other respects, have this property. In very simple situations, the old way of applying the function c^* leads to the required result, as I have shown in ([5], §110 D). This holds likewise for any other c-function of the lambda-system. However, this is not the case for more complicated situations, e.g. those is Achinstein's example with rhodium. This is due to the failure of the old methods to take account of the different degrees of similarity between Q-predicates. If (as in ([5], p. 125, Table A31-1)) Q_1 is defined by $P_1.P_2.P_3$, Q_2 by $P_1.P_2. \sim P_3$, Q_4 by $P_1. \sim P_2. \sim P_3$, and Q_8 by $\sim P_1. \sim P_2. \sim P_3$, then an object with Q_8 is most dissimilar to one with Q_1; one with Q_4 is somewhat more similar, and one with Q_2 still more.

This defect of the old methods is overcome by what we now call the method of several families. We now regard the formulas for functions of the lambda-system, including c^*, as exactly valid if they are applied to predicates of one family. This is explained in the new Preface to ([5], second edition). When predicates of two or more families are involved, the old methods are at best approximately valid, and new methods are required. For example, the old methods are appropriate for a law of the form: "For every x, if x is green or blue, then x is blue", which says in effect that the property Green is empty. On the other hand, for a law like ***1*** in §2, according to the new approach, we should use a new kind of c-function for two families, one family of kinds of substance, including K_1, ..., K_p, and one family of three predicates: "good conductor of electricity", "bad conductor", and "non-conductor".

Soon after writing [6], I found a c-function appropriate for two families. Then in 1953, John G. Kemeny and I together worked out methods for any number n of families. The formula for two families is given in ([7], *Anhang* B, sec. VIII). These methods take account of the various degrees of similarity among the Q-predicates. They would, I suppose, lead to intuitively plausible results in Achinstein's examples.

5. Coordinate-languages. In his earlier paper [1], Achinstein analyzes the problem of adequate c-functions for a coordinate language with natural numbers as coordinates (comp. [5], pp. 62 f., 74). He correctly rejects certain approaches as inadequate. In his requirements he makes use of my distinction between qualitative, positional, and mixed predicates. However, I would not require that only qualitative predicates occur in a law or other hypothesis; any sentence of the language may be taken as a hypothesis. What I mean is only this: in certain axioms and theorems referring to predicates, these predicates (or some of them) are required to be qualitative.

Achinstein states a very general Principle 1, which seems to him plausible, and which is supposed to justify his requirements. However, I have serious doubts about this principle. If I understand it correctly, it seems to me not valid in a coordinate language, not even if the hypothesis h contains only qualitative predicates. Let h be '$P(201)$'; let e say that the positions from 1 to 110 have P, and those from 111 to 200 non-P. *Thus e* contains 110 instances of h, and 90 of non-h. Principle 1 requires (if I understood it correctly) that $c(h, e) > c(\sim h, e)$. In contrast, I would think that $c(\sim h, e)$ should be near to 1, and therefore $c(h, e)$ near to 0. In a coordinate language, the numbers of positive instances and of negative instances have sometimes less effect on the c-value of a prediction h than the nearness of the instances to the position referred to in h. (I call this effect the proximity influence.)

Achinstein studies chiefly laws of periodicity. He analyzes various approaches, rejects each of them, and comes finally to an impasse. He believes that a solution will require a much stronger language containing predicates and quantified variables of higher order.

I do not share this belief. I will briefly indicate how I would approach the problem of a coordinate language in inductive logic. As an example, let us think of a family of five predicates for simple qualitative properties $P_1, ..., P_5$, say colors. An m-segment is a series of m consecutive positions. I introduce Q^m-predicates for the possible properties ("m-species") of m-segments. For example, I define:

(6) $$Q^3_{5,1,4}(n) =_{\mathrm{Df}} P_5(n) \cdot P_1(n + 1).P_4(n + 2).$$

Thus the sentence '$Q^3_{5,1,4}(8)$' ascribes to the 3-segment beginning with position 8 the 3-species consisting of P_5, P_1, P_4 in this order; but formally, '$Q^3_{5,1,4}$' is a one-place predicate of positions.

With the help of these Q^m-predicates, many kinds of regularity in the order in which the colors appear can be formulated. A universal sentence saying that p specified m-species never occur is called a law of span m and strength p (in analogy to ([5], D37-6a)). For example, a law saying that, if P_4 is followed by P_3, then P_1 always follows, followed in turn by either P_3 or P_5, is a law of span 4, and strength 12.

I should prefer to use the term "periodicity law" only for those laws which describe a regular recurrence with a fixed period length n. Such a law has a span n. For example, the sentence "P_1 is not empty and, for every i, $P_1(i)$ *iff* $P_1(i + 10)$" has a span 10; it excludes all and only those 10-species in which P_1 does not occur exactly once. All laws with finite span, whether periodical or not, can be dealt with in terms of Q^m-predicates. Inductive methods for coordinate languages are under investigation; they take into account both regularities of succession (of finite span) and the proximity influence. But there are difficult problems still to be solved. For the laws here involved no variables or predicates of higher order are necessary.

If a law refers to absolute values of the coordinate, then it is quite different from laws of finite span (which involve only coordinate *differences,* not absolute values). An example is the law: "For every n, $P_1(n)$ *iff* n is a prime number", discussed by Hilary Putnam [10] and Achinstein [1]. As I have explained in my reply to Putnam ([8], §29), I think that few physicists would even consider laws of this kind, and that it is hardly worthwhile to take account of such laws in adequacy conditions for c-functions for a coordinate language. It would perhaps be preferable to take as coordinates all integers, and not to give a distinguished role to any one position. This might diminish the temptation to refer in a law to absolute values of the coordinate.

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