The Logical Structure of the World

Pseudoproblems in Philosophy

THE LOGICAL STRUCTURE OF THE WORLD

‘|’

PSEUDOPROBLEMS IN PHILOSOPHY

BY RUDOLF CARNAP

Translated by
Rolf A. George

UNIVERSITY OF CALIFORNIA PRESS
BERKELEY AND LOS ANGELES 1967

University of California Press, Berkeley and Los Angeles, California
First published in German under the title Der Logische Aufbau Der Well

Library of Congress Catalog Card Number: 66-13604
Printed in the United States of America

PREFACE TO THE SECOND EDITION

Der Logische Aufbau der Welt was my first larger book, the first attempt to bring into systematic form my earlier philosophical reflections. The first version was written in the years 1922-1925. When I read the old formulations today, I find many a passage which I would now phrase differently or leave out altogether; but I still agree with the philosophical orientation which stands behind this book. This holds especially for the problems that are posed, and for the essential features of the method which was employed. The main problem concerns the possibility of the rational reconstruction of the concepts of all fields of knowledge on the basis of concepts that refer to the immediately given. By rational reconstruction is here meant the searching out of new definitions for old concepts. The old concepts did not ordinarily originate by way of deliberate formulation, but in more or less unreflected and spontaneous development. The new definitions should be superior to the old in clarity and exactness and above all, should fit into a systematic structure of concepts. Such a clarification of concepts, nowadays frequently called "explication", still seems to me one of the most important tasks of philosophy, especially if it is concerned with the main categories of human thought.

For a long time, philosophers of various persuasions have held the view that all concepts and judgments result from the coöperation of experience and reason. Basically, empiricists and rationalists agree in this view, even though both sides give a different estimation of the relative importance of the two factors, and obscure the essential agreement by carrying their viewpoints to extremes. The thesis which they have in common is frequently stated in the following simplified version: The senses provide the material of cognition, reason synthesizes the material so as to produce an organized system of knowledge. There arises then the problem of finding a synthesis of traditional empiricism and traditional rationalism. Traditional empiricism rightly emphasized the contribution of the senses, but did not realize the importance and peculiarity of logical and mathematical forms. Rationalism was aware of this importance, but believed that reason could not only provide the form, but could by itself (a priori) produce new content. Through the influence of Gottlob Frege, under whom I studied in Jena, but who was not recognized as an outstanding logician until after his death, and through the study of Bertrand Russell's work, I had realized, on the one hand, the fundamental importance of mathematics for the formation of a system of knowledge and, on the other hand, its purely logical, formal character to which it owes its independence from the contingencies of the real world. These insights formed the basis of my book. Later on, through conversations in Schlick's circle in Vienna and through the influence of Wittgenstein's ideas they developed into the mode of thought which characterized the "Vienna Circle." This orientation is sometimes called "logical empiricism" (or "logical positivism"), in order to indicate the two components.

In this book I was concerned with the indicated thesis, namely that it is in principle possible to reduce all concepts to the immediately given. However, the problem which I posed for myself was not to add to the number of general philosophical arguments which had already been advanced in support of this thesis. Rather, I wanted to attempt, for the first time, the actual formulation of a conceptual system of the indicated sort; that is to say, I was going to choose, to begin with, some simple basic concepts, for instance sensory qualities and relations, which are present in the raw material of experience; then I was going to formulate on this basis further definitions for concepts of various kinds. In order to handle this task, even if only in a few sample cases, it was necessary to have a logic available which was much superior to the traditional variety, especially as concerns the logic of relations. I could carry out my task thanks only to the modern logic which had been developed in the preceding decades, especially by Frege, Whitehead, and Russell; this logic contains a comprehensive theory of relations and their structural properties. Furthermore, through the definition of numbers and numerical functions on the basis of purely logical concepts, the entire conceptual structure of mathematics had been shown to be part of logic. I was much impressed by what this modern logic had already achieved, and I realized that further fruitful applications of its method were possible in the analysis and reformulation of concepts of all areas, including the empirical sciences. At the time, most philosophers did not even suspect the revolutionary importance of modern logic for philosophy and the investigation of the foundation of the sciences.

The system which is formulated in this book takes as basic elements the elementary experiences (§ 67). Only one basic concept is used, namely a certain relation between elementary experiences (recollection of similarity, § 78). It is then shown that the other concepts, e.g., the different senses, the visual sense, visual field places and their spatial relations, the colors and their relations of similarity, can be defined on this basis. It is certainly interesting that the restriction to a single basic concept is possible. However, nowadays this procedure appears to me to be too artificial. I should now prefer to use a larger number of basic concepts, especially since this would avoid some drawbacks which appear in the construction of the sense qualities (cf. the examples in §§ 70 and 72). I should now consider for use as basic elements, not elementary experiences, (in spite of the reasons which, in view of the findings of Gestalt psychology, speak for such a choice, cf. § 67), but something similar to Mach's elements, e.g., concrete sense data, as, for example, "a red of a certain type at a certain visual field place at a given time." I would then choose as basic concepts some of the relations between such elements, for example "x is earlier than y", the relation of spatial proximity in the visual field and in other sensory fields, and the relation of qualitative similarity, e.g., color similarity.

