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Testability and Meaning

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III. EMPIRICAL ANALYSIS OF CONFIRMATION AND TESTING

I. INTRODUCTION

I. Our Problem: Confirmation, Testing and Meaning

Two chief problems of the theory of knowledge are the question of meaning and the question of verification. The first question asks under what conditions a sentence has meaning, in the sense of cognitive, factual meaning. The second one asks how we get to know something, how we can find out whether a given sentence is true or false. The second question presupposes the first one. Obviously we must understand a sentence, i.e. we must know its meaning, before we can try to find out whether it is true or not. But, from the point of view of empiricism, there is a still closer connection between the two problems. In a certain sense, there is only one answer to the two questions. If we knew what it would be for a given sentence to be found true then we would know what its meaning is. And if for two sentences the conditions under which we would have to take them as true are the same, then they have the same meaning. Thus the meaning of a sentence is in a certain sense identical with the way we determine its truth or falsehood; and a sentence has meaning only if such a determination is possible.

If by verification is meant a definitive and final establishment of truth, then no (synthetic) sentence is ever verifiable, as we shall see. We can only confirm a sentence more and more. Therefore we shall speak of the problem of *confirmation* rather than of the problem of verification. We distinguish the *testing* of a sentence from its confirmation, thereby understanding a procedure—e. g. the carrying out of certain experiments—which leads to a confirmation in some degree either of the sentence itself or of its negation. We shall call a sentence *testable* if we know such a method of testing for it; and we call it *confirmable* if we know under what conditions the sentence would be confirmed. As we shall see, a sentence may be confirmable without being testable; e.g. if we know that our observation of such and such a course of events would confirm the sentence, and such and such a different course would confirm its negation without knowing how to set up either this or that observation.

In what follows, the problems of confirmation, testing and meaning will be dealt with. After some preliminary discussions in this Introduction, a logical analysis of the chief concepts connected with confirmation and testing will be carried out in Chapter I, leading to the concept of reducibility. Chapter II contains an empirical analysis of confirmation and testing, leading to a definition of the terms 'confirmable' and 'testable' mentioned before. The difficulties in discussions of epistemological and methodological problems are, it seems, often due to a mixing up of logical and empirical questions; therefore it seems desirable to separate the two analyses as clearly as possible. Chapter III uses the concepts defined in the preceding chapters for the construction of an empiricist language, or rather a series of languages. Further, an attempt will be made to formulate the principle of empiricism in a more exact way, by stating a requirement of confirmability or testability as a criterion of meaning. Different requirements are discussed, corresponding to different restrictions of the language; the choice between them is a matter of practical decision.

2. The Older Requirement of Verifiability

The connection between meaning and confirmation has sometimes been formulated by the thesis that a sentence is meaningful if and only if it is verifiable, and that its meaning is the method of its verification. The historical merit of this thesis was that it called attention to the close connection between the meaning of a sentence and the way it is confirmed. This formulation thereby helped, on the one hand, to analyze the factual content of scientific sentences, and, on the other hand, to show that the sentences of trans-empirical metaphysics have no cognitive meaning. But from our present point of view, this formulation, although acceptable as a first approximation, is not quite correct. By its oversimplification, it led to a too narrow restriction of scientific language, excluding not only metaphysical sentences but also certain scientific sentences having factual meaning. Our present task could therefore be formulated as that of a modification of the requirement of verifiability. It is a question of a modification, not of an entire rejection of that requirement. For among empiricists there seems to be full agreement that at least some more or less close relation exists between the meaning of a sentence and the way in which we may come to a verification or at least a confirmation of it.

The requirement of verifiability was first stated by *Wittgenstein,*[1] and its meaning and consequences were exhibited in the earlier publications of our *Vienna Circle;*[2] it is still held by the more conservative wing of this Circle.[3] The thesis needs both explanation and modification. What is meant by 'verifiability' must be said more clearly. And then the thesis must be modified and transformed in a certain direction.

Objections from various sides have been raised against the requirement mentioned not only by anti-empiricist metaphysicians but also by some empiricists, e.g. by *Reichenbach,*[4] *Popper*,[5] *Lewis,*[6] *Nagel,*[7] and *Stace.*[8] I believe that these criticisms are right in several respects; but on the other hand, their formulations must also be modified. The theory of confirmation and testing which will be explained in the following chapters is certainly far

[1] Wittgenstein [1].

[2] I use this geographical designation because of lack of a suitable name for the movement itself represented by this Circle. It has sometimes been called Logical Positivism, but I am afraid this name suggests too close a dependence upon the older Positivists, especially Comte and Mach. We have indeed been influenced to a considerable degree by the historical positivism, especially in the earlier stage of our development. But today we would like a more general name for our movement, comprehending the groups in other countries which have developed related views (see: Congress [1], [2]). The term '*Scientific Empiricism*' (proposed by *Morris* [1] p. 285) is perhaps suitable. In some historical remarks in the following, concerned chiefly with our original group I shall however use the term 'Vienna Circle'.

[3] Schlick [1] p. 150, and [4]; Waismann [1] p. 229.

[4] Reichenbach [1] and earlier publications; [3].

[5] Popper [1].

[6] Lewis [2] has given the most detailed analysis and criticism of the requirement of verifiability.

[7] Nagel [1].

[8] Stace [1].

from being an entirely satisfactory solution. However, by more exact formulation of the problem, it seems to me, we are led to a greater convergence with the views of the authors mentioned and with related views of other empiricist authors and groups. The points of agreement and of still existing differences will be evident from the following explanations.

A first attempt at a more detailed explanation of the thesis of verifiability has been made by *Schlick*[9] in his reply to *Lewis*' criticisms. Since 'verifiability' means 'possibility of verification' we have to answer two questions: 1) what is meant in this connection by 'possibility'? and 2) what is meant by 'verification'? Schlick—in his explanation of 'verifiability'—answers the first question, but not the second one. In his answer to the question: what is meant by 'verifiability of a sentence S', he substitutes the fact described by S for the process of verifying S. Thus he thinks e.g. that the sentence S_1: "Rivers flow up-hill," is verifiable, because it is logically possible that rivers flow up-hill. I agree with him that this fact is logically possible and that the sentence S_1 mentioned above is verifiable—or, rather, confirmable, as we prefer to say for reasons to be explained soon. But I think his reasoning which leads to this result is not quite correct. S_1 is confirmable, not because of the logical possibility of the fact described in S_1, but because of the physical possibility of the process of confirmation; it is possible to test and to confirm S_1 (or its negation) by observations of rivers with the help of survey instruments.

Except for some slight differences, e.g. the mentioned one, I am on the whole in agreement with the views of *Schlick* explained in his paper.[9] I agree with his clarification of some misunderstandings concerning positivism and so-called methodological solipsism. When I used the last term in previous publications I wished to indicate by it nothing more than the simple fact,[10] that everybody in testing any sentence empirically cannot do otherwise

[9] Schlick [4].
[10] Comp.: Erkenntnis 2, p. 461.

than refer finally to his own observations; he cannot use the results of other people's observations unless he has become acquainted with them by his own observations, e.g. by hearing or reading the other man's report. No scientist, as far as I know, denies this rather trivial fact. Since, however, the term `methodological solipsism'-in spite of all explanations and warnings-is so often misunderstood, I shall prefer not to use it any longer. As to the fact intended, there is, I think, no disagreement among empiricists; the apparent differences are due only to the unfortunate term. A similar remark is perhaps true concerning the term 'autopsychic basis' ('eigenpsychische Basis').

Another point may be mentioned in which I do not share *Schlick's* view. He includes in the range of meaningful sentences only synthetic and analytic sentences but not contradictory ones (for an explanation of these terms see §5). In my view—and perhaps also in his—this question is not a theoretical question of truth but a practical question of decision concerning the form of the language-system, and especially the formative rules. Therefore I do not say that Schlick is wrong, but only, that I am not inclined to accept his proposal concerning the limitation of the range of sentences acknowledged as meaningful. This proposal would lead to the following consequences which seem to me to be very inconvenient. In certain cases (namely if S_1 is analytic, S_2 is contradictory, S_3 and S_4 are synthetic and incompatible with each other) the following occurs: 1) the negation of a meaningful sentence S_1 is taken as meaningless; 2) the negation of a meaningless series of symbols S_2 is taken as a meaningful sentence; 3) the conjunction of two meaningful and synthetic sentences S_3 and S_4 is taken as meaningless. By the use of technical terms of logical syntax the objection can be expressed more precisely: if we decide to include in the range of (meaningful) sentences of our language only analytic and synthetic sentences (or even only synthetic sentences,[11] then the formative rules of our language become indefinite.[12] That means that in this case we have no fixed finite method of distinguishing between the meaningful and the meaningless, i.e. between sentences and expressions which are not sentences. And this would obviously be a serious disadvantage.

[11] Comp.: Carnap (6] p. 32, 34.

[12] Comp.: Carnap (4] §45.—About the indefinite character of the concepts 'analytic' and 'contradictory' comp.: Carnap [7] p. 163, or: (4b] §34a and 34d.

3. Confirmation instead of Verification

If verification is understood as a complete and definitive establishment of truth then a universal sentence, e.g. a so-called law of physics or biology, can never be verified, a fact which has often been remarked. Even if each single instance of the law were supposed to be verifiable, the number of instances to which the law refers—e.g. the space-time-points—is infinite and therefore can never be exhausted by our observations which are always finite in number. We cannot verify the law, but we can test it by testing its single instances i.e. the particular sentences which we derive from the law and from other sentences established previously. If in the continued series of such testing experiments no negative instance is found but the number of positive instances increases then our confidence in the law will grow step by step. Thus, instead of verification, we may speak here of gradually increasing *confirmation* of the law.

Now a little reflection will lead us to the result that there is no fundamental difference between a universal sentence and a particular sentence with regard to verifiability but only a difference in degree. Take for instance the following sentence: "There is a white sheet of paper on this table." In order to ascertain whether this thing is paper, we may make a set of simple observations and then, if there still remains some doubt, we may make some physical and chemical experiments. Here as well as in the case of the law, we try to examine sentences which we infer from the sentence in question. These inferred sentences are predictions about future observations. The number of such predictions which we can derive from the sentence given is infinite; and therefore the sentence can never be completely verified. To be sure, in many cases we reach a practically sufficient certainty after a small number of positive instances, and then we stop experimenting. But there is always the theoretical possibility of continuing the series of test-observations. Therefore here also *no complete verification is possible* but only a process of gradually increasing *confirmation.* We may, if we wish, call a sentence disconfirmed[13] in a certain degree if its negation is confirmed in that degree.

[13] "Erschüttert," Neurath [6].

The impossibility of absolute verification has been pointed out and explained in detail by *Popper.*[14] In this point our present views are, it seems to me, in, full accordance with *Lewis*[15] and *Nagel.*[16]

Suppose a sentence S is given, some test-observations for it have been made, and S is confirmed by them in a certain degree. Then it is a matter of practical decision whether we will consider that degree as high enough for our acceptance of S, or as low enough for our rejection of S, or as intermediate between these so that we neither accept nor reject S until further evidence will be available. Although our decision is based upon the observations made so far, nevertheless it is not uniquely determined by them. There is no general rule to determine our decision. Thus the acceptance and the rejection of a (synthetic) sentence always contains a *conventional component.* That does not mean that the decision—or, in other words, the question of truth and verification—is conventional. For, in addition to the conventional component there is always the non-conventional component—we may call it, the objective one—consisting in the observations which have been made. And it must certainly be admitted that in very many cases this objective component is present to such an overwhelming extent that the conventional component practically vanishes. For such a simple sentence as e.g. "There is a white thing on this table" the degree of confirmation, after a few observations have been made, will be so high that we practically cannot help accepting the sentence. But even in this case there remains still the theoretical possibility of denying the sentence. Thus even here it is a matter of decision or convention.

The view that no absolute verification but only gradual confirmation is possible, is sometimes formulated in this way: every sentence is a probability-sentence; e.g. by *Reichenbach*[17] and *Lewis.*[18] But it seems advisable to separate the two assertions.

[14] Popper [1].

[15] "Lewis [2) p. 137, note 12: "No verification of the kind of knowledge commonly stated in propositions is ever absolutely complete and final."

