BARKLEY ROSSER. *Gödel theorems for non-constructive logics.* Ibid., pp. 129-137. See *Errata* ibid., p. iv.

Gödel's theorem concerning the existence of undecidable sentences refers to systems which contain the system of ***Principia*** and in addition any other constructive rules. Rosser obtains in this paper some important analogous results with respect to systems which moreover contain non-constructive rules (referring to an infinite number of premisses). Let R be the most elementary of those rules (called Carnap's rule by the author): *If* '$f(0)$', '$f(1)$', '$f(2)$', ... *are all provable, then* '$(x)f(x)$' *shall be provable.* Let $P_\alpha$ (where $\alpha$ is any ordinal) be the system (i.e., the class of provable formulas) which contains Gödel's system $P_0$ (i.e., ***Principia*** plus Peano's axioms) and allows $\alpha$ uses of R (by $P_\omega$ is understood the logical sum of all $P_n$ for finite $n$). Let $\mathrm{Pv}_\alpha(x)$ be the formula which, if interpreted in arithmetized syntax, says that the formula whose number is $x$ is provable in $P_\alpha$. By extending the reasoning of Gödel, the author comes to the following results.

If $P_\alpha$, where $\alpha<\omega^2$, is simply consistent (in Rosser's sense), then: A. The formula stating the simple consistency of $P_\alpha$ cannot be proved in $P_\alpha$ (but can in $P_{\alpha+1}$). B. There are undecidable propositions in $P_\alpha$. C. $P_\alpha$ is not closed under Rule R.

$P_\Omega$ is closed under Rule R. Result A holds also for $P_\Omega$. In analogy to Gödel's concept '$\omega$-consistent' the author defines: A logic $L$ is called $\Omega$-consistent if, for each $x$, $\mathrm{Pv}_\Omega(x)$ is not provable in $L$ unless the formula whose number is $x$ is provable in $P_\Omega$. Then he shows: If $P_\Omega$ is simply consistent and $\Omega$-consistent, there are undecidable propositions in $P_\Omega$.

Finally the author studies rules of an interesting kind suggested by Kleene. While Rule R demands for '$(x)f(x)$' an infinite class of premisses, viz. '$f(0)$', '$f(1)$', etc., Rule K$\alpha$ demands only one premiss and is thus constructive; this premiss is the formula which says that every formula of the infinite class mentioned is provable in a certain sub-system (consisting of $P_0$ plus Rules K$\beta$ for every $\beta$ less than $\alpha$). These rules are related to the non-constructive rule R though somewhat weaker than it, but are stronger than the constructive rules of customary kinds.

The reviewer wishes to add a remark about a consequence of Rosser's results for his definition of 'consequence in Language I' (***Logical syntax,*** §14). Language I (as in §§11, 12) is a constructive sub-system of $P_0$; the non-constructive rule DC2 corresponds to Rule R; thus the system of §14 would roughly correspond to $P_\omega$ and, according to Rosser's result C, not be closed with respect to the rules. In order to make the system, if restricted to logical symbols, complete, the definition of 'consequence' given in §14 must be replaced by the following (in analogy to the correction already made in §48): $S_1$ is called a consequence of $K_1$ if $S_1$ belongs to every class containing $K_1$ and closed with respect to direct consequence.

R.CARNAP