ON THE COMPARATIVE CONCEPT OF CONFIRMATION*

RUDOLF CARNAP

1 *The Problem of an Explication of the Comparative Concept*

THE problem consists in finding an explicit definition for a relation MC such that

(1) MC (h, e, h', e')

may be interpreted as follows:

(2) The hypothesis h is confirmed by the evidence $e$equally strongly or more strongly than h' by e'.

This condition (2) can be expressed in terms of degree of confirmation c as follows:

(3) $c(h, e) \geq c(h', e')$.

However, we do not take (3) as a definition for (1), because our aim is to find an explication in a non-quantitative form, i.e. not involving such quantitative concepts as c-functions.
On the basis of MC we define Gr (meaning roughly : 'Greater confirmation') as follows:

(4) $\mathrm{Gr}(h, e, h', e') =_{\mathrm{Df}} \mathrm{MC}(h, e, h', e')$ and not $\mathrm{MC}(h', e', h, e)$.

Then the comparative condition

(5) $\mathrm{Gr}(h, e, h', e')$

corresponds to the following quantitative condition

(6) $c(h, e) > c(h', e')$.

I have constructed[1] a definition of MC as a tentative explication for (2) and proved[2] that (in any finite system) the condition (1) holds if and only if

(7)for every regular c, $c(h, e) \geq c(h', e')$.

Now Yehoshua Bar-Hille1[3] has shown that my definition is too narrow.

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[1] R. Carnap, *Logical Foundations of Probability,* Chicago, 1950, henceforth referred to by '[Prob.]'. The definition is D81-1, p. 436.

[2] L. c., T81-1, p. 440

[3] Y. Bar-Hillel, 'A note on comparative inductive logic', this volume, pp. 308-10.

Let us consider for a moment the classificatory concept of confirmation:

(8) The additional information i constitutes confirming evidence for the hypothesis h on the basis of the prior evidence e, or, more specifically:

(9) The hypothesis h is confirmed by e and i together more strongly than by e alone.

This clearly corresponds to the quantitative condition:

(10) $c(h, e \cdot i) > c(h, e)$.

In my book I discussed a possible explicatum C′ for (8) and (9), defined in a non-quantitative form. I showed[1] that C′ (h, i,e) if and only if

(11) for every regular c, $c(h, e \cdot i) > c(h, e)$.

I did not adopt C′ because it is too narrow. In particular, I mentioned the following two counter-examples in which (9) holds, but C′ does not hold

(12) h is a law '(x) $(Mx \supset M'x)$' (e.g. 'all swans are white') in a finite domain of N individuals ($N > 1$) ; e is tautological ; i is '$Mb \cdot M'b$' ('b is a swan and b is white').

(13) e is '$Pa_1 \cdot \sim Pa_2$', where 'P' is a primitive predicate ; i is '$Pa_3 \cdot Pa_4 \cdot \ldots \cdot Pa_{12}$' ; h is 'Pa_{13}'.

I indicated (1. c., p. 467) that the defect of C′ is due to its being defined in such a way that it is in accord with all regular c-functions (in the sense of (11)). Now Bar-Hillel points out, correctly, that my definition of MC is too narrow in just the same respect as C′. In the two examples (12) and (13), the relation (9) seems intuitively to hold. Therefore we should require of an adequate explicatum MC that

(14) MC (h, $e \cdot i$, h, e).

However, on the basis of my definition, this does not hold. There are regular c-functions for which neither (10) nor the weaker condition

(15) $c(h, e \cdot i) \geq c(h, e)$

holds in the cases (12) and (13). It is due to this fact that neither C′ nor MC covers the two examples, because C′ is in accord with all regular c-functions in the sense of (11), and likewise MC in the sense of (7).

Thus the problem of finding an adequate explicatum in non-quantitative terms for the comparative concept of confirmation is at

[1] L. c., p. 465

present unsolved. I shall not try here to construct an explicit definition of which would constitute an adequate explication ; but I shall indicate the direction in which a solution might be found.

2 *Requirements for c-Functions*

Let us leave aside for a moment the comparative concept and discuss *c*-functions. There are many relations among values of *c* which, although intuitively valid, do not hold for all regular *c*-functions, e.g. the inequality (10) in cases like (12) and (13). In order to make such relations provable, additional assumptions concerning *c* must be made. I shall list here six requirements R1-R6, which seem to me indispensable for adequacy. That is to say that any *c*-function which does not fulfil all of these requirements seems to me unacceptable as an explicatum for degree of confirmation, or, in other words, as a valid basis for inductive reasoning. On the other hand, these requirements do not constitute a sufficient condition of adequacy. There is an infinite number of *c*-functions which fulfil all of these requirements but nevertheless would generally be regarded as inadequate.