[16] Nagel [1 p. 144f.

[17] Reichenbach [1].

[18] Lewis [2] p. 133.

Most empiricists today will perhaps agree with the first thesis, but the second is still a matter of dispute. It presupposes the thesis that the degree of confirmation of a hypothesis can be interpreted as the degree of probability in the strict sense which this concept has in the calculus of probability, i.e. as the limit of relative frequency. Reichenbach[19] holds this thesis. But so far he has not worked out such an interpretation in detail, and today it is still questionable whether it can be carried out at all. *Poppers*[20] has explained the difficulties of such a frequency interpretation of the degree of confirmation; the chief difficulty lies in how we are to determine for a given hypothesis the series of "related" hypotheses to which the concept of frequency is to apply. It seems to me that at present it is not yet clear whether the concept of degree of confirmation can be defined satisfactorily as a quantitative concept, i.e. a magnitude having numerical values. Perhaps it is preferable to define it as a merely topological concept, i.e. by defining only the relations: "S_1 has the same (or, a higher) degree of confirmation than S_2 respectively," but in such a way that most of the pairs of sentences will be incomparable. We will use the concept in this way—without however defining it—only in our informal considerations which serve merely as a preparation for exact definitions of other terms. We shall later on define the concepts of complete and incomplete reducibility of confirmation as syntactical concepts, and those of complete and incomplete confirmability as descriptive concepts.

4. The Material and the Formal Idioms

It seems to me that there is agreement on the main points between the present views of the *Vienna Circle,* which are the basis of our following considerations, and those of *Pragmatism,* as interpreted e.g. by *Lewis.*[21] This agreement is especially marked with respect to the view that every (synthetic) sentence is a hypothesis, i.e. can never be verified completely and defini-

[19] Reichenbach [2] p. 271 ff. ; (3] p. 154 ff.

[20] Popper [1] Chapter VIII; for the conventional nature of the problem compare my remark in "Erkenntnis" vol. 5, p. 292.

[21] Lewis (2], especially p. 133.

tively. One may therefore expect that the views of these two empiricist movements will continue to converge to each other in their further development; *Morris*[22] believes that this convergence is a fact and, moreover, tries to promote it.

However, in spite of this agreement on many important points, there is a difference between our method of formulation and that which is customary in other philosophical movements, especially in America and England. This difference is not as unimportant as are the differences in formulation in many other cases. For the difference in formulation depends on the difference between the material and the formal idioms.[23] The use of the material idiom is very common in philosophy; but it is a dangerous idiom, because it sometimes leads to pseudo-questions. It is therefore advisable to translate questions and assertions given in the material idiom into the formal idiom. In the material idiom occur expressions like 'facts', 'objects', 'the knowing subject', 'relation between the knowing subject and the known subject', 'the given', 'sense-data', 'experiences' etc. The formal idiom uses syntactical terms instead, i.e. terms concerning the formal structure of linguistic expressions. Let us take an example. It is a pseudo-thesis of idealism and older positivism, that a physical object (e.g. the moon) is a construction out of sense-data. Realism on the other hand asserts, that a physical object is not constructed but only cognized by the knowing subject. We—the Vienna Circle—neither affirm nor deny any of these theses, but regard them as pseudo-theses, i.e. as void of cognitive meaning. They arise from the use of the material mode, which speaks about 'the object'; it thereby leads to such pseudo-questions as the "nature of this

[22] Morris [1], [2].

[23] Here I can give no more than some rough indications concerning the material and the formal idioms. For detailed explanations compare Carnap [4]. Ch. V. A shorter and more easily understandable exposition is contained in [5] p. 85-88.—What I call the formal and the material idioms or modes of speech, is not the same as what Morris ([1], p. 8) calls the formal and the empirical modes of speech. To Morris's empirical mode belong what I call the real object-sentences; and these belong neither to the formal nor to the material mode in my sense (comp. Carnap [4], §74, and [5], p. 61). The distinction between the formal and the material idioms does not concern the usual sentences of science but chiefly those of philosophy, especially those of epistemology or methodology.

object", and especially as to whether it is a mere construction or not. The formulation in the formal idiom is as follows: "A physical object-name (e.g. the word 'moon') is reducible to sense-data predicates (or perception predicates)." *Lewis*[24] seems to believe that logical positivism—the Vienna Circle—accepts the idealistic pseudo-thesis mentioned. But that is not the case. The misunderstanding can perhaps be explained as caused by an unintentional translation of our thesis from the formal into the more accustomed material idiom, whereby it is transformed into the idealistic pseudo-thesis.

The same is true concerning our thesis: "My testing of any sentence, even one which contains another man's name and a psychological predicate (e.g. "Mr. X is now cheerful"), refers back ultimately to my own observation-sentences." If we translate it into: "Your mind is nothing more than a construction which I put upon certain data of my own experience," we have the pseudo-thesis of solipsism, formulated in the material idiom. But this is not our thesis.

The formulation in the material idiom makes many epistemological sentences and questions ambiguous and unclear. Sometimes they are meant as psychological questions. In this case clearness could be obtained by a formulation in the psychological language. In other cases questions are not meant as empirical, factual questions, but as logical ones. In this case they ought to be formulated in the language of logical syntax. In fact, however, epistemology in the form it usually takes—including many of the publications of the Vienna Circle—is an unclear mixture of psychological and logical components. We must separate it into its two kinds of components if we wish to come to clear, unambiguous concepts and questions. I must confess that I am unable to answer or even to understand many epistemological questions of the traditional kind because they are formulated in the material idiom. The following are some examples taken from customary discussions: "Are you more than one of my ideas"?, "Is the past more than the present recollection"?, "Is the future more than

[24] Lewis [2], p. 127-128.

the present experience of anticipation"?, "Is the self more than one of those ideas I call mine"?, "If a robot is exhibiting all the behavior appropriate to tooth-ache, is there a pain connected with that behavior or not" ? etc.

I do not say that I have not the least understanding of these sentences. I see some possibilities of translating them into unambiguous sentences of the formal idiom. But unfortunately there are several such *translations,* and hence I can only make conjectures as to the intended meaning of the questions. Let me take another example. I find the following thesis[25] formulated in the material mode: "Any reality must, in order to satisfy our empirical concept of it, transcend the concept itself. A construction imposed upon given data cannot be identical with a real object; the thing itself must be more specific, and in comparison with it the construction remains abstract." As a conjecture, selected from a great number of possibilities, I venture the following translation into the formal idiom: "For any object-name and any given finite class C of sentences (or: of sentences of such and such a kind), there are always sentences *containing* that name such that neither their confirmation nor that of their negation is completely reducible to that of C (in syntactical terminology: there are sentences each of which is neither a consequence of C nor incompatible with C)."Our present views, by the way,—as distinguished from our previous ones—are in agreement with this thesis, provided my interpretation hits the intended meaning. The translation shows that the thesis concerns the structure of language and therefore depends upon a convention, namely the choice of the language-structure.This fact is concealed by the formulation in the material mode. There the thesis seems to be independent of the choice of language, it seems to concern a certain character which `reality' either does or does not possess. Thus the use of the material idiom leads to a certain absolutism, namely to the neglect of the fact that the thesis is relative to the chosen language-system. The use of the formal idiom reveals that fact. And indeed our present agreement with the thesis

[25] Lewis [2], p. 138.

mentioned is connected with our admission of incompletely confirmable sentences, which will be explained later on.

The dangers of the material idiom were not explicitly noticed by our Vienna Circle in its earlier period. Nevertheless we used this idiom much less frequently than is customary in traditional philosophy; and when we used it, we did so in most cases in such a way that it was not difficult to find a translation into the formal idiom. However, this rather careful use was not deliberately planned, but was adopted intuitively, as it were. It seems to me that most of the formulations in the material idiom which are considered by others as being theses of ours have never been used by us. In recent years we have become increasingly aware of the disadvantages of the material idiom. Nevertheless we do not try to avoid its use completely. For sometimes its use is preferable practically, as long as this idiom is still more customary among philosophers. But perhaps there will come a time when this will no longer be the case. Perhaps some day philosophers will prefer to use the formal idiom—at least in those parts of their works which are intended to present decisive arguments rather than general preliminary explanations.

II. LOGICAL ANALYSIS OF CONFIRMATION AND TESTING

5. *Some Terms and Symbols of Logic*

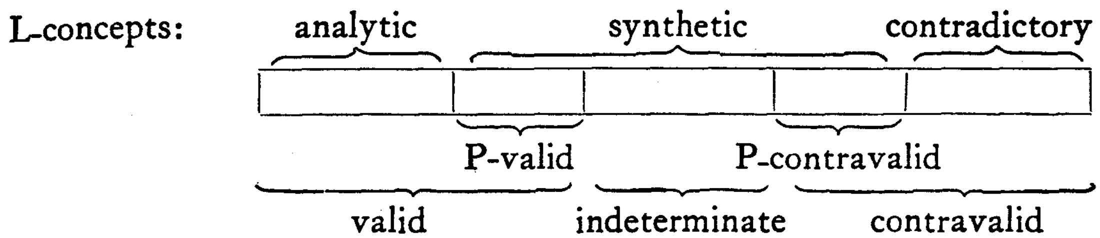
In carrying out methodological investigations especially concerning verification, confirmation, testing etc., it is very important to distinguish clearly between logical and empirical, e.g. psychological questions. The frequent lack of such a distinction in so-called epistemological discussions has caused a great deal of ambiguity and misunderstanding. In order to make quite clear the meaning and nature of our definitions and explanations, we will separate the two kinds of definitions. In this Chapter II we are concerned with logical analysis. We shall define concepts belonging to logic, or more precisely, to logical syntax, although the choice of the concepts to be defined and of the way in which they are defined is suggested in some respects by a consideration of empirical questions—as is often the case in laying down logical definitions. The logical concepts defined here will be applied later on, in Chapter III, in defining concepts of an empirical analysis of confirmation. These descriptive, i.e. non-logical, concepts belong to the field of biology and psychology, namely to the theory of the use of language as a special kind of human activity.

In the following logical analysis we shall make use of some few *terms of logical syntax,* which may here be explained briefly.[26] The terms refer to a language-system, say L, which is supposed to be given by a system of rules of the following two kinds. The formative rules state how to construct sentences of L out of the symbols of L. The transformative rules state how to deduce a sentence from a class of sentences, the so-called premisses, and which sentences are to be taken as true unconditionally, i.e., without reference to premisses. The transformative rules are divided into those which have a logico-mathematical nature; they are called logical rules or L-rules (this 'L-' has nothing to do with the name 'L' of the language); and those of an empirical nature, e.g. physical or biological laws stated as postulates; they are called physical rules or P-rules.

We shall take here 'S', 'S_1', 'S_2' etc. as designations of sentences (not as abbreviations for sentences). We use '~S' as designation of the negation of S. (Thus, in this connection, '~' is not a symbol of negation but a syntactical symbol, an abbreviation for the words 'the negation of'.) If a sentence S can be deduced from the sentences of a class C according to the rules of L, S is called a *consequence* of C; and moreover an L-consequence, if the L-rules are sufficient for the deduction, otherwise a P-consequence. S_1 and S_2 are called *equipollent* (with each other) if each is a consequence of the other. If S can be shown to be true on the basis of the rules of L, S is called *valid* in L; and moreover L-valid or *analytic,* if true on the basis of the L-rules alone, otherwise P-valid. If, by application of the rules of L, S can be shown to be false, S is called *contravalid;* and L-contravalid or *contradictory,* if by L-rules alone, otherwise P-contravalid. If S is neither valid

[26]For more exact explanations of these terms see Carnap [4]; some of them are explained also in [5].

nor contravalid S is called *indeterminate.* If S is neither analytic nor contradictory, in other words, if its truth or falsehood cannot be determined by logic alone, but needs reference either to P-rules or to the facts outside of language, S is called *synthetic.* Thus the totality of the sentences *of* L is classified in the following way:



A sentence S_1 is called incompatible with S_2 (or with a class C of sentences), if the negation ~S, is a consequence of S_2 (or of C, respectively). The sentences of a class are called mutually independent if none of them is a consequence of, or incompatible with, any other of them.

