I. STATISTICAL AND INDUCTIVE PROBABILITY
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If you ask a scientist whether the term 'probability' as used in science has always the same meaning, you will find a curious situation. Practically everyone will say that there is only one scientific meaning; but when you ask that it be stated, two different answers will come forth. The majority will refer to the concept of probability used in mathematical statistics and its scientific applications. However, there is a minority of those who regard a certain non-statistical concept as the only scientific concept of probability. Since either side holds that its concept is the only correct one, neither seems willing to relinquish the term 'probability'. Finally, there are a few people—and among them this author—who believe that an unbiased examination must come to the conclusion that both concepts are necessary for science, though in different contexts.

I will now explain both concepts—distinguishing them as 'statistical probability' and 'inductive probability'—and indicate their different functions in science. We shall see, incidentally, that the inductive concept, now advocated by a heretic minority, is not a new invention of the 20th century, but was the prevailing one in an earlier period and only forgotten later on.

The <u>statistical concept of probability</u> is well known to all those who apply in their scientific work the customary methods of mathematical statistics. In this field, exact methods for calculations employing statistical probability are developed and rules for its application are given. In the simplest cases, probability in this sense means the relative frequency with which a certain kind of event occurs within a given reference class, customarily called the "population". Thus, the statement "The probability that an inhabitant of the United States belongs to blood group A is p" means that a fraction p of the inhabitants belongs to this group. Sometimes a statement of statistical probability refers, not to an actually existing or observed frequency, but to a potential one, i.e. to a frequency that would occur under certain specifiable circumstances. Suppose, for example, a physicist carefully examines a newly made die and finds it is a geometrically perfect and materially homogeneous cube. He may then assert that the probability of obtaining an ace by a throw of this die is 1/6. This means that if a sufficiently long series of throws with this die were made, the relative frequency of aces would be 1/6. Thus, the probability statement here refers to a potential frequency rather than to an actual one. Indeed, if the die were destroyed before any throws were made, the assertion would still be valid. Exactly speaking, the statement refers to the physical microstate of the die; without specifying its details (which presumably are not known), it is characterized as being such that certain results would be obtained if the die were subjected to certain experimental procedures. Thus the statistical concept of probability is not essentially different from other disposition concepts which characterize the objective state of a thing by describing reactions to experimental conditions, as, for example, the I. Q. of a person, the elasticity of a material object, etc.

Inductive probability occurs in contexts of another kind; it is ascribed to a hypothesis with respect to a body of evidence. The hypothesis may be any statement concerning unknown facts, say, a prediction of a future event, e. g., tomorrow's weather or the outcome of a planned experiment or of a presidential election, or a presumption concerning the unobserved cause of an observed event. Any set of known or assumed facts may serve as evidence; it consists usually in results of observations which have been made. To say that the hypothesis h has the probability p (say, 3/5) with respect to the evidence e, means that for anyone to whom this evidence but no other relevant knowledge is available, it would be reasonable to believe in h to the degree p or, more exactly, it would be unreasonable for him to bet on h at odds higher than p: (1-p) (in the example, 3:2). Thus inductive probability measures the strength of support given to h by e or the degree of confirmation of h on the basis of e. In most cases in ordinary discourse, even among scientists, inductive probability is not specified by a numerical value but merely as being high or low or, in a comparative judgment, as being higher than another probability. It is, important to recognize that every inductive probability judgment is relative to some evidence. In many cases no explicit reference to evidence is made; it is then to be understood that the totality of relevant information available to the speaker is meant as evidence. If a member of a jury says that the defendant is very probably innocent or that, of two witnesses A and B who have made contradictory statements, it is more probable that A lied than that B did, he means it with respect to the evidence that was presented in the trial plus any psychological or, other relevant knowledge of a general nature he may possess. Probability as understood in contexts of this kind is not frequency. Thus, in our example, the evidence concerning the defendant, which was presented in the trial, may be such that it cannot be ascribed to any other person; and if it could be ascribed to several people, the juror would not know the relative frequency of innocent persons among them. Thus the probability concept used here cannot be the statistical one. While a statement of statistical probability asserts a matter of fact, a statement of inductive probability is of a purely logical nature. If hypothesis and evidence are given, the probability can be determined by logical analysis and mathematical calculation.

One of the basic principles of the theory of inductive probability is the principle of indifference. It says that, if the evidence does not contain anything that would favor either of two or more possible events, in other words, if our knowledge situation is symmetrical with respect to these events, then they have equal probabilities relative to the evidence. For example, if the evidence e_1, available to an observer X_1, contains nothing else about a given die than the information that it is a regular cube, then the symmetry condition is fulfilled and therefore each of the six faces has the same probability 1/6 to appear uppermost at the next throw. This means that it would be unreasonable for X_1, to bet more than one to five on any one face. If X_2 is in possession of the evidence e_2 which, in addition to e_1, contains the knowledge that the die is heavily loaded in favor of one of the faces without specifying which one, the probabilities for X_2 are the same as for X_1. If, on the other hand, X_3, knows e_3 to the effect that the load favors the ace, then the probability of the ace on the basis of e_3 is higher than 1/6. Thus, inductive probability, in contradistinction to statistical probability, cannot be ascribed to a material object by itself, irrespective of an observer. This is obvious in our example; the die is the same for all three observers and hence cannot have different properties for them. Inductive probability characterizes a hypothesis relative to available information; this information may differ from person to person and vary for any person in the course of time.