A system such as the one I have just indicated, as well as the one given in this book has its basis in the "autopsychological domain". However, in the book I have already indicated the possibility of another system form whose basic concepts refer to physical objects (§ 59). In addition to the three forms which are there given as examples for a physical basis (§ 62) I would now consider especially a form which contains as basic elements physical things, and as basic concepts observable properties and relations of such things. One of the advantages of this basis is the fact that relative to the properties and relations of the indicated sort, there is a greater degree of intersubjective agreement. All concepts which scientists use in their presystematic linguistic communication are of this sort. Hence a constructional system with such a basis seems particularly suitable for a rational reconstruction of the concept systems of the empirical sciences. In the discussions within the Vienna Circle, Otto Neurath and I subsequently developed the possibility of a unified system of concepts on a physical basis. This "physicalism" was presented in its first fairly rough form in several articles by Neurath and myself, which appeared in vols. 2-4 of *Erkenntnis* (1931-1934). Subsequently it has been modified and refined in several respects.

In the sequel I want to indicate in what respects I have changed my position since I wrote the Aufbau I shall concentrate on the most important points. A detailed description of the development of my philosophical thought and position is given in my intellectual autobiography [Autob.]. (The expressions in [ ] refer to some of my later publications and to writings of outer authors; cf. the "Bibliography 1961" below.) One of the most important changes is the realization that the reduction of higher level concepts to lower level ones cannot always take the form of explicit definitions; generally more liberal forms of concept introduction must be used. Actually, without clearly realizing it, I already went beyond the limits of explicit definitions in the construction of the physical world. For example, for the correlation of colors with space-time points, only general principles, but no clear operating rules were given (§ 127). This procedure is related to the method of introducing concepts through postulates, to which I shall return later. The positivist thesis of the reducibility of thing concepts to autopsychological concepts remains valid, but the assertion that the former can be defined in terms of the latter must now be given up and hence also the assertion that all statements about things can be translated into statements about sense data. Analogous considerations hold for the physicalist thesis of the reducibility of scientific concepts to thing concepts and the reducibility of heteropsychological concepts to thing concepts. These changes have been explained in [Test.] § 15. In that article I suggested the so-called reduction sentences as a more liberal form for the introduction of concepts, which is especially suitable for dispositional concepts.

Later on I considered a method which was already used in science, especially in theoretical physics, namely the introduction of "theoretical concepts" through theoretical postulates and correspondence rules, and investigated the logical and methodological character of these concepts (cf. [Theor.]). The correspondence rules connect the theoretical terms with observation terms. Thus the theoretical terms are interpreted, but this interpretation is always incomplete. Herein lies the essential difference between theoretical terms and explicitly defined terms. The concepts of theoretical physics and of other advanced branches of science are best envisaged in this way. At present I am inclined to think that the same holds for all concepts referring to heteropsychological objects whether they occur in scientific psychology or in daily life.

A comprehensive exposition of our present physicalist position has been given by Feigl [Mental]; cf. also his article [Phys.] and my replies [Feigl] and [Ayer].

I am no longer satisfied with my discussion of the *extensional method* (§§ 43-45 of this book). The then customary version of the thesis of extensionality, as it was maintained by Russell, Wittgenstein, and myself, claimed that all statements are extensional. However, in this form the thesis is not correct. Hence I have later proposed a weaker version which claims that every nonextensional statement can be translated into a logically equivalent statement of an extensional language. It seems that this thesis holds for all hitherto known examples of nonextensional statements, but this has not yet been demonstrated; we can propose it only as a conjecture (cf. [Syntax] § 67; [Meaning] § 32, Method V). Fundamentally, the method which I have called the "extensional method" in § 43 simply consists in using an extensional language for the entire constructional system. This is unobjectionable. However, my description of the procedures is unclear in some points. One could gel the impression that for the reconstruction of a given concept A through concept B it is sufficient that B have the same extension as A. Actually, a stronger requirement must be fulfilled: the coextensiveness of A and B must not be accidental, but necessary, i.e. it must rest either on the basis of logical rules or on the basis of natural laws (cf. my article [Goodman]). This condition is not mentioned in this book. However, it was my intention to formulate the reconstruction in such a way that the coextensiveness holds for any person (provided that he has normal senses and that circumstances are not "particularly unfavorable," §§ 70 and 72), hence is independent of the accidental selection of his observations and the course of his wanderings through the world. Hence the definitions of my system (to the extent in which they do not have to be disregarded as erroneous) fulfill the indicated conditions. For example, the characterization of the visual sense by the dimension number 5 rests upon the biologic-psychological laws which state that the visual sense is the only sense of a (normal, not color-blind) person for which the order of qualities has five dimensions.