The most important kind of predicates occurring in a language of science is that of the predicates attributed to space-time-points (or to small space-time-regions). For the sake of simplicity we shall restrict the following considerations—so far as they deal with predicates—to those of this kind. The attribution of a certain value of a physical function, e.g. of temperature, to a certain space-time-point can obviously also be expressed by a predicate of this kind. The following considerations, applied here to such predicates only, can easily be extended to descriptive terms of any other kind.

In order to be able to formulate examples in a simple and exact way we will use the following symbols. We take 'a', 'b', etc. as names of space-time-points (or of small space-time-regions), i.e. as abbreviations for quadruples of space-time-coördinates; we call them *individual constants.* 'x', 'y', etc. will be used as corresponding variables; we will call them *individual variables.* We shall use 'P', 'P_1', 'P_2' etc., and 'Q', 'Q_1' etc. as *predicates;* if no other indication is given, they are supposed to be predicates of the kind described. The sentence 'Q_1 (b)' is to mean: "The space-time-point b has the property Q_1." Such a sentence consisting of a predicate followed by one or several individual constants as arguments, will be called a full *sentence* of that predicate.

Connective symbols: '$\sim$' for 'not' (negation), '$\vee$' for 'or' (disjunction), '.' for 'and' (conjunction), '$\supset$' for 'if - then' (implication), '$\equiv$' for 'if - then - , and if not - then not –' (equivalence). '$\sim Q(a)$' is the negation of a full sentence of 'Q'; it is sometimes also called a full sentence of the predicate '$\sim Q$'.

Operators: '$(x)P(x)$' is to mean: "every point has the property P" (*universal* sentence; the first '(x)' is called the universal *operator,* and the sentential function '$P(x)$' its *operand*). '$(\exists x)P(x)$' is to mean: "There is at least one point having the property P" (*existential* sentence; '$(\exists x)$' is called the existential operator and '$P(x)$' its operand). (In what follows, we shall not make use of any other operators than universal and existential operators with individual variables, as described here.) In our later examples we shall use the following abbreviated notation for universal sentences of a certain form occurring very frequently. If the sentence '(x) [- - -]' is such that '- - -' consists of several partial sentences which are connected by '$\sim$', '$\vee$' etc. and each of which consists of a predicate with 'x' as argument, we allow omission of the operator and the arguments. Thus e.g. instead of '$(x)\ (P_1(x) \supset P_2(x))$' we shall write shortly '$P_1 \supset P_2$'; and instead of '$(x)\ [Q_1(x) \supset (Q_3(x) \equiv Q_2(x))]$' simply '$Q_1 \supset (Q_3 \equiv Q_2)$'. The form '$P_1 \supset P_2$' is that of the simplest physical laws; it means: "If any space-time-point has the property P_1, it has also the property P_2."

6. Reducibility of Confirmation

The number of sentences for which, at a certain moment, we have found a confirmation of some degree or other, is always finite. If now a class C of sentences contains a finite sub-class C′ such that the sentence S is a consequence of C′, then, if the sentences of C′ are found to be confirmed to a certain degree, S will be confirmed to at least the same degree. In this case we have, so to speak, a complete confirmation of S by C′. (It is to be noticed that "complete" is not meant here in an absolute sense, but in a relative sense with respect to certain premisses.) On the other hand, suppose that S is not a consequence of any finite sub-class of C, but each sentence of an infinite sub-class C″ of C is a consequence of S, - e.g. if S is a universal sentence and C″ the class of its instances. In this case, no complete confirmation of S by sentences of C is possible; nevertheless, S will be confirmed by the confirmation of sentences of C″ at least to some degree, though not necessarily to the same degree. Suppose moreover that the sentences of C″ are mutually independent. Since their number is infinite, they cannot be exhausted. Therefore the degree of confirmation of S will increase by the confirmation of more and more sentences of C″ but without ever coming to a complete confirmation. On the basis of these considerations we will lay down the definitions 1 to 6. In Definitions 1 and 2 C is a class of sentences. The terms defined in Definitions 1 a, b and c are only auxiliary terms for Definition 2.

Definition 1. a. We will say that the confirmation of S is completely reducible to that of C, if S is a consequence of a finite subclass of C.

b. We will say that the confirmation of S is directly incompletely reducible to that of C, if the confirmation of S is not completely reducible to that of C but if there is an infinite sub-class C′ of C such that the sentences of C′ are mutually independent and are consequences of S.

c. We will say that the confirmation of S is directly reducible to that of C, if it is either completely reducible or directly incompletely reducible to that of C.

Definition 2. a. We will say that the *confirmation* of S is re*ducible* to that of C, if there is a finite series of classes C_1, C_2, . . . C_n such that the relation of directly reducible confirmation subsists 1) between S and C_1, 2) between every sentence of C_i and C_{i+1} ($i = 1$ to $n - 1$), and 3) between every sentence of C_n and C.

b. We will say that the *confirmation* of S is *incompletely reducible* to that of C, if it is reducible but not completely reducible to that of C.

Definition 3. We will say that the confirmation of S is reducible (or completely reducible, or incompletely reducible) to that of a class C of predicates (or to that of its members) if it is reducible (or completely reducible, or incompletely reducible, respectively) to a not contravalid sub-class of the class which contains the full sentences of the predicates of C and the negations of these sentences. -The sub-class is required not to be contravalid because any sentence whatever is a consequence of a contravalid class, as e.g. {'P(a)', '~P(a)' }, and hence its confirmation is reducible to that of this class.

The following definitions concerning predicates are analogous to the previous ones concerning sentences.

Definition 4. We will say that the confirmation of a *predicate* 'Q' is reducible (or completely reducible, or incompletely reducible) to that of a class C of predicates, say 'P_1', 'P_2', etc., if the confirmation of every full sentence of 'Q' with a certain argument, e.g. 'Q(a)', is reducible (or completely reducible, or incompletely reducible, respectively) to that of the class C′ consisting of the full sentences of the predicates of C with the same argument and the negations of those sentences ('P_1(a)', '~P_1(a)', 'P_2(a)', '~P_2 (a)', etc.).

Definition 5. A *predicate* 'Q' is called *reducible* (or completely reducible, or incompletely reducible) to a class C of predicates or to its members, if the confirmation both of 'Q' and of '~Q' is reducible (or completely reducible, or incompletely reducible, respectively) to C.

When we speak of sentential functions, sentences are understood to be included because a sentence may be taken as a special case of a sentential function with the number zero of free variables. Therefore the following definitions are also applied to sentences.

Definition 6. A sentential function is said to have *atomic form* if it consists of one predicate followed by one or several arguments (individual constants or variables). (Examples: 'P(x)', 'Q(a, x)', 'P(a)'.

Definition 7. A sentential function is said to have *molecular form* if it is constructed out of one or several sentential functions with the help of none, one or several connective symbols (but without operators).

Definition 8. a. A sentential function is said to have *generalized form* if it contains at least one (unrestricted) operator.

b. A sentential function is said to have *essentially generalized form* if it has generalized form and cannot be transformed into a molecular form containing the same descriptive predicates.

We have to distinguish between a sentence of atomic form and an atomic sentence (see Definition 15a, §9; here the predicate occurring must fulfill certain conditions); and likewise between a sentence of molecular form and a molecular sentence (see Definition 15b, §9). Since the sentences of atomic form are included in those of molecular form, the important distinction is that between molecular and (essentially) generalized form.

In what follows we will apply the concepts of reducibility of confirmation, defined before, first to molecular sentences and then to generalized sentences.

Theorem 1. If the confirmation both of S_1 and of S_2 is completely reducible to that of a class C of predicates, then the confirmation both of their disjunction and of their conjunction is also completely reducible to that of C.

Proof. The disjunction is a consequence of S_1; the conjunction is a consequence of S_1 and S_2.

Theorem 2. If S is a sentence of *molecular form* and the descriptive predicates occurring in S belong to C, the confirmation of S is completely reducible to that of C.

Proof. Let C′ be the class of the full sentences of the predicates of C and their negations. According to a well known theorem of logic, S can be transformed into the so-called disjunctive normal form,[27] i.e. into a disjunction of conjunctions of sentences of C′. Now, the confirmation of a sentence of C′ is completely reducible to that of C. Therefore, according to Theorem I, the confirmation of each of the conjunctions is also completely reducible to that of C, and, again according to Theorem I, the same is true for the disjunction of these conjunctions, and hence for S.

The application of the concepts defined before to sentences of generalized form may be explained by the following examples.

S_1: ‘(x)P(x)’

S_2: ‘(x)~P(x)’ (in words: every point has the property not-P; in other words: no point has the property P).

[27] Compare *Hilbert* [1] p. 13.

C_1 may be taken as the class of the full sentences of 'P', i.e. the class of the particular sentences 'P(a)', 'P(b)', etc.; C_2 as the class of the negations of these sentences: '~ P(a)', etc.; and C as the sum of C_1 and C_2. Then, according to a well known result (see §3), the confirmation of S_1 is directly reducible to that of C_1 and hence to that of C, but only incompletely, because S_1 is not a consequence of any finite sub-class of C, however large this may be. On the other hand, ~ S_1 is a consequence of each sentence of C_2, e.g. of '~ P(a)'. Therefore the confirmation of ~S_1 is completely reducible to that of C_2 and hence to that of C.

S_2 bears the same relation to C_2 as S_1 does to C_1. Therefore the confirmation of S_2 is incompletely reducible to that of C_2, and the confirmation of ~S_2 is completely reducible to that of C_1. This can easily be seen when we transform ~S_2 into the existential sentence '(∃x)P(x)' which is a consequence of each sentence of C_1, e.g. of 'P(a)'. The results of these considerations may be exhibited by the following table which gives two formulations for each of the four sentences, one containing a universal operator and the other an existential operator. Some of the results, which we need later on, are formulated in the following Theorems 3 and 4.

	two formulations	The confirmation of S is reducible					
		to that of C_1 ('P(a)' etc.)		to that of C_2 ('~P(a)' etc.)		to that of C (= C_1 + C_2)	
		compl.	incompl.	compl.	incompl.	compl.	incompl.
S_1	$(x)P(x)$; $\sim(\exists x)\sim P(x)$	−	+	−	−	−	+
$\sim S_1$	$\sim(x)P(x)$; $(\exists x)(\sim P(x))$	−	−	+	−	+	−
S_2	$(x)\sim P(x)$; $\sim(\exists x)P(x)$	−	−	−	+	−	+
$\sim S_2$	$\sim(x)\sim P(x)$; $(\exists x)P(x)$	+	−	−	−	+	−

Theorem 3. Let S be the *universal sentence* '(x)P(x)'. The confirmation of S is incompletely reducible to that of the full sentences of 'P' and hence to that of 'P'. The confirmation of

~ S is completely reducible to that of the negation of any full sentence of ‘P’ and hence to that of ‘P’.

Theorem 4. Let S be the *existential sentence* ‘(∃x)P(x)’. The confirmation of S is completely reducible to that of any full sentence of ‘P’ and hence to that of ‘P’. The confirmation of ~ S is incompletely reducible to that of the negations of the full sentences of ‘P’ and hence to that of ‘P’.

The Theorems 3 and 4 correspond to the following usual, but not quite correct formulations: 1) “A universal sentence is not verifiable but falsifiable,” 2) “An existential sentence is verifiable but not falsifiable.” Still closer corresponding theorems will be stated later on (Theorems 19 and 20, §24).

7. Definitions

By an (explicit) definition of a descriptive predicate ‘Q’ with one argument we understand a sentence of the form

(D:) $$Q(x) \equiv \ldots x \ldots$$

where at the place of ‘. . . x . . .’ a sentential function — called the definiens — stands which contains ‘x’ as the only free variable. For several arguments the form is analogous. We will say that a definition D is based upon the class C of predicates if every descriptive symbol occurring in the definiens of D belongs to C. If the predicates of a class C are available in our language we may introduce other predicates by a chain of definitions of such a kind that each definition is based upon C and the predicates defined by previous definitions of the chain.

Definition 9. A definition is said to have atomic (or molecular, or generalized, or essentially generalized) form, if its definiens has atomic (or molecular, or generalized, or essentially generalized, respectively) form.