A brief look at the historical development of the concept of probability will give us a better understanding of the present controversy. The mathematical study of problems of probability began when some mathematicians of the sixteenth and seventeenth centuries were asked by their gambler friends about the odds in various games of chance. They wished to learn about probabilities as a guidance for their betting decisions. In the beginning of its scientific career, the concept of probability appeared in the form of inductive probability. This is clearly reflected in the title of the first major treatise. on probability, written by Jacob Bernoulli and published posthumously in 1713; it was called <u>Ars Conjectandi</u>, the art of conjecture, in other words, the art of judging hypotheses on the basis of evidence. This book may be regarded as marking the beginning of the so-called classical period of the theory of probability. This period culminated in the great systematic work by Laplace, <u>Theorie analytique des probabilites</u> (1812). According to Laplace, the purpose of the theory of probability is to guide our judgments and to protect us from illusions. His explanations show clearly that he is mostly concerned, not with actual frequencies, but with methods for judging the acceptability of assumptions, in other words, with inductive probability.

In the second half of the last century and still more in our century, the application of statistical methods gained more and more ground in science. Thus attention was increasingly focussed on the statistical concept of probability. However, there was no clear awareness of the fact that this development constituted a transition to a fundamentally different meaning of the word 'probability'. In the nineteen twenties the first probability theories based on the frequency interpretation were proposed by men like the statistician R. A. Fisher, the mathematician R. von Mises, and the physicist-philosopher H. Reichenbach. These authors and their followers did not explicitly suggest to abandon that concept of probability which had prevailed since the classical period, and to replace it by a new one. They rather believed that their concept was essentially the same as that of all earlier authors. They merely claimed that they had given a more exact definition for it and had developed more comprehensive theories on this improved foundation. Thus, they interpreted Laplace's word 'probability' not in his inductive sense, but in their own statistical sense. Since there is a strong, though by far not complete analogy between the two concepts, many mathematical theorems hold in both interpretations, but others do not. Therefore these authors could accept many of the classical theorems but had to reject others. In particular, they objected strongly to the principle of indifference. In the frequency interpretation, this principle is indeed absurd. In our earlier example with the observer X_1, who knows merely that the die has the form of a cube, it would be rather incautious for him to assert that the six faces will appear with equal frequency. And if the same assertion were made by X_2, who has information that the die is biased, although he does not know the direction of the bias, he would contradict his own knowledge. In the inductive interpretation, on the other hand, the principle is valid even in the case of X_2, since in this sense it does not predict frequencies but merely says in effect, that it would be arbitrary for X_2 to have more confidence in the appearance of one face than in that of any other face and therefore it would be unreasonable for him to let his betting decisions be guided by such arbitrary expectations. Therefore it seems much more plausible to assume that Laplace meant the principle of indifference in the inductive sense rather than to assume that one of the greatest minds of the eighteenth century in mathematics, theoretical physics, astronomy, and philosophy chose an obvious absurdity as a basic principle.

The great economist John Maynard Keynes made the first attempt in our century to revive the old but almost forgotten inductive concept of probability. In his Treatise on Probability (1921) he made clear that the inductive concept is implicitly used in all our thinking on unknown events both in every-day life and in science. He showed that the classical theory of probability in its application to concrete problems was understandable only if it was interpreted in the inductive sense. However, he modified and restricted the classical theory in several important points. He rejected the principle of indifference in its classical form. And he did not share the view of the classical authors that it should be possible in principle to assign a numerical value to the probability of any hypothesis whatsoever. He believed that this could be done only under very special, rarely fulfilled conditions, as in games of chance where there is a well determined number of possible cases, all of them alike in their basic features, e.g., the six possible results of a throw of a die, the possible distributions of cards among the players, the possible final positions of the ball on a roulette table, and the like. He thought that in all other cases at best only comparative judgments of probability could be made, and even these only for hypotheses which belong, so to speak, to the same dimension. Thus one might come to the result that, on the basis of available knowledge, it is more probable that the next child of a specified couple will be male rather than female; but no comparison could be made between the probability of the birth of a male child and the probability of the stocks of General Electric going up tomorrow.