I want to consider briefly the most important expositions and critical discussions of the *Aufbau*. Nelson Goodman has made the most thorough study of the problems dealt with in this book. In his book [Structure] he gives an explicit exposition of my theory, and a thorough and penetrating critical analysis, which even concerns itself with technical questions of the method used. He then describes the construction of his own system, which has essentially the same goal as my own, but deviates very considerably in some respects. In his contribution [Aufbau] Goodman briefly states his opinion about my system; I have replied to this in [Goodman]. Anybody concerned with the construction of a similar conceptual system will find valuable suggestions in Goodman's work even if he cannot agree with him in all points. Victor Kraft and J¢rgen Jorgensen consider the *Aufbau* in connection with discussions of the positions of the Vienna Circle and logical empiricism. A more comprehensive exposition is given in Francesco Barone's book [Neopos.]. His pamphlet [Carnap] is a brief nontechnical summary for the nonspecialist. It also contains a bibliography of writings of other authors about various aspects of my philosophical views. Wolfgang Stegmuller ([Gegenw.] Chapter IX, sect. 5) gives a good account and discussion of the main ideas of my book, also of physicalism and related problems.

The article "Pseudoproblems in Philosophy," which is reprinted in this volume, appeared in 1928 at roughly the same time as the Aufbau. However, I did not write it until the end of 1927, the end of my first year in Vienna. Hence it shows a stronger influence of the Vienna discussions and Wittgenstein's book. It was written for the nonspecialist and hence is less technical than the Aufbau. The main theme is the aim of eliminating pseudoproblems from epistemology. To begin with, a general criterion of meaningfulness is formulated. Then this criterion is applied to the recognition of the heteropsychological. My position at the time represents an early phase of physicalism, about whose subsequent development I have made some general remarks above.

On the basis of this meaning criterion, several theses concerning reality are tested. It is shown that the thesis of realism, asserting the reality of the external world, as well as the thesis of idealism, denying this reality, are pseudostatements, sentences without factual content. The same is shown for theses about the reality or irreality of the heteropsychological. This condemnation of all theses about metaphysical reality (which is clearly distinguished from empirical reality) is more radical than that in the *Aufbau*, where such theses were merely excluded from the domain of science. My more radical orientation was due, in part, to Wittgenstein's conception that metaphysical sentences are meaningless since they are in principle unverifiable. This position was held by the majority of the members of the Vienna Circle and other empiricists. On the other hand, the rejection of the theses of reality was not generally accepted. Wittgenstein had not explicitly included these theses among the metaphysical doctrines that were to be rejected; Schlick called himself a realist and accepted my position only later; Reichenbach did not share it at all. I myself have maintained these views even after the empirical meaning criterion had undergone several changes and had become considerably more liberal (cf. [Empir.] and [Ontol.]).

The *Aufbau* has not been available since the war, since not only the printed copies, but also the plates were destroyed in the war. I want to express my thanks to the publisher, Dr. Felix Meiner, for making the book available again. In behalf of myself and my friends I wish to thank Dr. Meiner for continuing to publish as long as possible our journal *Erkenntnis* in spite of all the political difficulties during the 1930's.

University of California, Los Angeles *RUDOLF CARNAP*

March 1961

BIBLIOGRAPHY 1961

This is a list of those publications by myself and other philosophers to which I have referred in the preface. Bibliographies of Carnap, the Vienna Circle and logical empiricism can be found in Ayer [Posit.] 66 pages, Barone [Carnap] 4 pages, Del Pra, 17 pages, Feigl [Mental] 14 pages.

AYER, ALFRED J. (ed.)

[Posit.] *Logical Positivism*. Glencoe, Illinois, 1958. BARONE, FRANCESCO

BARONE, FRANCESCO

[Carnap] *Rudolf Carnap*. Torino, 1953. Reprinted from: Filosofia, 4 (1953), 353-392.

[Neopos.] *Il neopositivismo logico*. Torino, 1953.

CARNAP, RUDOLF

[Syntax] *Logische Syntax der Sprache*. Vienna, 1934.

[Test.] "Testability and Meaning," *Philosophy of Science*, 3 (1936) 419-471; 4 (1937), 1-40. Also published separately, New Haven, Conn., 1950.

[Meaning] Meaning and Necessity. A study in semantics and modal logic. Chicago (1947), 2nd rev. ed., 1956.

[Empir.] "Empiricism, Semantics, and Ontology;" Revue Int. de Philosophie, 4 (1950), 20-40. Reprinted in: [Meaning] 2nd ed.

[Beob.] "Beobachtungssprache and theoretische Sprache," *Dialectica*, 12 (1958), 236-248. Reprinted in *Logica: Studio Paul Bernays dedicata*. Bibliotheque Scientifique, vol. 34. Neuchatel, 1959.