Theorem 5. If ‘P’ is defined by a definition D based upon C, ‘P’ is reducible to C. If D has molecular form, ‘P’ is completely reducible to C. If D has essentially generalized form, ‘P’ is incompletely reducible to C.

Proof. ‘P’ may be defined by ‘P(x) ≡ . . . x. . .’. Then, for any b, ‘P(b)’ is equipollent to ‘. . . b . . .’ and hence in the case of molecular form, according to Theorem 2, completely reducible to C, and in the other case, according to Theorems 3 and 4, reducible to C.

Let us consider the question whether the so-called *disposition-concepts* can be defined, i.e. predicates which enunciate the disposition of a point or body for reacting in such and such a way to such and such conditions, e.g. 'visible', 'smellable', 'fragile', 'tearable', 'soluble', 'indissoluble' etc. We shall see that such disposition-terms cannot be defined by means of the terms by which these conditions and reactions are described, but they can be introduced by sentences of another form. Suppose, we wish to introduce the predicate 'Q_3' meaning "soluble in water." Suppose further, that 'Q_1' and 'Q_2' are already defined in such a way that '$Q_1(x, t)$' means "the body x is placed into water at the time t," and '$Q_2(x,t)$' means "the body x dissolves at the time t." Then one might perhaps think that we could define 'soluble in water' in the following way: "x is soluble in water" is to mean "whenever x is put into water, x dissolves," in symbols:

(D:) $$Q_3(x) \equiv (t)\ [Q_1(x,t) \supset Q_2(x, t)].$$

But this definition would not give the intended meaning of 'Q_3'. For, suppose that c is a certain match which I completely burnt yesterday. As the match was made of wood, I can rightly assert that it was not soluble in water; hence the sentence '$Q_3(c)$' (S_1) which asserts that the match c is soluble in water, is false. But if we assume the definition D, S_1 becomes equipollent with '$(t)\ [Q_1(c, t) \supset Q_2 (c,t)]$' ($S_2$). Now the match c has never been placed and on the hypothesis made never can be so placed. Thus any sentence of the form '$Q_1(c,t)$' is false for any value of 't'. Hence S_2 is true, and, because of D, S_1 also is true, in contradiction to the intended meaning of S_1. 'Q_3' cannot be defined by D, nor by any other definition. But we can introduce it by the following sentence:

(R:) $$(x)(t)\ [Q_1(x)\ t) \supset Q_3\ (x) \equiv Q_2\ (x,t))],$$

in words: "if any thing x is put into water at any time t, then, if x is soluble in water, x dissolves at the time t, and if x is not soluble in water, it does not." This sentence belongs to that kind of sentences which we shall call reduction sentences.

8. Reduction Sentences

Suppose, we wish to introduce a new predicate 'Q_3' into our language and state for this purpose a pair of sentences of the following form:

(R_1) $$Q_1 \supset (Q_2 \supset \sim Q_3)$$
(R_2) $$Q_4 \supset (Q_5 \supset \sim Q_3)$$

Here, 'Q_1' and 'Q_4' may describe experimental conditions which we have to fulfill in order to find out whether or not a certain space-time-point b has the property Q_3, i.e. whether '$Q_3(b)$' or '$\sim Q_3(b)$' is true. 'Q_2' and 'Q_5' may describe possible results of the experiments. Then R_1 means: if we realize the experimental condition Q_1 then, if we find the result Q_2, the point has the property Q_3. By the help of R_1, from '$Q_1(b)$' and '$Q_2(b)$', 'Q_3 (b)' follows. R_2 means: if we satisfy the condition Q_4 and then find Q_5 the point has not the property Q_3. By the help of R_2, from '$Q_4(b)$' and 'Q_5 (b)', '$\sim Q_3(b)$' follows. We see that the sentences R_1 and R_2 tell us how we may determine whether or not the predicate 'Q_3' is to be attributed to a certain point, provided we are able to determine whether or not the four predicates 'Q_1', 'Q_2', 'Q_4'; and 'Q_5' are to be attributed to it. By the statement of R_1 and R_2 'Q_3' is reduced in a certain sense to those four predicates; therefore we shall call R_1 and R_2 reduction sentences for 'Q_3' and '$\sim Q_3$' respectively. Such a pair of sentences will be called a reduction pair for 'Q_3'. By R_1 the property Q_3 is attributed to the points of the class $Q_1 \bullet Q_2$, by R_2 the property $\sim Q_3$ to the points of the class $Q_4 \bullet Q_5$. If by the rules of the language — either logical rules or physical laws — we can show that no point belongs to either of these classes (in other words, if the universal sentence '$\sim [(Q_1 \bullet Q_2) \vee (Q_4 \bullet Q_5)]$' is valid) then the pair of sentences does not determine Q_3 nor $\sim Q_3$ for any point and therefore does not give a reduction for the predicate Q_3. Therefore, in the definition of 'reduction pair' to be stated, we must exclude this case.

In special cases 'Q_4' coincides with 'Q_1', and 'Q_5' with '$\sim Q_2$'.

In that case the reduction pair is '$Q_1 \supset (Q_2 \supset Q_3)$' and '$Q_1 \supset (\sim Q_2 \supset \sim Q_3)$'; the latter can be transformed into '$Q_1 \supset (Q_3 \supset Q_2)$'. Here the pair can be replaced by the one sentence '$Q_1 \supset (Q_3 \equiv Q_2)$' which means: if we accomplish the condition Q_1, then the point has the property Q_3 if and only if we find the result Q_2. This sentence may serve for determining the result '$Q_3(b)$' as well as for '$\sim Q_3(b)$'; we shall call it a bilateral reduction sentence. It determines Q_3 for the points of the class $Q_1 \bullet Q_2$, and $\sim Q_3$ for those of the class $Q_1 \bullet \sim Q_2$; it does not give a determination for the points of the class $\sim Q_1$. Therefore, if '$(x)(\sim Q_1(x))$' is valid, the sentence does not give any determination at all. To give an example, let '$Q'_1(b)$' mean "the point b is both heated and not heated", and '$Q''_1(b)$': "the point b is illuminated by light-rays which have a speed of 400,000 km/sec". Here for any point c, '$Q'_1(c)$' and '$Q''_1(c)$' are contravalid — the first contradictory and the second P-contravalid; therefore, '$(x)\ (\sim Q_1'\ (x))$' and '$(x)\ (\sim Q_1''\ (x))$' are valid — the first analytic and the second P-valid; in other words, the conditions Q'_1 and Q''_1 are impossible, the first logically and the second physically. In this case, a sentence of the form '$Q'_1 \supset (Q_3 \equiv Q_2)$' or '$Q''_1 \supset (Q_3 \equiv Q_2)$' would not tell us anything about how to use the predicate 'Q_3' and therefore could not be taken as a reduction sentence. These considerations lead to the following definitions.

Definition 10. a. A universal sentence of the form

(R) $$Q_1 \supset (Q_2 \supset Q_3)$$

is called a *reduction sentence* for 'Q_3' provided '$\sim (Q_1 \bullet Q_2)$' is not valid.

b. A pair of sentences of the forms

(R_1) $$Q_1 \supset (Q_2 \supset Q_3)$$

(R_2) $$Q_4 \supset (Q_5 \supset \sim Q_3)$$

is called a *reduction pair* for 'Q_3' provided '$\sim [(Q_1 \bullet Q_2) \vee (Q_4 \bullet Q_5)]$' is not valid.

c. A sentence of the form

(R_b) $$Q_1 \supset (Q_3 \equiv Q_2)$$

is called a *bilateral reduction sentence* for 'Q_3' provided '$(x)\ (\sim Q_1(x))$' is not valid.

Every statement about reduction pairs in what follows applies also to bilateral reduction sentences, because such sentences are comprehensive formulations of a special case of a reduction pair.

If a reduction pair for 'Q_3' of the form given above is valid — i.e. either laid down in order to introduce 'Q_3' on the basis of 'Q_1', 'Q_2', 'Q_4', and 'Q_5', or consequences of physical laws stated beforehand — then for any point c '$Q_3(c)$' is a consequence of '$Q_1(c)$' and '$Q_2(c)$', and $\sim Q_3(c)$' is a consequence of '$Q_4(c)$' and '$Q_5(c)$'. Hence 'Q_3' is completely reducible to those four predicates.

Theorem 6. If a reduction pair for 'Q' is valid, then 'Q' is completely reducible to the four (or two, respectively) other predicates occurring.

We may distinguish between logical reduction and physical reduction, dependent upon the reduction sentence being analytic or P-valid, in the latter case for instance a valid physical law. Sometimes not only the sentence '$Q_1 \supset (Q_3 \equiv Q_2)$ is valid, but also the sentence '$Q_3 \equiv Q_2$'. (This is e.g. the case if '$(x)Q_1(x)$' is valid.) Then for any b, '$Q_3(b)$' can be transformed into the equipollent sentence '$Q_2(b)$', and thus 'Q_3' can be eliminated in any sentence whatever. If '$Q_3 \equiv Q_2$' is not P-valid but analytic it may be considered as an explicit definition for 'Q_3'. Thus an *explicit definition* is a special kind of a logical bilateral reduction sentence. A logical bilateral reduction sentence which does not have this simple form, but the general form '$Q_1 \supset (Q_3 \equiv Q_2)$', may be considered as a kind of conditioned definition.

If we wish to construct a language for science we have to take some descriptive (i.e. non-logical) terms as primitive terms. Further terms may then be introduced not only by explicit definitions but also by other reduction sentences. The possibility of *introduction by laws,* i.e. by physical reduction, is, as we shall see, very important for science, but so far not sufficiently noticed in the logical analysis of science. On the other hand the terms introduced in this way have the disadvantage that in general it is not possible to eliminate them, i.e. to translate a sentence containing such a term into a sentence containing previous terms only.

Let us suppose that the term 'Q_3' does not occur so far in our language, but 'Q_1', 'Q_2', 'Q_4', and 'Q_5' do occur. Suppose further that either the following reduction pair R_1, R_2 for 'Q_3':

$$(R_1) \qquad Q_1 \supset (Q_2 \supset Q_3)$$
$$(R_2) \qquad Q_4 \supset (Q_5 \supset \sim Q_3)$$

or the following bilateral reduction sentence for 'Q_3':

$$(R_b) \qquad Q_1 \supset (Q_3 \equiv Q_2)$$

is stated as valid in order to introduce 'Q_3', i.e. to give meaning to this new term of our language. Since, on the assumption made, 'Q_3' has no antecedent meaning, we do not assert anything about facts by the statement of R_b. This statement is not an assertion but a convention. In other words, the factual content of R_b is empty; in this respect, R_b is similar to a definition. On the other hand, the pair R_1, R_2 has a positive content. By stating it as valid, beside stating a convention concerning the use of the term 'Q_3', we assert something about facts that can be formulated in the following way without the use of 'Q_3'. If a point c had the property $Q_1 \bullet Q_2 \bullet Q_4 \bullet Q_5$, then both '$Q_3(c)$' and '$\sim Q_3(c)$' would follow. Since this is not possible for any point, the following universal sentence S which does not contain 'Q_3', and which in general is synthetic, is a consequence of R_1 and R_2:

$$(S{:}) \qquad \sim (Q_1 \bullet Q_2 \bullet Q_4 \bullet Q_5).$$

In the case of the bilateral reduction sentence R_b 'Q_4' coincides with 'Q_1' and 'Q_5' with '$\sim Q_2$'. Therefore in this case S degenerates to '$\sim(Q_1 \bullet Q_2 \bullet Q_1 \bullet \sim Q_2)$' and hence becomes analytic. Thus a bilateral reduction sentence, in contrast to a reduction pair, has no factual content.

9. Introductive Chains

For the sake of simplicity we have considered so far only the introduction of a predicate by one reduction pair or by one bilateral reduction sentence. But in most cases a predicate will be introduced by either several reduction pairs or several bilateral reduction sentences. If a property or physical magnitude can be determined by different methods then we may state one reduction pair or one bilateral reduction sentence for each method. The intensity of an electric current can be measured for instance by measuring the heat produced in the conductor, or the deviation of a magnetic needle, or the quantity of silver separated out of a solution, or the quantity of hydrogen separated out of water etc. We may state a set of bilateral reduction sentences, one corresponding to each of these methods. The factual content of this set is not null because it comprehends such sentences as e.g. "If the deviation of a magnetic needle is such and such then the quantity of silver separated in one minute is such and such, and vice versa" which do not contain the term 'intensity of electric current', and which obviously are synthetic.