Every *c*-function is assumed to be based on an m-function (in the sense that $c(h, e)$ is defined as $\mathrm{m}(e \cdot h)/\mathrm{m}(e)$). The requirements may therefore be formulated in terms of m-functions. It is here assumed that all primitive predicates designate properties, not relations, and that the number of individuals is finite.

R1. m is *regular* (i.e. the m-values for the state-descriptions are positive and their sum is 1, see [Prob.] § 55A).

R2. m is *symmetrical with respect to individual constants.* If *i'* is formed from the state-description *i* by exchanging two individual constants (i.e. by replacing each occurrence of the one constant in *i* by the other constant and vice versa), then *i* and *i'* have equal m-values.

[If this condition is fulfilled, then any two isomorphic state-descriptions have equal m-values.] The requirement R2 means in effect that all individuals are treated on a par.

R3. For any sentence *i* without variables, m(*i*) has the same value in all language-systems in which *i* occurs, irrespective of the number of individuals in the system. (In technical terms, the functions ${}_N\mathrm{m}$ for $N = 1, 2$, etc. form a fitting sequence, see [Prob.] § 57C.) R4. If one state-description is formed from another by exchanging two primitive predicates (i.e. by replacing each occurrence of the one predicate by the other and vice versa), then both state-descriptions have the same m-value.

This means that all primitive properties are treated on a par.

R5. If one state-description is formed from another by exchanging one primitive predicate and its negation (i.e. by adding a sign of negation to each component containing this predicate and then deleting double negation signs), then both state-descriptions have the same m-value.

This means that every primitive property is treated on a par with its contradictory.

R6. *Requirement of instantial relevance*. Let '*M*' be a factual, molecular predicate. Let e be a non-L-false sentence without variables. Let i and h be full sentences of '*M*' with two distinct individual constants which do not occur in e. Let k_o be $e \cdot \sim i \cdot \sim h$, k_1 be $e \cdot i \cdot \sim h$, and k_2 be $e \cdot i \cdot h$. Then $\mathrm{m}(k_0)/\mathrm{m}(k_1) > \mathrm{m}(k_1)/\mathrm{m}(k_2)$

This requirement will hardly appear immediately plausible. However, it leads to the following theorem T1a (from which its name is derived) ; and this theorem seems plausible. (T1a is, moreover, equivalent to R6 on the basis of R1 and R2 ; this can easily be shown by reversing the subsequent proof.)

T1. *Theorem of instantial relevance*. Let c be based on m, and m fulfil R1, R2, and R6. Let '*M*', e, i, and h be as in R6.

a. $c(h, e \cdot i) > c(h, e)$; in other words, i is positively relevant to h on e.
b. $c(h, e \cdot \sim i) < c(h, e)$; $\sim i$ is negatively relevant to h on e.
c. $c(h, e \cdot i) > c(h, e \cdot \sim i)$.

Proof. a. Let k_o, k_1, and k_2 be as in R6; let k_1' be $e \cdot \sim i \cdot h$. Then, by R2, $\mathrm{m}(k_1) = \mathrm{m}(k_1')$. We put $m_o = \mathrm{m}(k_0)$, $rn_1 = \mathrm{m}(k_1) = \mathrm{m}(k_1')$, and $m_2 = \mathrm{m}(k_2)$. These three values are positive (R1). According to R6,

$$m_0/m_1 > m_1/m_2.$$

Hence

$$m_o m_2 > m_1^2 \ ; \quad m_o m_2 + 2m_1 m_2 + m_2^2 > m_1^2 + 2m_1 m_2 + m_2^2 \ ;$$
$$m_2\,(m_0 + 2m_1 + m_2) > (m_1 + m_2)^2 \ ;$$
$$m_2/\,(m_1 + m_2) > (m_1 + m_2)/\,(m_0 + 2m_1 + m_2)$$

Now

$$c(h, e \cdot i) = \mathrm{m}(e \cdot i \cdot h)/\mathrm{m}(e \cdot i) = m_2/(m_1 + m_2).$$

And

$$c(h, e) = \mathrm{m}(e \cdot h)/\mathrm{m}(e) = (m_1 + m_2)/(m_0 + 2m_1 + m_2)$$

(see [Prob.] T65-1). Hence the assertion (a).—*b*. From (a), according to [Prob.] T65-6e.—*c*. From (a) and (b).