[Einf.] *Einführung in die symbolische Logik*, mit besonderer Berucksichtigung
ihrer Anwendungen. Vienna (1954), 2nd rev. ed., 1960.
[Theor.] "Theoretische Begriffe der Wissenschaft; eine logische and methodologische Untersuchung," *Zeitschr. f. philos*. Forschung, 4 (1960-1961), 209-233 and 571-596. Translated by A. Scheibal in: Feigl [Minn. St.] vol. 1, 38-76.
[Autob.] "Intellectual Autobiography." In: Schilpp.
[Replies] "Replies and Systematic Expositions." In: Schilpp.
[Ontol.] "My Views on Ontological Problems of Existence." [Replies] § 4.
[Feigl] "Herbert Feigl on Physicalism." [Replies] § 7.
[Ayer] "A. J. Ayer on Other Minds." [Replies] § 8.
[Goodman] "Nelson Goodman on *Der Logische Aufbau der Welt*." [Replies] § 21.

DEL PRA, MARIO (ed.)
Rivista Critica della Storia di Filosofza 10 (1955), Fasc. V-VI (a double number on Rudolf Carnap).

FEIGEL, HERBERT
[Empir.] "Logical Empiricism," in: D. D. Runes, ed., *Twentieth Century Philosophy*. New York, 1943. Reprinted and somewhat abbreviated in: H. Feigl and W. Sellars, *Readings in Philosophical Analysis*. New York, 1949, 3-26.
[Minn. St.] (ed. with others) *Minnesota Studies in Philosophy of Science*, vol. 1, 1956, vol. 2, 1958.
[Mental] "The 'Mental' and the `Physical'." In [Minn. St.] vol. 2.
[Phys.] "Physicalism, unity of Science, and the Foundations of Psychology In: Schilpp.

GOODMAN, NELSON
[Structure] *The Structure of Appearance*. Cambridge, Mass., 195 I.
[Aufbau] "The Significance of *Der Logische Aufbau der Welt*," In: Schilpp. Reprinted in: Sidney Hook (ed.), *American Philosophers at Work*. New York, 1956.

JØRGENSEN, JØRGEN
"The Development of Logical Empiricism." her. Encyclopedia of Unified Science, II/9, Chicago, 1951.

KRAFT, VICTOR
Der Wiener Kreis. Der Ursprung des Neupositivismus. Vienna 1050.

SCHIILPP, PAUL A. (ed.)
The Philosophy of Rudolf Carnap. The Library of Living Philosopher.,. La Salle, 1965.

STEGMULLER, WOLFGANG
[Gegenw.] Haupströmungen der Gegenwartsphilosophie. 2nd ed., Stuttgart, 1960.

PREFACE TO THE FIRST EDITION

What is the purpose of a scientific book? It is meant to convince the reader of the validity of the thoughts which it presents. However, this may not completely satisfy the reader; he may want to know, in addition, whence these thoughts came and where they lead, whether there are movements in other areas of inquiry with which they are connected. Only the book as a whole can demonstrate that the thoughts are correct. Here, outside of the framework of the theory, a brief answer to the second question may be attempted: what position in contemporary philosophy and contemporary life in general does this book occupy?

In the last few decades mathematicians have developed a new logic. They were forced to do this from necessity, namely by the foundation crisis of mathematics, in which traditional logic had proved an utter failure. It not only proved incapable of dealing with these difficult problems but something much worse happened, the worst fate that can befall a scientific theory: it led to contradictions. This was the strongest motive for the development of the new logic. The new logic avoided the contradictions of traditional logic, but aside from this purely negative virtue, it has already given proof of its positive capabilities, though only by examining and reestablishing the foundations of mathematics.

It is understandable that the new logic has, to begin with, found attention only in the narrow circle of mathematicians and logicians. Its outstanding importance for philosophy as a whole has been realized only by a few; its application to this wider field has hardly begun. As soon as philosophers are willing to follow a scientific course (in the strict sense), they will not be able to avoid using this penetrating and efficient method for the clarification of concepts and the purification of problems. This book is to go a step along this road and to encourage further steps in the same direction.

We are here concerned, in the main, with questions of epistemology, that is with questions of the reduction of cognitions[1] to one another. The fruitfulness of the new method is shown by the fact that the answer to the question of reduction can be provided through a uniform reductional system of the concepts which occur in science. This system is much like a genealogy; it requires only a few root concepts. It can be expected that such a clarification of the relation of the scientific concepts to one another will also put in a new light several of the more general problems of philosophy. It will turn out that some problems are considerably simplified through the epistemological insights which are obtained in this way; others turn out to be mere pseudoproblems. But these additional tasks are only briefly mentioned in this book. Here is a wide, largely untilled, field which awaits our attention.