If we establish one reduction pair (or one bilateral reduction sentence) as valid in order to introduce a predicate 'Q_3', the meaning of 'Q_3' is not established completely, but only for the cases in which the test condition is fulfilled. In other cases, e.g. for the match in our previous example, neither the predicate nor its negation can be attributed. We may diminish this region of indeterminateness of the predicate by adding one or several more laws which contain the predicate and connect it with other terms available in our language. These further laws may have the form of reduction sentences (as in the example of the electric current) or a different form. In the case of the predicate 'soluble in water' we may perhaps add the law stating that two bodies of the same substance are either both soluble or both not soluble. This law would help in the instance of the match; it would, in accordance with common usage, lead to the result "the match c is not soluble," because other pieces of wood are found to be insoluble on the basis of the first reduction sentence. Nevertheless, a region of indeterminateness remains, though a smaller one. If a body b consists of such a substance that for no body of this substance has the test-condition — in the above example: "being placed into water" — ever been fulfilled, then neither the predicate nor its negation can be attributed to b. This region may then be diminished still further, step by step, by stating new laws. These laws do not have the conventional character that definitions have;

rather are they discovered empirically within the region of meaning which the predicate in question received by the laws stated before. But these laws are extended by convention into a region in which the predicate had no meaning previously; in other words, we decided to use the predicate in such a way that these laws which are tested and confirmed in cases in which the predicate has a meaning, remain valid in other cases.

We have seen that a new predicate need not be introduced by a definition, but may equally well be introduced by a set of reduction pairs. (A bilateral reduction sentence may here be taken as a special form of a reduction pair.) Consequently, instead of the usual chain of definitions, we obtain a chain of sets of sentences, each set consisting either of one definition or of one or several reduction pairs. By each set a new predicate is introduced.

Definition 11. A (finite) chain of (finite) sets of sentences is called an *introductive chain* based upon the class C of predicates if the following conditions are fulfilled. Each set of the chain consists either of one definition or of one or more reduction pairs for one predicate, say 'Q'; every predicate occurring in the set, other than 'Q', either belongs to C or is such that one of the previous sets of the chain is either a definition for it or a set of reduction pairs for it.

Definition 12. If the last set of a given introductive chain based upon C either consists in a definition for 'Q' or in a set of reduction pairs for 'Q', 'Q' is said to be *introduced* by this chain on the basis of C.

For our purposes we will suppose that a reduction sentence always has the simple form '$Q_1 \supset (Q_2 \supset Q_3)$' and not the analogous but more complicated form '(x) [---x--- $\supset$ (... x... $\supset Q_3(x)$)]' where '---x---' and '...x...' indicate sentential functions of a non-atomic form. This supposition does not restrict the generality of the following considerations because a reduction sentence of the compound form indicated may always be replaced by two definitions and a reduction sentence of the simple form, namely by:

$Q_1 \equiv$ ---x---

$Q_2 \equiv$ ...x...

$Q_1 \supset (Q_2 \supset Q_3)$

The above supposition once made, the nature of an introductive chain is chiefly dependent upon the form of the definitions occurring. Therefore we define as follows.

Definition 13. An introductive chain is said to have atomic form (or molecular form) if every definition occurring in it has atomic form (or molecular form, respectively); it is said to have generalized form (or essentially generalized form) if at least one definition of generalized form (or essentially generalized form, respectively) occurs in it.

Theorem 7. If 'P' is introduced by an introductive chain based upon C, 'P' is reducible to C. If the chain has molecular form, 'P' is completely reducible to C; if the chain has essentially generalized form, 'P' is incompletely reducible to C. — This follows from Theorems 5 (§ 7) and 6 (§ 8).

We call *primitive symbols* those symbols of a language L which are introduced directly, i.e. without the help of other symbols. Thus there are the following kinds of symbols of L:

1) *primitive symbols* of L,
2) *indirectly introduced symbols,* i.e. those introduced by introductive chains based upon primitive symbols; here we distinguish:
   a) *defined symbols,* introduced by chains of definitions,
   b) *reduced symbols,* i.e. those introduced by introductive chains containing at least one reduction sentence; here we may further distinguish:
      α) *L-reduced symbols,* whose chains contain only L-reduction pairs,
      β) *P-reduced symbols,* whose chains contain at least one P-reduction pair.

Definition 14. a. An *introductive chain* based upon primitive predicates of a language L and having atomic (or molecular, or generalized, or essentially generalized, respectively) form is called an atomic (or molecular, or generalized, or essentially generalized, respectively) introductive chain of L.

b. A *predicate* of L is called an *atomic* (or *molecular)* predicate if it is either a primitive predicate of L or introduced by an atomic (or molecular, respectively) introductive chain of L; it is called a *generalized* (or essentially generalized) predicate if it is introduced by a generalized (or essentially generalized, respectively) introductive chain of L.

Definition 15. a. A sentence S is called an *atomic sentence* if S is a full sentence of an atomic predicate. — b. S is called a *molecular sentence* if S has molecular form and contains only molecular predicates. — c. S is called a *generalized sentence* if S contains an (unrestricted) operator or a generalized predicate. — d. S is called an essentially generalized sentence if S is a generalized sentence and is not equipollent with a molecular sentence.

It should be noticed that the term 'atomic sentence', as here defined, is not at all understood to refer to ultimate facts.[28] Our theory does not assume anything like ultimate facts. It is a matter of convention which predicates are taken as primitive predicates of a certain language L; and hence likewise, which predicates are taken as atomic predicates and which sentences as atomic sentences.

10. Reduction and Definition

In § 8 the fact was mentioned that in some cases, for instance in the case of a disposition-term, the reduction cannot be replaced by a definition. We now are in a position to see the situation more clearly. Suppose that we introduce a predicate 'Q' into the language of science first by a reduction pair and that, later on, step by step, we add more such pairs for 'Q' as our knowledge about 'Q' increases with further experimental investigations. In the course of this procedure the range of indeterminateness for 'Q', i.e. the class of cases for which we have not yet given a meaning to 'Q', becomes smaller and smaller. Now at each stage of this development we could lay down a definition for 'Q' corresponding to the set of reduction pairs for 'Q' established up to that stage. But, in stating the definition, we should have to make an arbitrary decision concerning the cases which are not determined by the set of reduction pairs. A definition determines the meaning of the new term once for all. We could either decide to attribute 'Q' in the cases not determined by the set, or to

[28] In contradistinction to the term 'atomic sentence' or 'elementary sentence' as used by Russell or Wittgenstein.

attribute '~Q' in these cases. Thus for instance, if a bilateral reduction sentence R of the form '$Q_1 \supset (Q_3 \equiv Q_2)$' is stated for '$Q_3$', then the predicate '$Q_3$' is to be attributed to the points of the class $Q_1 \bullet Q_2$, and '$\sim Q_3$' to those of the class $Q_1 \bullet \sim Q_2$, while for the points of the class $\sim Q_1$, the predicate 'Q_3' has no meaning. Now we might state one of the following two definitions:

(D_1) $$Q_3 \equiv (Q_1 \bullet Q_2)$$
(D_2) $$Q_3 \equiv (\sim Q_1 \vee Q_2)$$

If *c* is a point of the undetermined class, on the basis of D_1, '$Q_3(c)$' is false, and on the basis of D_2 it is true. Although it is possible to lay down either D_1 or D_2, neither procedure is in accordance with the intention of the scientist concerning the use of the predicate 'Q_3'. The scientist wishes neither to determine all the cases of the third class positively, nor all of them negatively; he wishes to leave these questions open until the results of further investigations suggest the statement of a new reduction pair; thereby some of the cases so far undetermined become determined positively and some negatively. If we now were to state a definition, we should have to revoke it at such a new stage of the development of science, and to state a new definition, incompatible with the first one. If, on the other hand, we were now to state a reduction pair, we should merely have to add one or more reduction pairs at the new stage; and these pairs will be compatible with the first one. In this latter case we do not correct the determinations laid down in the previous stage but simply supplement them.

Thus, if we wish to introduce a new term into the language of science, we have to distinguish two cases. If the situation is such that we wish to fix the meaning of the new term once for all, then a definition is the appropriate form. On the other hand, if we wish to determine the meaning of the term at the present time for some cases only, leaving its further determination for other cases to decisions which we intend to make step by step, on the basis of empirical knowledge which we expect to obtain in the future, then the method of reduction is the appropriate one rather than that of a definition. A set of reduction pairs is a partial determination of meaning only and can therefore not be replaced by a definition. Only if we reach, by adding more and more reduction pairs, a stage in which all cases are determined, may we go over to the form of a definition.

We will examine in greater detail the situation in the case of several reduction pairs for 'Q_3':

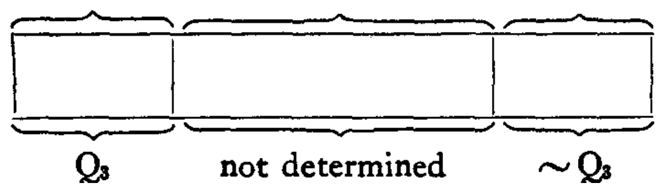
(R_1)	$Q_1 \supset (Q_2 \supset Q_3)$
(R_2)	$Q_4 \supset (Q_5 \supset {\sim}Q_3)$
(R'_1)	$Q'_1 \supset (Q'_2 \supset Q_3)$
(R'_2)	$Q'_4 \supset (Q'_5 \supset {\sim}Q_3)$ etc.

Then 'Q_3' is determined by R_1 for the points of the class $Q_1 \bullet Q_2$, by R'_1 for the class $Q'_1 \bullet Q'_2$, etc., and therefore, by the totality of reduction sentences for 'Q_3', for the class $(Q_1 \bullet Q_2) \vee (Q'_1 \bullet Q'_2) \vee \ldots$. This class may shortly be designated by '$Q_{1,2}$'. Analogously '$\sim Q_3$' is determined by the reduction sentences for '$\sim Q_3$' for the points of the class $(Q_4 \bullet Q_5) \vee (Q'_4 \bullet Q'_5) \vee \ldots$, which we designate by '$Q_{4,5}$'. Hence 'Q_3' is determined either positively or negatively for the class $Q_{1,2} \vee Q_{4,5}$. Therefore the universal sentence '$Q_{1,2} \vee Q_{4,5}$' means, that for every point either 'Q_3' or '${\sim}Q_3$' is determined. If this sentence is true, the set of reduction sentences is complete and may be replaced by the definition '$Q_3 \equiv Q_{1,2}$'. For the points of the class ${\sim}(Q_{1,2} \vee Q_{4,5})$, '$Q_3$' is not determined, and hence, in the stage in question, 'Q_3' is without meaning for these points. If on the basis of either logical rules or physical laws it can be shown that all points belong to this class, in other words, if the universal sentence '${\sim}(Q_{1,2} \vee Q_{4,5})$' is valid — either analytic or P-valid — then neither 'Q_3' nor '${\sim}Q_3$' is determined for any point and hence the given set of reduction pairs does not even partly determine the meaning of 'Q_3' and therefore is not a suitable means of introducing this predicate.

The given set of reduction pairs asserts that a point belonging to the class $Q_{4,5}$ has the property ${\sim}Q_3$ and hence not the property Q_3, and therefore cannot belong to $Q_{1,2}$ because every point of this class has the property Q_3. What the set asserts can therefore be formulated by the universal sentence saying that no point belongs to both $Q_{1,2}$ and $Q_{4,5}$, i.e. the sentence '$\sim(Q_{1,2} \bullet Q_{4,5})$'. This sentence represents, so to speak, the factual content of the set. In the case of one reduction pair this representative sentence is '$\sim(Q_1 \bullet Q_2 \bullet Q_4 \bullet Q_5)$'; in the case of one bilateral reduction sentence this becomes '$\sim (Q_1 \bullet Q_2 \bullet Q_1 \bullet \sim Q_2)$' or '$(x)(Q_2(x) \vee \sim Q_2(x))$', which is analytic.