A much more comprehensive theory of inductive probability was constructed by the geophysicist Harold Jeffreys (Theory of Probability, 1939). He agreed with the classical view that probability can be expressed numerically in all cases. Furthermore, in view of the fact that science replaces statements in qualitative terms (e.g., "the child to be born will be very heavy") more and more by those in terms of measurable quantities ("the weight of the child will be more than eight pounds"), Jeffreys wished to apply probability also to hypotheses of quantitative form. For this reason, he set up an axiom system for probability much stronger than that of Keynes. In spite of Keynes' warning, he accepted the principle of indifference in a form quite similar to the classical one: "If there is no reason to believe one hypothesis rather than another, the probabilities are equal". However, it can easily be seen that the principle in this strong form leads to contradictions. Suppose, for example, that it is known that every ball in an urn is either blue or red or yellow but that nothing is known either of the color of any particular ball or of the numbers of blue, red, or yellow balls in the urn. Let B be the hypothesis that the first ball to be drawn from the urn will be blue, R, that it will be red, and Y, that it will be yellow. Now consider the hypotheses B and non-B. According to the principle of indifference as used by Laplace and again by Jeffreys, since nothing is known concerning B and non-B, these two hypotheses have equal probabilities, i.e., one half. Non-B means that the first ball is not blue, hence either red or yellow. Thus “R or Y” has probability one half. Since nothing is known concerning R and Y, their probabilities are equal and hence must be one fourth each. On the other hand, if we start with the consideration of R and non-R, we obtain the result that the probability of R is one half and that of B one fourth, which is incompatible with the previous result. Thus Jeffreys’ system as it stands is inconsistent. This defect cannot be eliminated by simply omitting the principle of indifference. It plays an essential role in the system; without it, many important results can no longer be derived. In spite of this defect, Jeffreys’ book remains valuable for the new light it throws on many statistical problems by discussing them for the first time in terms of inductive probability.

Both Keynes and Jeffreys discussed also the statistical concept of probability, and both rejected it. They believed that all probability statements could be formulated in terms of inductive probability and that therefore there was no need for any probability concept interpreted in terms of frequency. I think that in this point they went too far. Today an increasing number of those who study both sides of the controversy which has been going on for thirty years, are coming to the conclusion that here, as often before in the history of scientific thinking, both sides are right in their positive theses, but wrong in their polemic remarks about the other side. The statistical concept, for which a very elaborate mathematical theory exists, and which has been fruitfully applied in many fields in science and industry, need not at all be abandoned in order to make room for the inductive concept. Both concepts are needed for science, but they fulfill quite different functions. Statistical probability characterizes an objective situation, e. g., a state of a physical, biological or social system. Therefore it is this concept which is used in statements concerning concrete situations or in laws expressing general regularities of such situations. On the other hand, inductive probability, as I see it, does not occur in scientific statements, concrete or general, but only in judgments about such statements; in particular, in judgments about the strength of support given by one statement, the evidence, to another, the hypothesis, and hence about the acceptability of the latter on the basis of the former. Thus, strictly speaking, inductive probability belongs not to science itself but to the methodology of science, i.e., the analysis of concepts, statements, theories, and methods of science.

The theories of both probability concepts must be further developed. Although a great deal of work has been done on statistical probability, even here some problems of its exact interpretation and its application, e.g., in methods of estimation, are still controversial. On inductive probability, on the other hand, most of the work remains still to be done. Utilizing results of Keynes and Jeffreys and employing the exact tools of modern symbolic logic, I have constructed the fundamental parts of a mathematical theory of inductive probability or inductive logic (Logical Foundations of Probability, 1950). The methods developed make it possible to calculate numerical values of inductive probability ("degree of confirmation") for hypotheses concerning either single events or frequencies of properties and to determine estimates of frequencies in a population on the basis of evidence about a sample of the population. A few steps have been made towards extending the theory to hypotheses involving measurable quantities such as mass, temperature, etc.

It is not possible to outline here the mathematical system itself. But I will explain some of the general problems that had to be solved before the system could be constructed and some of the basic conceptions underlying the construction. One of the fundamental questions to be decided by any theory of induction is, whether to accept a principle of indifference and, if so, in what form. It should be strong enough to allow the derivation of the desired theorems, but at the same time sufficiently restricted to avoid the contradictions resulting from the classical form.

The problem will become clearer if we use a few elementary concepts of inductive logic. They will now be explained with the help of the first two columns of the accompanying diagram. We consider a set of four individuals,

	Statistical Distributions: Number of Blue	Statistical Distributions: Number of White	Individual Distributions		Method I: Initial Probability of Individual Distributions	Method II: Initial Probability of: Statistical Distributions	Method II: Initial Probability of: Individual Distributions
1.	4	0	1	● ● ● ●	1/16	1/5	1/5 = 12/60
2.	3	1	2.	● ● ● ○	1/16	1/5	1/20 = 3/60
			3.	● ● ○ ●	1/16		1/20 = 3/60
			4.	● ○ ● ●	1/16		1/20 = 3/60
			5	○ ● ● ●	1/16		1/20 = 3/60
3	2	2	6	● ● ○ ○	1/16	1/5	1/30 = 2/60
			7.	● ○ ● ○	1/16		1/30 = 2/60
			8.	● ○ ○ ●	1/16		1/30 = 2/60
			9	○ ● ● ○	1/16		1/30 = 2/60
			10	○ ● ○ ●	1/16		1/30 = 2/60
			11.	○ ○ ● ●	1/16		1/30 = 2/60
4.	1	3	12	● ○ ○ ○	1/16	1/5	1/20 = 3/60
			13	○ ● ○ ○	1/16		1/20 = 3/60
			14	○ ○ ● ○	1/16		1/20 = 3/60
			15	○ ○ ○ ●	1/16		1/20 = 3/60
5	0	4	16.	○ ○ ○ ○	1/16	1/5	1/5 = 12/60

say four balls drawn from an urn. The individuals are described with respect to a given division of mutually exclusive properties; in our example, the two properties black (B) and white (W). An individual distribution is specified by ascribing to each individual one property. In our example, there are sixteen individual distributions; they are pictured in the second column (e.g., in the individual distribution No. 3, the first, second, and fourth ball are black, the third is white). A statistical distribution, on the other hand, is characterized by merely stating the number of individuals for each property. In the example, we have five statistical distributions, listed in the first column (e.g., the statistical distribution No. 2 is described by saying that there are three B and one W, without specifying which individuals are B and which W).