T1a says in effect that one instance of a property is positively relevant to the prediction of another instance of the same property. This seems a basic feature of all inductive reasoning concerning the prediction of a future event. We can easily derive from T1a the result that a conjunction of one or more instances of a property is positively relevant to a conjunction of one or more other instances. Thus; the second counter-example (13) is covered by any *c* which fulfils our requirements.

The following is a corollary of T1c : If e_M describes a sample of *s* individuals, of which s_M are *M* and the remaining $s - s_M$ are non-*M*, then $c(h, e_M)$ increases with increasing s_M. This means that the inductive probability of a future instance of *M* is the higher, the greater the relative frequency of *M* among past observations. T1c is equivalent to T1a (on the basis of R1 and R2) ; hence it could likewise be taken as a requirement instead of R6.

3 *Axioms for the Comparative Concept*

Various axiom systems for the comparative concept of confirmation or inductive probability have been constructed. Those by Keynes[1] and Jeffreys[2] form the basic part of a quantitative system. The most elaborate and strongest comparative system so far constructed is that by B. O. Koopman.[3] But even the latter system covers only those cases in which the quantitative condition (3) is fulfilled for all regular *c*-functions (see [Prob.] p. 342 and § 83B) ; thus, e.g. it does not cover the two examples (12) and (13) and the cases referred to in T1. (This is easily seen from the fact that Koopman's axioms refer, not to the internal structure of the sentences involved, but only to deductive relations among them; for T1, on the other hand, it is essential that the sentences *i* and *h*, which are deductively independent, are full sentences of the same predicate.) For Koopman this restriction is not accidental ; it is in line with his basic conception. According to this conception, 'it is the unique function of

[1] John Maynard Keynes, *A Treatise on Probability,* London and New York; 1921 (see my comments, [Prob.] pp. 338f.)

[2] Harold Jeffreys, *Theory of Probability,* Oxford, 1939 (see [Prob.] pp. 340-342.)

[3] B. O. Koopman, 'The axioms and algebra of intuitive probability', *Annals of Math.,* Ser. 2, 1940, **41**, 269-292 ; 'The bases of probability', *Bull. Amer. Math. Soc.,* 1940, **46**, 763-774

such a theory [a rational mathematical theory of probability] to develop the rules for the derivation of comparisons in probability from other comparisons in probability previously given'.[1] In distinction to this conception, I would regard it as the task of a comparative axiom system to supply also direct comparative statements in addition to merely conditional statements.

If we wish to construct an axiom system which is not limited in this way, then we have to add axioms which are not in accord with all regular *c*-functions. I would suggest to add for this purpose to an axiom system of the weak kind, e.g. the one by Koopman, the subsequent axioms A1-A4. They correspond to our earlier requirements R2, R4, R5, and T1a (as an equivalent to R6) for *c*-functions. (We omit R3, because the present axiom system is meant to apply to one language-system only.) We use 'MC' as the primitive term. For the sake of convenience we make use of the definitions (4) and (16):

(16) $\mathrm{Cq}(h, e, h', e') =_{\mathrm{Df}} \mathrm{MC}\,(h, e, h', e')$ and $\mathrm{MC}\,(h', e', h, e)$.

In the intended interpretation, the comparative concept hereby defined corresponds to the quantitative condition

(17) $c(h, e) = c(h', e')$.

A1. *Symmetry with respect to individual constants.* Let *e* be not L-false. Let *h′* and *e′* be formed from *h* and *e,* respectively, by exchanging two individual constants. Then $\mathrm{Cq}(h, e, h', e')$.

A2. *Symmetry with respect to primitive predicates.* Let *e* be not L-false. Let *h′* and *e′* be formed from *h* and *e,* respectively, by exchanging two primitive predicates. Then $\mathrm{Cq}(h, e, h', e')$.

A3. *Symmetry with respect to a primitive predicate and its negation.* Let *e* be not L-false. Let *h′* and *e′* be formed from *h* and *e,* respectively, by exchanging one primitive predicate and its negation. Then $\mathrm{Cq}(h, e, h', e')$.