The basic orientation and the line of thought of this book are not property and achievement of the author alone but belong to a certain scientific atmosphere which is neither created nor maintained by any single individual. The thoughts which I have written down here are supported by a group of active or receptive collaborators. This group has in common especially a certain basic scientific orientation. That they have turned away from traditional philosophy is only a negative characteristic. The positive features are more important; it is not easy to describe them, but I shall try to give a loose characterization. The new type of philosophy has arisen in close contact with the work of the special sciences, especially mathematics and physics. Consequently they have taken the strict and responsible orientation of the scientific investigator as their guideline for philosophical work, while the attitude of the traditional philosopher is more like that of a poet. This new attitude not only changes the style of thinking but also the type of problem that is posed. The individual no longer undertakes to erect in one bold stroke an entire system of philosophy. Rather, each works at his special place within the one unified science. For the physicist and

1. Erkenntnisse

the historian this orientation is commonplace, but in philosophy we witness the spectacle (which must be depressing to a person of scientific orientation) that one after another and side by side a multiplicity of incompatible philosophical systems is erected. If we allot to the individual in philosophical work as in the special sciences only a partial task, then we can look with more confidence into the future: in slow careful construction insight after insight will be won. Each collaborator contributes only what he can endorse and justify before the whole body of his co-workers. Thus stone will be carefully added to stone and a safe building will be erected at which each following generation can continue to work.

This requirement for justification and conclusive foundation of each thesis will eliminate all speculative and poetic work from philosophy. As soon as we began to take seriously the requirement of scientific strictness, the necessary result was that all of metaphysics was banished from philosophy, since its theses cannot be rationally justified. It must be possible to give a rational foundation for each scientific thesis, but this does not mean that such a thesis must always be discovered rationally, that is, through an exercise of the understanding alone. After all, the basic orientation and the direction of interests are not the result of deliberation, but are determined by emotions, drives, dispositions, and general living conditions. This does not only hold for philosophy but also for the most rational of sciences, namely physics and mathematics. The decisive factor is, however, that for the justification of a thesis the physicist does not cite irrational factors, but gives a purely empirical-rational justification. We demand the same from ourselves in our philosophical work. The practical handling of philosophical problems and the discovery of their solutions does not have to be purely intellectual, but will always contain emotional elements and intuitive methods. The *justification*, however, has to take place before the forum of the understanding; here we must not refer to our intuition or emotional needs. We too, have "emotional needs" in philosophy, but they are filled by clarity of concepts, precision of methods, responsible theses, achievement through coöperation in which each individual plays his part.

We do not deceive ourselves about the fact that movements in metaphysical philosophy and religion which are critical of such an orientation have again become very influential of late. Whence then our confidence that our call for clarity, for a science that is free from metaphysics, will be heard? It stems from the knowledge or, to put it some what more carefully, from the belief that these opposing powers belong to the past. We feel that there is an inner kinship between the attitude on which our philosophical work is founded and the intellectual attitude which presently manifests itself in entirely different walks of life; we feel this orientation in artistic movements, especially in architecture, and in movements which strive for meaningful forms of personal and collective life, of education, and of external organization in general. We feel all around us the same basic orientation, the same style of thinking and doing. It is an orientation which demands clarity everywhere, but which realizes that the fabric of life can never quite be comprehended. It makes us pay careful attention to detail and at the same time recognizes the great lines which run through the whole. It is an orientation which acknowledges the bonds that tie men together, but at the same time strives for free development of the individual. Our work is carried by the faith that this attitude will win the future.

Vienna RUDOLF
CARNAP
May 1928

TRANSLATOR'S PREFACE

A few remarks about some details of this translation are in order.

The German editions of the *Aufbau* contain no footnotes. All footnotes appearing in the present edition were added by me; most of them give the German original of certain expressions; two (in §§ 3 and 88) are based upon suggestions by Professor Carnap.

I have frequently omitted italics where the German text is spaced, since in German spacing is much more common as a mark of emphasis than italics are in English.

In the original, quotation marks are used for several purposes: to indicate the unusual employment of an expression, to mark out the first occurrence of a technical expression, or to show that a term is mentioned rather than used. Except for the last case, I have frequently deleted quotation marks, especially where they occurred in conjunction with spacing. I have italicized all expressions that are both spaced and enclosed in quotation marks in the German text. Neither quotation marks nor italics have been added.

The translation of several expressions warrants special mention. The word '*Beziehung*' has been translated either as 'relation' or as 'many place attribute', while '*Relation*' has been translated as 'relation extension'. An exception is 'basic relation' for '*Grundrelation*', and 'theory of relations' for '*Relationstheorie*'; other nontrivial exceptions are indicated in the footnotes. '*Aussage*' has been translated both as 'statement' and as 'proposition', whichever seemed more appropriate in a given context; '*Wesen*' is rendered both as 'essence' and as 'nature'; ' . . . of physics' is the translation of '*physikalisch*', 'physical' that of 'physisch' Other translations have been listed in the index.