By the initial probability of a hypothesis ("probability a priori" in traditional terminology) we understand its probability before any factual knowledge concerning the individuals is available. Now we shall see that, if any initial probabilities which sum up to one are assigned to the individual distributions, all other probability values are thereby fixed. To see how the procedure works, put a slip of paper on the diagram alongside the list of individual distributions and write down opposite each distribution a fraction as its initial probability; the sum of the sixteen fractions must be one, but otherwise you may choose them just as you like. We shall soon consider the question whether some choices might be preferable to others. But for the moment we are only concerned with the fact that any arbitrary choice constitutes one and only one inductive method in the sense that it leads to one and only one system of probability values which contain an initial probability for any hypothesis (concerning the given individuals and the given properties) and a relative probability for any hypothesis with respect to any evidence. The procedure is as follows. For any given statement we can, by perusing the list of individual distributions, determine those in which it holds (e.g., the statement "among the first three balls there is exactly one W" holds in distributions No. 3, 4, 5, 6, 7, 9). Then we assign to it as initial probability the sum of the initial probabilities of the individual distributions in which it holds. Suppose that an evidence statement e (e.g., "The first ball is B, the second W, the third B") and a hypothesis h (e.g., “The fourth ball is B”) are given. We ascertain first the individual distributions in which e holds (in the example, No. 4 and 7), and then those among them in which also h holds (only No. 4). The former ones determine the initial probability of e; the latter ones determine that of e and h together. Since the latter are among the former, the latter initial probability is a part (or the whole) of the former. We now divide the latter initial probability by the former and assign the resulting fraction to h as its relative probability with respect to e. (In our example, let us take the values of the initial probabilities of individual distributions given in the diagram for methods I and II, which will soon be explained. In method I the values for No. 4 and 7—as for all other individual distributions—are 1/16; hence the initial probability of e is 2/16. That of e and h together is the value of No. 4 alone, hence 1/16. Dividing this by 2/16, we obtain 1/2 as the probability of h with respect to e. In method II, we find for No. 4 and 7 in the last column the values 3/60 and 2/60 respectively. Therefore the initial probability of e is here 5/60, that of e and h together 3/60; hence the probability of h with respect to e is 3/5.)

The problem of choosing an inductive method is closely connected with the problem of the principle of indifference. Most authors since the classical period have accepted some form of the principle and have thereby avoided the otherwise unlimited arbitrariness in the choice of a method. On the other hand, practically all authors in our century agree that the principle should be restricted to some well-defined class of hypotheses. But there is no agreement as to the class to be chosen. Many authors advocate either method I or method II, which are exemplified in our diagram. Method I consists in applying the principle of indifference to individual distributions, in other words, in assigning equal initial probabilities to individual distributions. In method II the principle is first applied to the statistical distributions and then, for each statistical distribution, to the corresponding individual distributions. Thus, in our example, equal initial probabilities are assigned in method II to the five statistical distributions, hence 1/5 to each; then this value 1/5 or 12/60 is distributed in equal parts among the corresponding individual distributions, as indicated in the last column.

If we examine more carefully the two ways of using the principle of indifference, we find that either of them leads to contradictions if applied without restriction to all divisions of properties. (The reader can easily check the following results by himself. We consider, as in the diagram, four individuals and a division D_2 into two properties; blue (instead of black) and white. Let h be the statement that all four individuals are white. We consider, on the other hand, a division D_3 into three properties: dark blue, light blue, and white. For division D_2, as used in the diagram, we see that h is an individual distribution (No. 16) and also a statistical distribution (No. 5). The same holds for division D_3. By setting up the complete diagram for the latter division, one finds that there are fifteen statistical distributions, of which h is one, and 81 individual distributions (viz., 3x3x3x3), of which h is also one. Applying method I to division D_2, we found as the initial probability of h 1/16; if we apply it to D_3, we find 1/81; these two results are incompatible. Method II applied to D_2 led to the value 1/5; but applied to D_3 it yields 1/15. Thus this method likewise furnishes incompatible results.) We therefore restrict the use of either method to one division, viz. the one consisting of all properties which can be distinguished in the given universe of discourse (or which we wish to distinguish within a given context of investigation). If modified in this way, either method is consistent. We may still regard the examples in the diagram as representing the modified methods I and II, if we assume that the difference between black and white is the only difference among the given individuals, or the only difference relevant to a certain investigation.