The following diagram shows the tripartition of the class of all points by a reduction pair (or a bilateral reduction sentence, or a set of reduction pairs, respectively). For the first class 'Q_3' is determined, for the second class '$\sim Q_3$'. The third class lies between them and is not yet determined; but some of its points may be determined as belonging to Q_3 and some others as belonging to $\sim Q_3$ by reduction pairs to be stated in the future.

reduction pair:	$Q_1 \bullet Q_2$	$\sim[(Q_1 \bullet Q_2) \vee (Q_4 \bullet Q_5)]$	$Q_4 \bullet Q_5$
bilat. reduction sentence:	$Q_1 \bullet Q_2$	$\sim Q_1$	$Q \bullet \sim Q_2$
set of reduction pairs:	$Q_{1,2}$	$\sim(Q_{1,2} \vee Q_{4,5})$	$Q_{4,5}$



If we establish a set of *reduction pairs* as new valid sentences for the introduction of a new predicate 'Q'_3', are these valid sentences *analytic or P-valid?* Moreover, which other sentences containing 'Q_3' are analytic? The distinction between analytic and P-valid sentences refers primarily to those sentences only in which all descriptive terms are primitive terms. In this case the criterion is as follows:[29] a valid sentence S is analytic if and only if every sentence S′ is also valid which is obtained from S when any descriptive term wherever it occurs in S is replaced by any other term whatever of the same type; otherwise it is P-valid. A sentence S containing defined terms is analytic if the sentence S′ resulting from S by the elimination of the defined terms is analytic; otherwise it is P-valid. A definition, e.g. '$Q(x) \equiv . . . x ...$'

[29] Carnap [4] §51.

is, according to this criterion, itself analytic; for, after it has been stated as a valid sentence, by the elimination of 'Q' we get from it '…x …≡…x…', which is analytic.

In the case of a new descriptive term introduced by a set of reduction pairs, the situation is not as simple as in the case of a definition because elimination is here not possible. Let us consider the question how the criterion is to be stated in this case. The introduction of a new term into a language is, strictly speaking, the construction of a new language on the basis of the original one. Suppose that we go over from the language L_1, which does not contain 'Q', to the language L_2 by introducing 'Q' by a set R of reduction pairs, whose representative sentence (in the sense explained before) may be taken to be S. Then S as not containing 'Q' is a sentence of L_1 also; its logical character within L_1 does not depend upon 'Q' and may therefore be supposed to be determined already. By stating the sentences of R as valid in L_2, S becomes also valid in L_2 because it is a consequence of R in L_2. If now S is analytic in L_1, it is also analytic in L_2; in this case R does not assert anything about facts, and we must therefore take its sentences as analytic. According to this, every bilateral reduction sentence is analytic, because its representative sentence is analytic, as we have seen before. If S is either P-valid or indeterminate in L_1, it is valid and moreover P-valid in L_2 in consequence of our stating R as valid in L_2. In this case every sentence of R is valid; it is P-valid unless it fulfills the general criterion of analyticity stated before (referring to all possible replacements of the descriptive terms, see p. 451). If S is either P-contravalid or contradictory in L_1, it has the same property in L_2 and is simultaneously valid in L_2. It may be analytic in L_2, if it fulfills the general criterion. In this case every sentence of R is both valid and contravalid, and hence L_2 is inconsistent.[30] If S is contradictory in L_1 and at least one sentence of R is analytic according to the general criterion, then L_2 is not only inconsistent but also L-inconsistent. The results of these considerations may be exhibited by the following table; column (1) gives a complete classification of the sentences of a language (see the diagram in § 5).

[30] Compare Carnap [4] §59.

The representative sentence S		a reduction in sentence of R (in L_2)	L_2
in L_1	in L_2		
1. analytic	analytic	analytic	consistent (if L_1 is consistent)
2. P-valid	P-valid	valid*	
3. indeterminate	P-valid	valid*	
4. P-contravalid	valid and P-contravalid	valid* and P-contravalid	inconsistent
5. contradictory	valid and contradictory	valid* and contradictory	inconsistent†

* analytic if fulfilling the general criterion (p. 451); otherwise P-valid.

† and moreover L-inconsistent if at least one sentence of R is analytic on the basis of the general criterion (p. 451).

Now the *complete criterion for 'analytic'* can be stated as follows:

Nature of S	Criterion for S being *analytic*
1. S does not contain any descriptive symbol.	S is valid.
2. All descriptive symbols of S are primitive.	Every sentence S′ which results from S when we replace any descriptive symbol at all places where it occurs in S by any symbol whatever of the same type—and hence S itself also—is valid.
3. S contains a defined descriptive symbol 'Q'.	The sentence S′ resulting from S by the elimination of 'Q' is valid.
4. S contains a descriptive symbol 'Q' introduced by a set R of reduction pairs; let L′ be the sub-language of L not containing 'Q', and S′ the representative sentence of R (comp. p. 451)	S′ is analytic in L′, and S is an L-consequence of R (e.g. one of the sentences of R); in other words, the implication sentence containing the conjunction of the sentences of R as first part and S as second part is analytic (i.e. every sentence resulting from this implication sentence where we replace 'Q' at all places by any symbol of the same type occurring in L′ is valid in L′).

III. EMPIRICAL ANALYSIS OF CONFIRMATION AND TESTING

II. Observable and Realizable Predicates

In the preceding chapter we analyzed logically the relations which subsist among sentences or among predicates if one of them may be confirmed with the help of others. We defined some concepts of a syntactical kind, based upon the concept ‘consequence’ as the chief concept of logical syntax. In what follows we shall deal with *empirical methodology.* Here also we are concerned with the questions of confirming and testing sentences and predicates. These considerations belong to a theory of language just as the logical ones do. But while the logical analysis belongs to an analytic theory of the formal, syntactical structure of language, here we will carry out an empirical analysis of the application of language. Our considerations belong, strictly speaking, to a biological or psychological theory of language as a kind of human behavior, and especially as a kind of reaction to observations. We shall see, however, that for our purposes we need not go into details of biological or psychological investigations. In order to make clear what is understood by empirically testing and confirming a sentence and thereby to find out what is to be required for a sentence or a predicate in a language having empirical meaning, we can restrict ourselves to using very few concepts of the field mentioned. We shall take two descriptive, i.e. non-logical, terms of this field as *basic terms* for our following considerations, namely *‘observable’* and ‘*realizable’.* All other terms, and above all the terms ‘confirmable’ and ‘testable’, which are the chief terms of our theory, will be defined on the basis of the two basic terms mentioned; in the definitions we shall make use of the logical terms defined in the foregoing chapter. The two basic terms are of course, as basic ones, not defined within our theory. Definitions for them would have to be given within psychology, and more precisely, within the behavioristic theory of language.We do not attempt such definitions, but we shall give at least some rough explanations for the terms, which will make their meaning clear enough for our purposes.

Explanation 1. A predicate ‘P’ of a language L is called *observable* for an organism (e.g. a person) N, if, for suitable arguments, e.g. ‘b’, N is able under suitable circumstances to come to a decision with the help of few observations about a full sentence, say ‘P(b)’, i.e. to a confirmation of either ‘P(b)’ or ‘~P(b)’ of such a high degree that he will either accept or reject ‘P(b)’.

This explanation is necessarily vague. There is no sharp line between observable and non-observable predicates because a person will be more or less able to decide a certain sentence quickly, i.e. he will be inclined after a certain period of observation to accept the sentence. For the sake of simplicity we will here draw a sharp distinction between observable and non-observable predicates. By thus drawing an arbitrary line between observable and non-observable predicates in a field of continuous degrees of observability we partly determine in advance the possible answers to questions such as whether or not a certain predicate is observable by a given person. Nevertheless the general philosophical, i.e. methodological question about the nature of meaning and testability will, as we shall see, not be distorted by our oversimplification. Even particular questions as to whether or not a given sentence is confirmable, and whether or not it is testable by a certain person, are affected, as we shall see, at most to a very small degree by the choice of the boundary line for observable predicates.

According to the explanation given, for example the predicate ‘red’ is observable for a person N possessing a normal colour sense. For a suitable argument, namely a space-time-point *c* sufficiently near to N, say a spot on the table before N, N is able under suitable circumstances — namely, if there is sufficient light at *c* — to come to a decision about the full sentence “the spot *c* is red” after few observations — namely by looking at the table. On the other hand, the predicate ‘red’ is not observable by a colour-blind person. And the predicate ‘an electric field of such and such an amount’ is not observable to anybody, because, although we know how to test a full sentence of this predicate, we cannot do it directly, i.e. by a few observations; we have to apply certain instruments and hence to make a great many preliminary observations in order to find out whether the things before us are instruments of the kind required.

Explanation 2. A predicate ‘P’ of a language L is called '*realizable*' by N, if for a suitable argument, e.g. 'b', N is able under suitable circumstances to make the full sentence 'P(b)' true, i.e. to produce the property P at the point b.

When we use the terms 'observable' and 'realizable' without explicit reference to anybody, it is to be understood that they are meant with respect to the people who use the language L to which the predicate in question belongs.

Examples. Let 'P_1(b)' mean: 'the space-time-point b has the temperature 100°C'. 'P_1' is realizable by us because we know how to produce that temperature at the point b, if b is accessible to us. — 'P_2(b)' may mean: 'there is iron at the point b'. 'P_2' is realizable because we are able to carry a piece of iron to the point b if b is accessible. — If 'P_3(b)' means: 'at the point b is a substance whose index of light refraction is 10', 'P_3' is not realizable by anybody at the present time, because nobody knows at present how to produce such a substance.

12. Confirmability

In the preceding chapter we have dealt with the concept of reducibility of a predicate 'P' to a class C of other predicates, i.e. the logical relation which subsists between 'P' and C if the confirmation of 'P' can be carried out by that of predicates of C. Now, if confirmation is to be feasible at all, this process of referring back to other predicates must terminate at some point. The reduction must finally come to predicates for which we can come to a confirmation directly, i.e. without reference to other predicates. According to Explanation i, the observable predicates can be used as such a basis. This consideration leads us to the following definition of the concept 'confirmable'. This concept is a descriptive one, in contradistinction to the logical concept 'reducible to C' — which could be named also 'confirmable with respect to C'.

Definition 16. A *sentence* S is called *confirmable* (or completely confirmable, or incompletely confirmable) if the confirmation of S is reducible (or completely reducible, or incompletely reducible, respectively) to that of a class of observable predicates.

Definition 17. A sentence S is called *bilaterally confirmable* (or bilaterally completely confirmable) if both S and ~ S are confirmable (or completely confirmable, respectively).

Definition 18. A *predicate* 'P' *is* called *confirmable* (or completely confirmable, or incompletely confirmable) if 'P' is reducible (or completely reducible, or incompletely reducible, respectively) to a class of observable predicates.

Hence, if 'P' is confirmable (or completely confirmable) the full sentences of 'P' are bilaterally confirmable (or bilaterally completely confirmable, respectively).

When we call a sentence S confirmable, we do not mean that it is possible to arrive at a confirmation of S under the circumstances as they actually exist. We rather intend this possibility under some *possible circumstances,* whether they be real or not. Thus e.g. because my pencil is black and I am able to make out by visual observation that it is black and not red, I cannot come to a positive confirmation of the sentence "My pencil is red." Nevertheless we call this sentence confirmable and moreover completely confirmable for the reason that we are able to indicate the — actually non-existent, but possible — observations which would confirm that sentence. Whether the real circumstances are such that the testing of a certain sentence S leads to a positive result, i.e. to a confirmation of S, or such that it leads to a negative result, i.e. to a confirmation of ~ S, is irrelevant for the questions of confirmability, testability and meaning of the sentence though decisive for the question of truth i.e. sufficient confirmation.

Theorem 8. If 'P' is introduced on the basis of observable predicates, 'P' is confirmable. If the introductive chain has molecular form, 'P' is completely confirmable. — This follows from Theorem 7 (§ 9).