A4. *Instantial relevance.* Let '*M*' be a factual, molecular predicate. Let *e* be a non-L-false sentence without variables. Let *i* and *h* be full sentences of '*M*' with two distinct individual constants which do not occur in *e.* Then $\mathrm{Cq}(h, e, h', e')$.

The following theorem is based on A4 together with customary comparative axioms.

[1] L. c. (the first paper), p. 271

T2. Let. ‘*M*’, *e, i*, and *h* be as in A4.

a. Lemma. MC(h, $e \cdot i$, h, e). (From A4 and definition (4).)
b. Lemma. Not MC(h, e, h, $e \cdot i$). (From A4 and (4).)
c. Lemma. MC($\sim h$, $e \cdot \sim i$, $\sim h$, e). (From (a), with ‘$\sim M$’ for ‘*M*’.)
d. Lemma. Not MC($\sim h$, e, $\sim h$, $e \cdot \sim i$). (From (b), likewise.)
e. MC(h, e, h, $e \cdot \sim i$). (From (c) and the axiom of negated hypotheses (Koopman’s axiom A, Prob.] T83-33).)
f. Not MC(h, $e \cdot \sim i$, h, e). (From (d), likewise.)
g. MC (h, $e \cdot i$, h, $e \cdot \sim i$). (From (a), (e), and the axiom of transitivity (Koopman’s axiom T, Prob.] T83-22).)
h. Not MC(h, $e \cdot \sim i$, h, $e \cdot i$). [Indirect proof. Assume that MC(h, $e \cdot \sim i$, h, $e \cdot i$). Then with (a) and the axiom of transitivity : MC(h, $e \cdot \sim i$, h, e), which is impossible (*f*).]

T2(e) and (f) say that Gr(h, e, h, $e \cdot \sim i$), which corresponds to T1b : the negation of one instance of *M* is negatively relevant to another instance. The theorems (g) and (h) say that Gr(h, $e \cdot i$, h, $e \cdot \sim i$), which corresponds to T1c : a future instance of *M* is the more probable, the greater the observed relative frequency in the past.

4 *The Problem of an Explicit Definition for the Comparative Concept*

The definition of the comparative concept MC given in my book was too narrow, as we have seen, because it was constructed in such a way that the defined relation was in accord with all regular *c*-functions. The defined concept was ‘proposed merely as a tentative explicatum’ (p. 453). And I indicated what ought to be done in case the defined concept were found to be too narrow : ‘It would be of interest to investigate the possibility . . . of an explicatum which is wider than MC [as then defined] because based on a narrower class of *c*-functions’ (p. 467). Thus the task would be to choose a suitable narrower class *C* of *c*-functions and then try to find a definition of which, although it makes no reference to *c*-functions ‘or other numerical functions, is in accord with all *c*-functions in *C* and, moreover, is the most extensive concept in accord with all *c*-functions in *C*. In other words, MC would be required to fulfil the following condition:

(18) For any sentences h, e, h', e', if, for every function c in C, MC (h, e, h', e') if and only $\bar{c}(h, e) \geq c(h', e')$.

It seems to me that a suitable choice for *C* might be the class of those *c*-functions which fulfil the requirements R1-R6. In connection with the above quotation, I suggested for the explication of both the classificatory and the comparative concepts to add to the requirement of regularity (R1) at least those of symmetry with respect to individual constants (R2) and of symmetry with respect to basic matrices (R4 and R5). Now the discussions in this paper have shown that it is essential to add also the requirement of instantial relevance (R6), in order to make sure that the explicata cover the important class of cases of the predictive inference. To the passage quoted above I added : 'However, . . . it seems doubtful whether a simple definition in L-terms can be found'. The discussions in this article will make these doubts even stronger. At any rate, it seems clear, in view of the six requirements, that the definition cannot have a simple form. It will have to refer, in addition to deductive relations (L-concepts) between the sentences involved, also to the internal structure of the sentences, as the above requirements do. Whether it is at all possible to find a definition which is non-quantitative, i.e. does not contain numerical variables, must remain an open question at the present moment. I am in this respect not quite as pessimistic as Bar-Hillel. If we permit the use of variables for expressions and maybe for classes of expressions (though not for numbers), then I think the task may be possible although not easy. To find a solution is chiefly of interest to those who need a comparative concept because they believe that it is impossible to find an adequate explicatum for the quantitative concept of degree of confirmation.

Department of Philosophy
University of Chicago