How shall we decide which of the two methods to choose? Each of them is regarded as the reasonable method by prominent scholars. However, in my view, the chief mistake of the earlier authors was their failure to specify explicitly the main characteristic of a reasonable inductive method. It is due to this failure that some of them chose the wrong method. This characteristic is not difficult to find. Inductive thinking is a way of judging hypotheses concerning unknown events. In order to be reasonable, this judging must be guided by our knowledge of observed events. More specifically, other things being equal, a future event is to be regarded as the more probable, the greater the relative frequency of similar events observed so far under similar circumstances. This principle of learning from experience guides, or rather ought to guide, all inductive thinking in everyday affairs and in science. Our confidence that a certain drug will help in a present case of a certain disease is the higher the more frequently it has helped in past cases. We would regard a man's behavior as unreasonable if his expectation of a future event were the higher the less frequently he saw it happen in the past, and also if he formed his expectations for the future without any regard to what he had observed in the past. The principle of learning from experience seems indeed so obvious that it might appear superfluous to emphasize it explicitly. In fact, however, even some authors of high rank have advocated an inductive method that violates the principle.

Let us now examine the methods I and II from the point of view of the principle of learning from experience. In our earlier example we considered the evidence e saying that of the four balls drawn the first was B, the second W, the third B; in other words, that two B and one W were so far observed. According to the principle, the prediction h that the fourth ball will be black should be taken as more probable than its negation, non-h. We found, however, that method I assigns probability 1/2 to h, and therefore likewise 1/2 to non-h. And we see easily that it assigns to h this value 1/2 also on any other evidence concerning the first three balls. Thus method I violates the principle. A man following this method sticks to the initial probability value for a prediction, irrespective of all observations he makes. In spite of this character of method I, it was proposed as the valid method of induction by prominent philosophers, among them Charles Sanders Peirce (in 1883) and Ludwig Wittgenstein (in 1921), and even by Keynes in one chapter of his book, although in other chapters he emphasizes eloquently the necessity of learning from experience.

We saw earlier that method II assigns, on the evidence specified, to h the probability 3/5, hence to non-h 2/5. Thus the principle of learning from experience is satisfied in this case, and it can be shown that the same holds in any other case. (The reader can easily verify, for example, that with respect to the evidence that the first three balls are black, the probability of h is 4/5 and therefore that of non-h 1/5.) Method II in its modified, consistent form, was proposed by the author in 1945. Although it was often emphasized throughout the historical development that induction must be based on experience, nobody as far as I am aware, succeeded in specifying a consistent inductive method satisfying the principle of learning from experience. (The method proposed by Thomas Bayes (1763) and developed by Laplace—sometimes called "Bayes' rule" or "Laplace's rule of succession" —fulfills the principle. It is essentially method II, but in its unrestricted form; therefore it is inconsistent.) I found later that there are infinitely many consistent inductive methods which satisfy the principle (The Continuum of Inductive Methods, 1952). None of them seems to be as simple in its definition as method II, but some of them have other advantages.

Once a consistent and suitable inductive method is developed, it supplies the basis for a general method of estimation, i.e., a method for calculating, on the basis of given evidence, an estimate of an unknown value of any magnitude. Suppose that, on the basis of the evidence, there are n possibilities for the value of a certain magnitude at a given time, e.g., the amount of rain tomorrow, the number of persons coming to a meeting, the price of wheat after the next harvest. Let the possible values be $x_1, x_2, \ldots, x_n$, and their inductive probabilities with respect to the given evidence $p_1, p_2, \ldots, p_n$, respectively. Then we take the product $p_1 x_1$ as the expectation value of the first case at the present moment. Thus, if the occurrence of the first case is certain and hence $p_1=1$, its expectation value is the full value x_1; if it is just as probable that it will occur as that it will not, and hence $p_1=1/2$, its expectation value is half its full value ($p_1x_1=x_1/2$), etc. We proceed similarly with the other possible values.As estimate or total expectation value of the magnitude on the given evidence we take the sum of the expectation values for the possible cases, that is, $p_1x_1+ p_2 x_2 + \ldots + p_nx_n$. (For example, suppose someone considers buying a ticket for a lottery and, on the basis of his knowledge of the lottery procedure, there is a probability of 0.01 that the ticket will win the first prize of $200 and a probability of 0.03 that it will win $50; since there are no other prizes, the probability that it will win nothing is 0.96. Hence the estimate of the gain in dollars is 0. 01x200 + 0.03x50 + 0. 96x0 = 3.50. This is the value of the ticket for him and it would be irrational for him to pay more for it.) The same method may be used in order to make a rational decision in a situation where one among various possible actions is to be chosen. For example, a man considers several possible ways for investing a certain amount of money. Then he can—in principle, at least—calculate the estimate of his gain for each possible way. To act rationally, he should then choose that way for which the estimated gain is highest.

Bernoulli and Laplace and many of their followers envisaged the idea of a theory of inductive probability which, when fully developed, would supply the means for evaluating the acceptability of hypothetical assumptions in any field of theoretical research and at the same time methods for determining a rational decision in the affairs of practical life. In the more sober cultural atmosphere of the late nineteenth century and still more in the first half of the twentieth, this idea was usually regarded as a utopian dream. It is certainly true that those audacious thinkers were, not as near to their aim as they believed. But a few men dare to think today that the pioneers were not mere dreamers and that it will be possible in the future to make far-reaching progress in essentially that direction in which they saw their vision.