Theorem 9. If S is a sentence of molecular form and all predicates occurring in S are confirmable (or completely confirmable) S is bilaterally confirmable (or bilaterally completely confirmable, respectively). — From Theorem 2 (§ 6).

Theorem 10. If the sentence S is constructed out of confirmable predicates with the help of connective symbols and universal or existential operators, S is bilaterally confirmable. — From Theorems 2, 3, and 4 (§ 6).

If 'P' is confirmable then it is not impossible that for a suitable point b we may find a confirmation of 'P(b)' or of '~ P(b)'. But it is not necessary that we know a method for finding such a confirmation. If such a procedure can be given — we may call it a *method of testing* — then 'P' is not only confirmable but — as we shall say later on — testable. The following considerations will deal with the question how to formulate a method of testing and thereby will lead to a definition of 'testable'.

The description of a method of testing for 'Q_3' has to contain two other predicates of the following kinds:

1) A predicate, say 'Q_1', describing a *test-condition* for 'Q_3,' i.e. an experimental situation which we have to create in order to test 'Q_3' at a given point.

2) A predicate, say 'Q_2', describing a *truth-condition* for 'Q_3' with respect to 'Q_1', i.e. a possible experimental result of the test-condition Q_1 at a given point *b* of such a kind that, if this result occurs, 'Q_3' is to be attributed to *b*. Now the connection between 'Q_1', 'Q_2', and 'Q_3' is obviously as follows: if the test-condition is realized at the given point *b* then, if the truth-condition is found to be fulfilled at *b, b* has the property to be tested; and this holds for any point. Thus the method of testing for 'Q_3' is to be formulated by the universal sentence '$Q_1 \supset (Q_2 \supset Q_3)$', in other words, by a reduction sentence for 'Q_3'. But this sentence, beside being a reduction sentence, must fulfill the following two additional requirements:

1) 'Q_1' must be realizable because, if we did not know how to produce the test-condition, we could not say that we had a method of testing.

2) We must know beforehand how to test the truth condition Q_2; otherwise we could not test 'Q_3' although it might be confirmable. In order to satisfy the second requirement, 'Q_2' must be either observable or explicitly defined on the basis of observable predicates or a method of testing for it must have been stated. If we start from observable predicates — which, as we know, can be tested without a description of a method of testing being necessary — and then introduce other predicates by explicit

definitions or by such reduction sentences as fulfill the requirements stated above and hence are descriptions of a method of testing, then we know how to test each of these predicates. Thus we are led to the following definitions.

Definition 19. An introductive chain of such a kind that in each of its reduction sentences, say '$Q_1 \supset (Q_2 \supset Q_3)$' or '$Q_4 \supset (Q_5 \supset \sim Q_3)$', the first predicate — 'Q_1' or 'Q_4', respectively — is realizable, is called a *test chain.* A reduction sentence (or a reduction pair, or a bilateral reduction sentence) belonging to a test chain is called a *test sentence* (or a *test pair,* or a *bilateral test sentence,* respectively).

A test pair for 'Q', and likewise a bilateral test sentence for 'Q', describes a method of testing for both 'Q' and '~ Q'. A bilateral test sentence, e.g. '$Q_1 \supset (Q_3 \equiv Q_2)$' may be interpreted in words in the following way: "If at a space-time-point x the test-condition Q_1 (consisting perhaps in a certain experimental situation, including suitable measuring instruments) is realized then we will attribute the predicate 'Q_3' to the point *x* if and only if we find at *x* the state Q_2 (which may be a certain result of the experiment, e.g. a certain position of the pointer on the scale)". To give an example, let '$Q_3(b)$' mean: "The fluid at the space-time-point *b* has a temperature of 100°"; '$Q_1(b)$': "A mercury thermometer is put at *b;* we wait, while stirring the liquid, until the mercury comes to a standstill"; '$Q_2(b)$': "The head of the mercury column of the thermometer at *b* stands at the mark 100 of the scale." If here 'Q_3' is introduced by '$Q_1 \supset (Q_3 \equiv Q_2)$' obviously its testability is assured.

14. Testability

Definition 20. If a predicate is either observable or introduced by a test chain it is called *testable.* A testable predicate is called *completely testable* if it is either observable or introduced by a test chain having molecular form; otherwise *incompletely testable.*

Let us consider the question under what conditions a set of laws, e.g. of physics, which contain a predicate 'Q' can be transformed into a set of reduction-sentences or of test-sentences for 'Q'. Suppose a set of laws is given which contain 'Q' and have the following form. Each of the laws is a universal sentence containing only individual variables (no predicate variables); 'Q' is followed wherever it occurs in the sentence by the same set of variables, which are bound by universal operators applying to the whole sentence. Thus each of the laws has the form '(x) [...Q(x) ... Q(x) ...]'. The majority of the laws of classical physics can be brought into this form. Now the given set of laws can be transformed in the following way. First we write down the conjunction of the laws of the given set and transform it into one universal sentence '(x)[... Q(x) ... Q(x) ...]'. Then we transform the function included in square brackets into the so-called conjunctive normal form,[31] i.e. a conjunction of say *n* disjunctions of such a kind that 'Q' occurs only in partial sentences which are members of such disjunctions and have either the form 'Q(x)' or '~Q(x)'. Finally we dissolve the whole universal sentence into *n* universal sentences in accordance with the rule that '$(x)[P_1(x) \bullet P_2(x) \bullet \ldots \bullet P_n(x)]$' can be transformed into '$(x)P_1(x) \bullet (x)P_2(x) \ldots . (x)P_n(x)$'. Thus we have a set of *n* universal sentences; each of them is a disjunction having among its members either 'Q(x)' or '~Q(x)' or both. If we employ '~ P(x)' as abbreviation for the disjunction of the remaining members not containing 'Q' these sentences have one of the following forms:

1. $Q \lor \sim P$,
2. $\sim Q \lor \sim P$
3. $Q \lor \sim Q \lor \sim P$.

A sentence of the form (3) is analytic and can therefore be omitted without changing the content of the set. (1) can be given the form '$P \supset Q$' and, by analysing 'P' in some way or other into a conjunction '$P_1 \bullet P_2$', the form '$(P_1 \bullet P_2) \supset Q$' and hence '$P1 \supset (P_2 \supset Q)$' which is a reduction sentence of the first form. In the same way (2) can be transformed into '$P \supset \sim Q$' and hence into '$(P_1 \bullet P_2) \supset \sim Q$' and into '$P_1 \supset (P_2 \supset \sim Q)$' which is a reduction sentence of the second form. An analysis of 'P' into '$P_1 \bullet P_2$' is obviously always possible; if not otherwise then in the triv-

[31] Compare Hilbert [1] p. 13; Carnap [4b] §34b, RR 2.

ial way of taking an analytic predicate as 'P_1' and 'P' itself as 'P_2'. If 'P' is testable then we may look for such an analysis that 'P_1' is realizable. If we can find such a one then — since 'P_2' is also testable in this case — the reduction sentence '$P_1 \supset (P_2 \supset Q)$' or '$P_1 \supset (P_2 \supset Q)$' is a test-sentence.

Thus we have seen that a set of laws of the form here supposed can always be transformed into a set of reduction sentences for 'Q', and, if a special condition is fulfilled, into a set of test-sentences. This condition is fulfilled in very many and perhaps most of the cases actually occurring in physics because nearly all predicates used in physics are testable and perhaps most of them are realizable. —

Theorem 11. If a predicate is testable it is confirmable; if it is completely testable it is completely confirmable. — By Theorem 8, § 12.

On the other hand, 'P' may be *confirmable without being testable.* This is the case, if 'P' is introduced by an introductive chain based upon observable predicates but containing a reduction sentence ('$Q_1 \supset (Q_2 \supset Q_3)$' of such a kind that '$Q_1$', although it is of course confirmable and may even be testable, is not realizable. If this should be the case, there is a possibility that by a happy chance the property Q_3 will be found at a certain point, although we have no method which would lead us with certainty to such a result. Suppose that 'Q_1' and 'Q_2' are completely confirmable, i.e. completely reducible to observable predicates — they may even be observable themselves — and that 'Q_3' is introduced by '$Q_1 \supset (Q_3 \equiv Q_2)$'. Let *c* be a point in our spatio-temporal neighborhood such that we are able to observe its properties. Then by happy chance '$Q_1(c)$' may be true. If so, we are able to find this out by observation and then, by either finding '$Q_2(c)$' or '$\sim Q_2(c)$', to arrive at the conclusion either of '$Q_3(c)$' or of '$\sim Q_3(c)$'. But if that stroke of luck does not happen, i.e. if '$Q_1(c)$' is false — no matter whether we find that out by our observations or not — we are not in a position to determine the truth or falsehood of '$Q_3(c)$', and it is impossible for us to come to a confirmation of either '$Q_3(c)$' or '$\sim Q_3(c)$' in any degree whatsoever. To give an example, let '$Q_1(c)$' mean that at the space-time point *c* there is a person with a certain disease. We suppose that we know symptoms both for the occurrence of this disease as well as for its non-occurrence; hence 'Q_1' is confirmable. It may even be the case that we know a method by which we are able to find out with certainty whether or not a given person at a given time has this disease; if we know such a method 'Q_1' is not only confirmable but testable and moreover completely testable. We will suppose, however, that 'Q_1' is not realizable, i.e. we do not know at present any method of producing this disease; whether or not '$\sim Q_1$' is realizable, in other words, whether or not we are able to cure the disease, does not matter for our considerations. Let us suppose further that clinical observations of the cases of this disease show that there are two classes of such cases, one characterized by the appearance of a certain symptom, i.e. a testable or even observable predicate, say 'Q_2', the other by the lack of this symptom, i.e. by '$\sim Q_2$'. If this distinction turns out to be relevant for the further development of the disease and for its consequences, physicians may wish to classify all persons into two classes: those who are disposed to show the symptom Q_2 in case they acquire the disease Q_1, and those who do not, i.e. those who show $\sim Q_2$ if they get Q_1. The first class may be designated by 'Q_3' and hence the second by '$\sim Q_3$'. Then 'Q_3' can be introduced by the bilateral reduction sentence '$Q_1 \supset (Q_3 \equiv Q_2)$'. The classification by 'Q_3' and '$\sim Q_3$' will be useful if observations of a long series of cases of this disease show that a person who once belongs to the class Q_3 (or $\sim Q_3$) always belongs to this class. Moreover, other connections between Q_3 and other biological properties may be discovered; these connections will then be formulated by laws containing 'Q_3'; under suitable circumstances these laws can be given the form of supplementary reduction pairs for 'Q_3'. Thus 'Q_3' may turn out to be a useful and important concept for the formulation of the results of empirical investigation. But 'Q_3' is not testable, not even incompletely, because we do not know how to decide a given sentence '$Q_3(a)$', i.e. how to make experiments in order to find out whether a given person belongs to the class Q_3 or not; all we can do is to wait until this person happens to get the disease Q_1 and then to find out whether he shows the symptom Q_2 or not. It may happen, however, in the further development of our investigations, that we find that every person for whom we find 'Q_1' and 'Q_2' and to whom we therefore attribute 'Q_3' shows a certain constant testable property Q_4, e.g. a certain chemical property of the blood, and that every person for whom we find 'Q_1' and '$\sim Q_2$' and whom we therefore classify into $\sim Q_3$, does not show Q_4. On the basis of such results we would state the law '$Q_3 \equiv Q_4$'. By this law, 'Q_3' becomes synonymous — not L-synonymous, but P-synonymous — with the testable predicate 'Q_4' and hence becomes itself testable. But until we are in a position to state a law of this or a similar kind, 'Q_3' is not testable.

This example shows that a non-testable predicate can nevertheless be confirmable, and even completely confirmable, and its introduction and use can be helpful for the purposes of empirical scientific investigation.

Definition 2 1. If a *sentence* S is confirmable (or completely confirmable) and all predicates occurring in S are testable (or completely testable), S is called *testable* (or completely testable, respectively). If S is testable but not completely testable it is called incompletely testable. If S is bilaterally confirmable (or bilaterally completely confirmable) and all predicates occurring in it are testable (or completely testable), S is called *bilaterally testable* (or bilaterally completely testable, respectively).