The summary of the *Aufbau* contained in Nelson Goodman's Structure of Appearance proved very helpful in preparing this edition. The present text follows Professor Goodman in the translation of some technical expressions, but deviates in others.

I wish to acknowledge my special debt to Professor Carnap, whom I could consult on a number of occasions; since he has not supervised the translation in detail, any translation errors are entirely my responsibility.

I also wish to express my gratitude to my colleagues at San Fernando Valley State College, where I did most of the work on this translation, for allowing me to use some of their reader funds for the preparation of the manuscript.

Special thanks are due to Mrs. Billie Kiger, not only for her efforts in preparing the manuscript, but also for her advice in stylistic as well as philosophical matters, and to Mrs. Patricia Poggi for reading the manuscript and making many valuable suggestions.

East Lansing, Michigan
R. A. G.
December 1963

CONTENTS

PART II. PRELIMINARY DISCUSSIONS

PART III. THE FORMAL PROBLEMS OF THE CONSTRUCTIONAL SYSTEM

PART V. CLARIFICATION OF SOME PHILOSOPHICAL PROBLEMS ON THE BASIS OF CONSTRUCTION THEORY

PSEUDOPROBLEMS IN PHILOSOPHY

I. THE AIM OF EPISTEMOLOGY

THE LOGICAL STRUCTURE OF THE WORLD

PART ONE

INTRODUCTION: OBJECTIVE AND PLAN OF THE INVESTIGATION

CHAPTER
A

THE OBJECTIVE

> The supreme maxim in scientific philosophizing is this: Wherever possible, logical constructions are to be substituted for inferred entities. RUSSELL

1. *The Aim: A Constructional System of Concepts*

The present investigations aim to establish a "constructional system", that is, an epistemic-logical system of objects or concepts. The word "object" is here always used in its widest sense, namely, for anything about which a statement can be made. Thus, among objects we count not only things, but also properties and classes, relations in extension and intension, states and events, what is actual as well as what is not.

Unlike other conceptual systems, a constructional system undertakes more than the division of concepts into various kinds and the investigation of the differences and mutual relations between these kinds. In addition, it attempts a step-by-step derivation or "construction" of all concepts from certain fundamental concepts, so that a genealogy of concepts results in which each one has its definite place. It is the main thesis of construction theory that all concepts can in this way be derived from a few fundamental concepts, and it is in this respect that it differs from most other ontologies.[1]

1.Gegenstandstheorie

2. *What Does "Construction" Mean?*

In order to indicate more clearly the nature of our objective, the "constructional system", some important concepts of construction theory should first be explained. An object (or concept) is said to be *reducible* to one or more other objects if all statements about it can be transformed into statements about these other objects. (For the time being, the explanation in terms of the loose concept of "transformation" suffices. The following examples will make it sufficiently clear. The exact definitions of reducibility and construction will appear later;[2] they will not be given in terms of statements,[3] but of propositional functions.[4] If *a* is reducible to *b*, and *b* to *c*, then *a* is reducible to *c*. Thus, reducibility is transitive.

> EXAMPLE. All fractions are reducible to natural numbers (i.e., positive integers), since all statements about fractions can be transformed into statements about natural numbers. Thus, for example, 3/7 is reducible to 3 and 7, 2/5 to 2 and 5, and the statement, "3/7 > 2/5", when transformed into a statement about natural numbers, turns into "For any natural numbers x and y, if 7x = 5y, then 3x > 2y." Furthermore, all real numbers, even the irrationals, can be reduced to fractions. Finally, all entities of arithmetic and analysis are reducible to natural numbers.

According to the explanation given above, if an object *a* is reducible to objects *b*, *c*, then all statements about *a* can be transformed into statements about *b* and *c*. To reduce *a* to *b*, *c* or to *construct a* out of *b*, *c* means to produce a general rule that indicates for each individual case how a statement about a must be transformed in order to yield a statement about b, c. This rule of translation we call a *construction rule* or *constructional definition* (it has the form of a definition; cf. § 38).

By a *constructional system* we mean a step-by-step ordering of objects in such a way that the objects of each level are constructed from those of the lower levels. Because of the transitivity of reducibility, all objects of the constructional system are thus indirectly constructed from objects of the first level. These *basic objects* form the *basis* of the system.

> EXAMPLE. A constructional system of arithmetical concepts can be established by deriving or "constructing" step-by-step (through chains

---

2 See § 35.
3 Aussage
4Aussagefunktion

of definitions) all arithmetical concepts from the fundamental concepts of natural number and immediate successor.

A theory is *axiomatized* when all statements of the theory are arranged in the form of a deductive system whose basis is formed by the axioms, and when all concepts of the theory are arranged in the form of a constructional system whose basis is formed by the fundamental concepts. So far, much more attention has been paid to the first task, namely, the deduction of statements from axioms, than to the methodology of the systematic construction of concepts. The latter is to be our present concern and is to be applied to the conceptual system of unified science. Only if we succeed in producing such a unified system of all concepts will it be possible to overcome the separation of unified science into unrelated special sciences.