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INDUCTIVE LOGIC AND SCIENCE

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The question of the usefulness of inductive logic for science and even the question of the very possibility of inductive logic are today still debated. The conception here explained differs from that of many other scholars.

First I wish to emphasize that inductive logic does not propose *new* ways of thinking, but merely to explicate *old* ways. It tries to make explicit certain forms of reasoning which implicitly or instinctively have always been applied both in everyday life and in science. This is analogous to the situation at the beginning of deductive logic. Aristotle did not invent deductive reasoning; that had gone on as long ago as there was human language. If somebody had said to Aristotle: "What good is your new theory to us? We have done well enough without it. Why should we change our ways of thinking and accept your new invention?", he might have answered: "I do not propose new ways of thinking, I merely want to help you to do consciously and hence with greater clarity and safety from pitfalls what you have always done. I merely want to replace common sense by exact rules."

It is the same with inductive logic. Inductive reasoning is likewise as old as human language. I mean there by inductive reasoning all forms of reasoning or inference where the conclusion goes beyond the content of the premises, and therefore cannot be stated with certainty.

Thus, for example, if a physicist states a new law or a theory as a system of laws on the basis of the experimental results he has found, he makes an inductive inference. So does a scientist who assumes an unknown single fact on the basis of known facts; for example, the meteorologist who predicts the weather for tomorrow, the physicist who assumes a certain distribution of the velocities of gas molecules which he cannot directly observe, a historian who tries to explain a reported act of Abraham Lincoln by hypothetically assuming a certain

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motivation which is not reported, or a statistician who makes an estimate for the unknown value of a parameter in a population on the basis of an observed sample from the population. Since inductive logic merely intends to explicate common ways of inductive reasoning, the question of its usefulness leads back to the general question: Is it desirable that procedures which are generally applied, though only intuitively or instinctively, are brought into the clear daylight, analyzed and systematized in the form of exact rules? Whoever gives an affirmative answer to this general question will acknowledge the importance of the special problem of explicating inductive reasoning, that is, of constructing a system of inductive logic with rules as exact as those of the older, well-established system of deductive logic. Whether any particular system proposed as a solution for this problem is workable and fruitful is, of course, another question.

As I see it, the fundamental concept of inductive logic is *probability*. All inductive reasoning is probability reasoning. However, the word ‘probability’ is not unambiguous. I refer here to probability in one particular sense, the logical sense, which we might call inductive probability. This concept must be clearly distinguished from probability in the statistical sense. The distinction is practically important and theoretically fundamental.

Statistical probability is a certain quantitative physical characteristic of physical systems. Like any other physical magnitude it is to be established empirically, by observations. In this case the observations are of a statistical nature. They consist in counting frequencies. Statistical probability is obviously very closely connected with frequency, but it is not just the same as frequency. When we say that for a given die the probability of throwing an ace is 0.158, then this statement refers to a physical characteristic of the die and thus is not fundamentally different from statements about its mass, temperature, electric conductivity, etc. Imagine a fictitious physicist who, like the Laplacean superman, knows the present microstate of the die in terms of the distribution of the particles and of the fields and, in addition, all the relevant laws. This physicist could, by purely mathematical calculations, find not only the present temperature of the die, its conductivity, etc., but also the probability of its yielding an ace if thrown under specified conditions. Since the micro-state is actually not known, the question arises how to test a statement about the probability, how to confirm or disconfirm it. The answer in the case of probability is not fundamentally different from that in the case of temperature or other physical magnitudes. The statement is to be tested by making experimental arrangements which lead to observable phenomena connected with the magnitude in question, whose value itself is not directly observable. To test the probability statement, we determine the relative frequency of aces in a sufficiently long series of throws of the die. This frequency is itself not the probability; it is rather a consequence of the probability state of the die, a consequence which is observable and therefore may serve for its as a symptom for the probability state, just as the expansion of the mercury column in the thermometer is not itself the temperature but an observable consequence of the temperature state and therefore a suitable means of testing a statement about the temperature. It is sometimes said that the statistical concept of probability involves a peculiar difficulty, since obviously no finite series of throws is sufficient to determine the probability with absolute precision and certainty. This is indeed true, but the same holds for all physical magnitudes. There is likewise no possible procedure for determining the temperature with absolute precision and certainty. The answer to the question: "How long then shall we make the series of throws with the die in order to determine the probability?" is the same as the answer to the question: "How fine a thermometer should we use to measure the temperature?" In both cases the answer depends, on the one hand, on the time and money available and, on the other hand, on the desired degree of precision. More specifically, it depends on the theoretical or practical advantages to be expected from higher precision. The finer the thermometer and the longer the series of throws, the higher the precision which is achieved. In neither case is there a perfect procedure. The concept of statistical probability may be introduced either by an explicit definition in terms of a limit as done by Mises and Reichenbach, or by an axiom system with rules of application as done by the majority of contemporary statisticians. In either case, the concept is logically legitimate and practically useful for work in statistics and in all branches of science which apply statistical methods. Thus I do not agree with those representatives of the inductive concept of probability, like Keynes and Jeffreys, who reject the statistical concept. On the other hand, I do not agree with Mises, Reichenbach, and the statisticians, who reject the inductive concept. Both concepts of probability are important for scientific work, each in its own field, the one within science itself, the other in inductive logic, which gives rules for certain operations with the statements of the language of science. The statistical concept is today generally recognized. Although certain problems connected with it are still under investigation, a defense of its legitimacy and usefulness is no longer necessary. The status of the inductive concept, however, is still debated. Therefore, today it is still necessary to defend its right of existence and to show its usefulness.