Theorem 12. If S is a full sentence of a testable (or completely testable) predicate, S is bilaterally testable (or bilaterally completely testable, respectively).

Theorem 13. If S is a sentence of molecular form and all predicates occurring in S are testable (or completely testable) S is bilaterally testable (or bilaterally completely testable, respectively). — By Theorem 11 and 9 (§ 12).

Theorem 14. If the sentence S is constructed out of testable predicates with the help of connective symbols and universal or existential operators, S is bilaterally testable. — From Theorem 11 and 10 (§ 12).

15. A Remark about Positivism and Physicalism

One of the fundamental theses of *positivism* may perhaps be formulated in this way: every term of the whole language L of science is reducible to what we may call sense-data terms or perception terms. By a perception term we understand a predicate 'P' such that 'P(b)' means: "the person at the space-time-place *b* has a perception of the kind P". (Let us neglect here the fact that the older positivism would have referred in a perception sentence not to a space-time-place, but to an element of "consciousness"; let us here take the physicalistic formulation given above.) I think that this thesis is true if we understand the term 'reducible' in the sense in which we have defined it here. But previously reducibility was not distinguished from definability. Positivists therefore believed that every descriptive term of science could be defined by perception terms, and hence, that every sentence of the language of science could be translated into a sentence about perceptions. This opinion is also expressed in the former publications of the Vienna Circle, including mine of 1928 (Carnap [I]), but I now think, that it is not entirely adequate. Reducibility can be asserted, but not unrestricted possibility of elimination and re-translation; the reason being that the method of introduction by reduction pairs is indispensable.

Because we are here concerned with an important correction of a widespread opinion let us examine in greater detail the reduction and retranslation of sentences as positivists previously regarded them. Let us take as an example a simple sentence about a physical thing:

(1) "On May 6, 1935, at 4 P.M., there is a round black table in my room."

According to the usual positivist opinion, this sentence can be translated into the conjuncton of the following conditional sentences (2) about (possible) perceptions. (For the sake of simplicity we eliminate in this example only the term "table" and continue to use in these sentences some terms which are not perception terms e.g. "my room", "eye" etc., which by further reduction would have to be eliminated also.)

(2a) "If on May … somebody is in my room and looks in such and such direction, he has a visual perception of such and such a kind."

(2a′), (2a″), etc. Similar sentences about the other possible aspects of the table.
(2b) "If … somebody is in my room and stretches out his hands in such and such a direction, he has touch perceptions of such and such a kind."
(2b′), (2b″), etc. Similar sentences about the other possible touchings of the table.
(2c) etc. Similar sentences about possible perceptions of other senses.

It is obvious that no single one of these sentences (2) nor even a conjunction of some of them would suffice as a translation of (1); we have to take the whole series containing all possible perceptions of that table. Now the first difficulty of this customary positivistic reduction consists in the fact that it is not certain that the series of sentences (2) is finite. If it is not, then there exists no conjunction of them; and in this case the original sentence (1) cannot be translated into one perception sentence. But a more serious objection is the following one. Even the whole class of sentences (2) — no matter whether it be finite or infinite — is not equipollent with (1), because it may be the case that (1) is false, though every single sentence of the class (2) is true. In order to construct such a case, suppose that at the time stated there is neither a round black table in my room, nor any observer at all. (1) is then obviously false. (2a) is a universal implication sentence:

"(x) [(x is … in my room and looks …) ⊃ (x perceives …)]",

which we may abbreviate in this way:

(3) $$(x)\ [P(x) \supset Q(x)]$$

which can be transformed into

(4) $$(x)\ [\sim P(x) \vee Q(x)]$$

((2) can be formulated in words in this way: "For anybody it is either not the case that he is in my room on May… and looks … or he has a visual perception of such and such a kind".) Now, according to our assumption, for every person *x* it is false that *x* is at that time in my room and looks ... ; in symbols:

(5) $$(x)(\sim P(x)).$$

Therefore (4) is true, and hence (2a) also, and analogously every one of the other sentences of the class (2), while (1) is false. In this way the positivistic reduction in its customary form is shown to be invalid. The example dealt with is a sentence about a directly perceptible thing. If we took as examples sentences about atoms, electrons, electric field and the like, it would be even clearer that the positivistic translation into perception terms is not possible.

Let us look at the consequences which these considerations have for the construction of a scientific language on a positivistic basis, i.e. with perception terms as the only primitive terms. The most important consequence concerns the method of introduction of further terms. In introducing simple terms of perceptible things (e.g. 'table') and *a fortiori* the abstract terms of scientific physics, we must not restrict the introductive method to definitions but must also use reduction. If we do this the positivistic thesis concerning reducibility above mentioned can be shown to be true.

Let us give the name *'thing-language'* to that language which we use in every-day life in speaking about the perceptible things surrounding us. A sentence of the thing-language describes things by stating their observable properties or observable relations subsisting between them. What we have called observable predicates are predicates of the thing-language. (They have to be clearly distinguished from what we have called perception terms; if a person sees a round red spot on the table the perception term 'having a visual perception of something round and red' is attributed to the person while the observable predicate 'round and red' is attributed to the space-time point on the table.) Those predicates of the thing-language which are not observable, e.g. disposition terms, are reducible to observable predicates and hence confirmable. We have seen this in the example of the predicate 'soluble' (§ 7).

Let us give the name *'physical language'* to that language which is used in physics. It contains the thing-language and, in addition, those terms of a scientific terminology which we need for a scientific description of the processes in inorganic nature. While the terms of the thing-language for the most part serve only for a qualitative description of things, the other terms of the physical language are designed increasingly for a quantitative description. For every term of the physical language physicists know how to use it on the basis of their observations. Thus every such term is reducible to observable predicates and hence confirmable. Moreover, nearly every such term is testable, because for every term — perhaps with the exception of few terms considered as preliminary ones — physicists possess a method of testing; for the quantitative terms this is a method of measurement.

The so-called thesis of *Physicalism*[32] asserts that every term of the language of science — including beside the physical language those sub-languages which are used in biology, in psychology, and in social science — is reducible to terms of the physical language. Here a remark analogous to that about positivism has to be made. We may assert reducibility of the terms, but not — as was done in our former publications — definability of the terms and hence translatability of the sentences.

In former explanations of physicalism we used to refer to the physical language as a basis of the whole language of science. It now seems to me that what we really had in mind as such a basis was rather the thing-language, or, even more narrowly, the observable predicates of the thing-language. In looking for a new and more correct formulation of the thesis of physicalism we have to consider the fact mentioned that the method of definition is not sufficient for the introduction of new terms. Then the question remains: can every term of the language of science be introduced on the basis of observable terms of the thing-language by using only definitions and test-sentences, or are reduction sentences necessary which are not test sentences? In other words, which of the following formulations of the thesis of physicalism is true?

[32] Comp. Neurath [1], [2], [3]; Carnap [2], [8].

1. Thesis of Physicalistic Testability: "Every descriptive predicate of the language of science is testable on the basis of observable thing-predicates."

2. Thesis of Physicalistic Confirmability: "Every descriptive predicate of the language of science is confirmable on the basis of observable thing-predicates."

If we had been asked the question at the time when we first stated physicalism, I am afraid we should perhaps have chosen the first formulation. Today I hesitate to do this, and I should prefer the weaker formulation (2). The reason is that I think scientists are justified to use and actually do use terms which are confirmable without being testable, as the example in § 14 shows.

We have sometimes formulated the thesis of physicalism in this way: "The language of the whole of science is a physicalistic language." We used to say: a language L is called a physicalistic language if it is constructed out of the physical language by introducing new terms. (The introduction was supposed to be made by definition; we know today that we must employ reduction as well.) In this definition we could replace the reference to the physical language by a reference to the thing-language or even to the observable predicates of the thing-language. And here again we have to decide whether to admit for the reduction only test-chains or other reduction chains as well; in other words, whether to define 'physicalistic language' as 'a language whose descriptive terms are testable on the basis of observable thing-predicates' or '. . . are confirmable . . .'.

16. Sufficient Bases

A class C of descriptive predicates of a language L such that every descriptive predicate of L is reducible to C is called a *sufficient reduction basis* of L; if in the reduction only definitions are used, C is called a *sufficient definition basis.* If C is a sufficient reduction basis of L and the predicates of C — and hence all predicates of L — are confirmable, C is called a *sufficient confirmation basis* of L; and if moreover the predicates of C are completely testable, for instance observable, and every predicate of L is reducible to C by a test chain — and hence is testable — C is called a *sufficient test basis* of L.

As we have seen, positivism asserts that the class of perception terms is a sufficient basis for the language of science; physicalism asserts the same for the class of physical terms, or, in our stronger formulation, for the class of observable thing-predicates. Whether positivism and physicalism are right or not, at any rate it is clear that there can be several and even mutually exclusive bases. The classes of terms which positivism and physicalism assert to be sufficient bases, are rather comprehensive. Nevertheless even these bases are not sufficient definition bases but only sufficient reduction bases. Hence it is obvious that, if we wish to look for narrower sufficient bases, they must be reduction bases. We shall find that there are sufficient reduction bases of the language of science which have a far narrower extension than the positivistic and the physicalistic bases.

Let L be the physical language. We will look for sufficient reduction bases of L. If physicalism is right, every such basis of L is also a basis of the total scientific language; but here we will not discuss the question of physicalism. We have seen that the class of the observable predicates is a sufficient reduction basis of L. In what follows we will consider only bases consisting of observable predicates; hence they are *confirmation bases of the physical language* L. Whether they are also test bases depends upon whether all confirmable predicates of L are also testable; this question may be left aside for the moment. The visual sense is the most important sense; and we can easily see that it is sufficient for the confirmation of any physical property. A deaf man for instance is able to determine pitch, intensity and timbre of a physical sound with the help of suitable instruments; a man without the sense of smell can determine the olfactory properties of a gas by chemical analysis; etc. That all physical functions (temperature, electric field etc.) can be determined by the visual sense alone is obvious. Thus we see that the predicates of the visual sense, i.e. the colour-predicates as functions of space-time-places, are a sufficient confirmation basis of the physical language L.

But the basis can be restricted still more. Consider a man who cannot perceive colours, but only differences of brightness. Then he is able to determine all physical properties of things or events which we can determine from photographs; and that means, all properties. Thus he determines e.g. the colour of a light with the help of a spectroscope or a spectrograph. Hence the class of predicates which state the degree of brightness at a space-time-place — or the class consisting of the one functor[33] whose value is the degree of brightness — is a sufficient basis of L.

Now imagine a man who's visual sense is still more restricted. He may be able to distinguish neither the different colours nor the different degree of brightness, but only the two qualities bright and dark (= not bright) with their distribution in the visual field. What he perceives corresponds to a bad phototype which shows no greys but only black and white. Even this man is able to accomplish all kinds of determinations necessary in physics. He will determine the degree of brightness of a light by an instrument whose scale and pointer form a black-white-picture. Hence the one predicate 'bright' is a sufficient basis of L.

But even a man who is completely blind and deaf, but is able to determine by touching the spatial arrangements of bodies, can determine all physical properties. He has to use instruments with palpable scale-marks and a palpable pointer (such e.g. as watches for the blind). With such a spectroscope he can determine the colour of a light; etc. Let 'Solid' be a predicate such that 'Solid(b)' means: "There is solid matter at the space-time-point b". Then this single predicate 'Solid' is a sufficient basis of L.

Thus we have found several very narrow bases which are sufficient confirmation bases for the physical language and simultaneously sufficient test bases for the testable predicates of the physical language. And, if physicalism is right, they are also sufficient for the total language of science. Some of these bases consist of one predicate only. And obviously there are many more sufficient bases of such a small extent. This result will be relevant for our further considerations. It may be noticed that this result cannot at all be anticipated a priori; neither the fact of the existence of so small sufficient bases nor the fact that just the predicates mentioned are sufficient, is a logical necessity.

[33] Compare Carnap [4] §3.

Reducibility depends upon the validity of certain universal sentences, and hence upon the system of physical laws; thus the facts mentioned are special features of the structure of that system, or — expressed in the material idiom — special features of the causal structure of the real world. Only after constructing a system of physics can we determine what bases are sufficient with respect to that system.

(To be continued)

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