Even though the subjective origin of all knowledge lies in the contents of experiences and their connections, it is still possible, as the constructional system will show, to advance to an intersubjective, objective world, which can be conceptually comprehended and which is identical for all observers.

3. The Method: The Analysis of Reality with the Aid of the Theory of Relations

The present investigations, as far as their method is concerned, are characterized by the fact that they attempt to bring to bear upon one another two branches of science that have so far been treated separately. Both branches have been developed independently to a considerable extent, but in our opinion they can make further progress only if they are conjoined. Logistics (symbolic logic) has been advanced by Russell and Whitehead to a point where it provides a theory of relations which allows almost all problems of the pure theory of ordering to be treated without great difficulty. On the other hand, the reduction of "reality" to the "given" has in recent times been considered an important task and has been partially accomplished, for example, by Avenarius, Mach, Poincaré, Külpe, and especially by Ziehen and Driesch (to mention only a few names). The present study is an attempt to apply the theory of relations to the task of analyzing reality. This is done in order to formulate the logical requirements which must be fulfilled by a constructional system of concepts, to bring into clearer focus the basis of the system, and to demonstrate by actually producing such a system (though part of it is

only an outline) that it can be constructed on the indicated basis and within the indicated logical framework.

REFERENCES. The fundamental concepts of the theory of relations are found as far back as Leibniz' ideas of a *mathesis universalis* and of an *ars combinatoria*. The application of the theory of relations to the formulation of a constructional system is closely related to Leibniz' idea of a *characteristica universalis* and of a *scientia generalis*.

Logistics. The most comprehensive system of logistics is that of Whitehead and Russell. At the moment it is the only one which contains a well-developed theory of relations and therefore the only one which can be considered a methodological aid to construction theory. It is based on the pioneer work of Frege, Schroder Peano, and others. It is contained *in toto* in [Princ. Math.]. An outline of the system with applications is given by Carnap [Logistik]. The concepts are explained (without symbolism) in Russell [Principles], [Math. Phil.], Dubislav [Wörterbuch]; with a different symbolism: Behmann [Math.]. A historical survey with a rich bibliography (up to 1917): Lewis [Survey].5

Applied theory of relations. Whitehead and Russell make some suggestions for the application of the theory of relations to nonlogical objects (without carrying them through in logical detail): Whitehead's "theory of extensive abstraction" and his "theory of occasions" in [Space], [Nat. Knowledge], [Nature]; Russell's construction of the external world [External W.], [Const. Matter], [Sense Data]. In questions of detail, construction theory diverges very considerably from Russell, but it, too, is based on his methodological principle: "The supreme maxim in scientific philosophizing is this: Wherever possible, logical constructions are to be substituted for inferred entities" [Sense Data] 155. We shall, however, employ this principle in an even more radical way than Russell (for example, through the choice of an autopsychological basis [§ 64], in the construction of that which is not seen from that which is seen [§ 124], and in the construction of heteropsychological objects [§ 140]). Carnap [Logistik] Part II, contains examples of the application of the theory of relations to various subjects (set theory, geometry, physics, theory of kinship relations, analysis of knowledge, analysis of language).

Construction theory. The most important suggestions for the solution

5. Prof. Camap suggests that it would be preferable to consult appropriate sections from his later work in symbolic logic, rather than the older [Logistik]; in particular his Einfuhrung in die symbolische Logik, Vienna: Springer, 1954, and his Introduction to Symbolic Logic and its Applications, New York: Dover, 1958. Further literature can be found through: Alonzo Church, "A Bibliography of Symbolic Logic;" Journal of Symbolic Logic, 1 (1936), and 111 (1938), and through reviews in subsequent issues of that journal.

of the problem as to how scientific concepts are to be reduced to the "given" have been made by Mach and Avenarius. In recent times, three different, independent attempts at a system of concepts have been made: Ziehen [Erkth.], Driesch [Ordnungsl.], Dubislav [Wörterbuch]. Only Dubislav's attempt has the form of a constructional system, since he is the only one who introduces chains of definitions. We will indicate agreements between our system and the just-mentioned systems on the few occasions when they occur, but our approach is, on the whole, quite different from those others because of the methodological tools which we shall employ.

There is also a connection with the goal which was proposed by Husserl, namely, his "mathesis of experiences" [Phanomenol.] 141, and with Meinong's theory of objects. More remotely connected are the classificatory systems of concepts (e.g., those of Ostwald, Wundt, Külpe, Tillich), since they do not derive concepts from one another.