A statement of inductive probability states a relation between a hypothesis and a given body of evidence, e.g., results of actual or possible observations. The asserted probability value means the degree to which the hypothesis is confirmed or supported by the evidence. It is important to notice that a statement on inductive probability or *degree of confirmation* is relative to the evidence. This does not merely mean that the statement is based on or derived from observations. That is the case for every scientific statement. For example, the statement "The probability that it will rain tomorrow is 1/5" is incomplete unless we add "with respect to such and such an evidence", e.g., certain meteorological observations. For the validity of the statement it does not matter whether the evidence referred to in the statement is true and whether it is known to the speaker. To be sure, in the practical application of any inference, whether deductive or inductive, the premises are usually known. But that is not necessary. They may be unknown or they may even be known to be false. If this is so, what can be the basis of the validity of the probability statement itself, as distinguished from the validity of the hypothesis or the evidence? It can obviously not be of an empirical nature. All relevant empirical knowledge or assumption is contained in the evidence statement. In our example, we can empirically reexamine the truth of the evidence concerning past meteorological events. If we wait until tomorrow, we can empirically test the truth of the hypothesis that it will rain tomorrow. But in neither way can we test the truth of the probability statement itself. We shall see tomorrow either rain or not-rain, we may observe a rain of short or of long duration, a rain of high or low intensity, but we shall not see a rain of probability 1/5. Some critics of inductive logic have pointed to this fact and drawn from it the conclusion that, since the inductive probability statement is not empirically testable, it must be scientifically meaningless. Their mistake was that they regarded the probability statement as a factual synthetic statement. The statement is, however, of a purely logical nature. Hence there is no need and no possibility of empirical testing. A statement of inductive probability is in one respect similar to a statement in deductive logic: the relation between the hypothesis and the evidence which it asserts is a logical relation, similar to the deductive relations of deducibility or incompatibility, though weaker than those. If the statement asserts a probability value close to 1, then the probability relation hereby expressed is very close to the relation of deducibility: the hypothesis is nearly deducible from the evidence but not quite. On the other hand, if the stated probability value is near to 0, then the probability relation is close to the deductive relation of incompatibility: the hypothesis is nearly incompatible with the evidence but not quite. For any intermediate probability value the probability relation is more remote from the deductive relations which are, so to speak, the extreme cases. Thus inductive probability means in a sense partial deducibility. It is a logical relation inasmuch as it can be established, just as a deductive relation, as soon as the two statements of hypothesis and evidence are given, by merely applying logical analysis, in this case the rules of inductive logic, without the use of observations. Although the statement expresses only a logical relation, it has nevertheless significance. It draws boundaries to reasonable conduct. For example, if the probability of rain tomorrow is 1/5 with respect to the evidence available to an observer, then it would not be reasonable for him to bet on rain tomorrow at odds higher than 1:4.

If we recognize that statements on inductive probability have a purely logical character, then we are in a position to clear up a question which has been debated for two hundred years the problem of the so-called principle of insufficient reason or *principle of indifference*. As I see it, the beginning of the development of inductive logic was made in the classical theory of probability by men like Bernoulli, Bayes, and Laplace. Many points of the classical theory, and among them also fundamental points, have been criticized for more than a hundred years and especially in our century. I think that this criticism is correct to a large extent. I agree with the critics that today it is impossible to go back to the classical conception. But I do not agree with those who say that the only way out is the total rejection of the classical conception. The classical principle of indifference states: “If no reasons are known which would favor one of several possible events, then the events are to be taken as equally probable." The usual objection against this principle is that it puts a premium on ignorance; if you do not know anything about the alternatives, then the principle allows you to make a certain statement about them; if you know certain things, then that statement is no longer permissible. To derive a statement from ignorance looks rather absurd. And it would indeed be absurd to apply this procedure to a factual statement. But the statement of equiprobability to which the principle of indifference leads is, like all statements on inductive probability, not a factual but a logical statement. If the knowledge of the observer does not favor any of the possible events, then with respect to this knowledge as evidence they are equally probable. The statement assigning equal probabilities in this case does not assert anything about the facts, but merely something about the logical relations between the given evidence and each of the hypotheses; namely, that these relations are logically alike. These relations are obviously alike if the evidence has a symmetrical structure with respect to the possible events. The statement of equiprobability asserts nothing more than the symmetry.

For these reasons I believe that the basic idea of the old principle of indifference is valid. On the other hand, there can be no doubt that many of the applications of this principle, especially in the earlier period, were invalid and that some of the conclusions drawn were even outright absurd. But I believe that the aim which the classical pioneers envisaged was valid. Our task is not to abandon entirely the classical conception, but to construct an exact concept of degree of confirmation explicating the classical conception of inductive probability on a more cautious and more solid foundation.

Now let us look at the function of inductive logic in the field of science. A scientist makes, on the one hand, observations of natural phenomena or of results of experiments. These constitute his evidence. On the other hand, he entertains hypotheses concerning facts not yet observed or even unobservable. The hypothesis may concern a single fact or it may be a conditional prediction of the form "If we were to do such and such things, then such and such events would happen", or it may have a general form, e.g., a statement about the value of a material constant, or a general law stating the relations between various physical magnitudes in terms of mathematical functions. The law may have a deterministic form or a statistical form, stating e.g., proportions, averages, or other statistical parameters of distributions of certain magnitudes. The purpose of the law is to explain known phenomena and predict new ones. The task of inductive logic is not to *find* a law for the explanation of given phenomena. This task cannot be solved by any mechanical procedure or by fixed rules; it is rather solved through the intuition, the inspiration, and the good luck of the scientist. The function of inductive logic begins *after* a hypothesis is offered for examination. Its task is to measure the support which the given evidence supplies for the tentatively assumed hypothesis. In particular, the task will often be to determine among several competing hypotheses the one which is most strongly confirmed by the given evidence. The competing hypotheses may, for example, concern the possible results of an experiment to be made, the possible causes of an observed event, or possible outcomes of a business investment. Or they may be various laws which are mutually incompatible, each of which might be regarded as an explanation of a given set of observational results concerning new phenomena not explained so far.

Sometimes an objection is raised against the idea of a system of inductive logic with exact rules for the determination of the degree of confirmation because of the fact that a scientist who chooses one among a number of considered hypotheses is influenced in this choice also by many non-rational factors and that he would not be willing to hand over the task of this choice to a machine or to have himself, so-to-speak, transformed into a machine which merely applies fixed rules. Now it is true that many non-rational factors affect the scientist's choice, and I believe that this will always be the case. The influence of some of these factors may be undesirable, for instance a bias in favor of a hypothesis previously maintained publicly or, in the case of a hypothesis in social science, a bias caused by moral or political preferences. But there are also non-rational factors whose effect is important and fruitful; for example, the influence of the "scientific instinct or hunch". Inductive logic does not intend to eliminate factors of *this* kind. Its function is merely to give to the scientist a clearer picture of the situation by demonstrating to what degree the various hypotheses considered are confirmed by the evidence. This logical picture supplied by inductive logic will (or should) influence the scientist, but it does not uniquely determine his decision of the choice of a hypothesis. He will be helped in this decision in the same way a tourist is helped by a good map. If he uses inductive logic, the decision still remains his; it will, however, be an enlightened decision rather than a more or less blind one. In addition to judging the status of hypotheses, inductive logic has also the task of supplying rules of *estimation*. There is much discussion and controversy among statisticians concerning the validity of particular methods of estimation and the choice of a suitable method of estimation in a given problem situation. I believe that, if the basis of inductive logic is constructed by laying down rules for calculating the degree of confirmation, then it is possible to define a general estimate function in terms of degree of confirmation, applicable to all kinds of magnitudes expressible in the language in question. The definition which I propose takes as the estimate of the magnitude on the basis of a given body of evidence the weighted mean of the possible values of the magnitude, the weight of each value being its degree of confirmation with respect to the given evidence. This is the same as the expectation value of the magnitude (if we understand the term 'expectation value' in the inductive sense based on inductive probability, in contrast to its statistical sense based on statistical probability). To obtain a general method of estimation would be of great importance not only from a *theoretical* point of view but also for the problem of determining *practical* decisions in a rational way. Suppose a man has to make a decision in a given economic situation, e.g., concerning investments. This means that he has to choose one among a number of alternative actions possible to him in the situation. For each possible action he considers the various possible outcomes in terms of money gained. If he is able to determine the degree of confirmation for each possible outcome in the case of the considered action, he may calculate the sum of these gains, each multiplied with its degree of confirmation. This will be his estimate of the gain in the case of the action considered. In the same way, he may calculate the estimate of the gain for each of the possible actions. Then, if he is a rational man, he will choose that one among the possible actions for which the estimated gain has its greatest value. (A more exact procedure would consider, not the gain in terms of money, but the utility of this gain, i.e., the measure of satisfaction derived by the person from the gain.) Thus inductive logic serves as an instrument for the determination of rational decisions.

As mentioned earlier, the development of inductive logic began with the classical theory of probability. However, its systematization as a branch of modern logic is of recent origin, beginning with John Maynard Keynes thirty years ago. I have constructed a set of rules of inductive logic for a simple language system, which is restricted to qualitative descriptions of things without the use of measurable magnitudes (like temperature, electric current, etc.), but including statements of frequencies. These rules make possible the calculation of the degree of confirmation for any hypothesis and any body of evidence expressible in that language system and the calculation of the estimate of a frequency on the basis of any given evidence. The further development of inductive logic for more comprehensive language systems and finally for the language of science as a whole remains a task for the future.

A theory of inductive logic is systematically developed in my book *Logical Foundations of Probability*, Chicago, 1950. The underlying conception of inductive probability is explained in *The Nature and Application of Inductive Logic*, Chicago, 1951, which is a reprint of six non-technical sections from the book mentioned.