4. *The Unity of the Object Domain*

If a constructional system of concepts or objects (it can be taken in either sense; cf. § 5) is possible in the manner indicated, then it follows that the objects do not come from several unrelated areas, but that *there is only one domain of objects and therefore only one science*. We can, of course, still differentiate various types of objects if they belong to different levels of the constructional system, or, in case they are on the same level, if their form of construction is different. Later on (III A), we shall show that the objects on higher levels are not constructed by mere summation, but that they are logical complexes. The object state,[6] for example, will have to be constructed in this constructional system out of psychological processes, but it should by no means be thought of as a sum of psychological processes. We shall distinguish between a *whole* and a *logical* complex. The whole is composed of its elements; they are its parts. An independent logical complex does not have this relation to its elements, but rather, it is characterized by the fact that all statements about it can be transformed into statements about its elements.
EXAMPLE. An analogy for the uniformity of objects and the multiplicity of different constructs[7], is found in synthetic geometry. It starts from points, straight lines, and surfaces as its elements; the higher constructs are constructed as complexes of these elements. The construction takes place in several steps, and the objects on the different levels

6. Staat
7. Gebilde

are essentially different from one another. Nevertheless, all statements about these constructs are ultimately statements about the elements. Thus we find different types of objects in this case, too, and yet a unified domain of objects from which they all arise.

5. *Concept and Object*

Since we always use the word "object" in its widest sense (§ 1), it follows that to every concept there belongs one and only one object: "its object" (not to be confused with the objects that fall *under* the concept). In opposition to the customary theory of concepts, it seems to us that the generality of a concept is relative, so that the borderline between general and individual concepts can be shifted, depending on the point of view (cf. § 158). Thus, we will say that even general concepts have their "objects". It makes no logical difference whether a given sign [8] denotes the concept or the object, or whether a sentence holds for objects or concepts. There is at most a psychological difference, namely, a difference in mental imagery.[9] Actually, we have here not two conceptions, but only two different interpretative modes of speech. Thus, in construction theory we sometimes speak of constructed objects, sometimes of constructed concepts, without differentiating.

These two parallel languages which deal with concepts and with objects and still say the same thing are actually the languages of realism and idealism. Does thinking "create" the objects, as the Neo-Kantian Marburg school teaches, or does thinking "merely apprehend" them, as realism asserts? Construction theory employs a neutral language and maintains that objects are neither "created" nor "apprehended" but constructed. I wish to emphasize from the beginning that the phrase "to construct" is always meant in a completely neutral sense. From the point of view of construction theory, the controversy between "creation" and "apprehension" is an idle linguistic dispute.

We can actually go even further (without here giving any reasons) and state boldly that the object and its concept are one and the same. This identification does not amount to a reification [10] of the concept, but, on the contrary, is a "functionalization" of the object.

8 Gegenstandszeichen
9 reprasentierende Vorstellung
10 Substantialisierung

CHAPTER
B

THE PLAN OF THE INVESTIGATION

6. *The Preliminary Discussions* (Part II)

The second part will be preparatory to the construction theory itself. Thus, the arguments given there do not presuppose the basic assumption of construction theory, namely, the possibility of a unified constructional system, but merely seek to clarify the scientific, or perhaps more exactly, the ontological [11] situation as it exists today.

In the first chapter (A) of Part II, the very important concept of a *structure* (in the sense of the purely formal aspects of a relation extension) will be explained, and its fundamental importance for science will be shown. It will be demonstrated that it is in principle possible to characterize all objects through merely structural properties (i.e., certain formal-logical properties of relation extensions or complexes of relation extensions) and thus to transform all scientific statements into purely structural statements.

In the second chapter (B), the most important *types of objects*, namely the physical, the psychological, and the cultural [12] will be briefly discussed as to their characteristics, differences, and mutual relations. We

11 gegenstandstheoretisch
12 das Geistige

will speak, not from the point of view and in the language of construction theory, but from the traditional viewpoint and in the (realistic) language of the empirical sciences. This discussion will give us, in a sense, a synopsis of the material which will be used in the formulation of the constructional system. This leads to a nonformal requirement which must be fulfilled, namely, the assignment of definite positions within the system for all the indicated objects.

7. *The Formal Problems of the Constructional System* (Part III)

The presentation of construction theory will begin with Part III. In the first chapter (A), the concept of construction will be discussed in more detail; in particular, it will be shown how it differs from composition by the summation of parts. It will be shown that the construction of an object must be given in the logical form of a definition: every object to be constructed will be introduced through its constructional definition either as a class or as a relation extension. Thus, in each step within the constructional system, one of these two forms will be produced. They are the ascension forms [13] of the constructional system. Others are not required.

In the second chapter (B), we shall undertake logical and factual investigations concerning the *object forms* and the *system form* of the constructional system. By the object form of a constructed object is meant the series of constructional steps which lead to it from the basic objects. We shall show in a general way how the object form can be established from the information found in the empirical sciences about this object, especially about its indicators.[14] By "system form" is meant the form of the system as a whole, i.e., the arrangement of the various steps in the system and the objects which are constructed by these steps. From the various logically and factually possible system forms, we shall select that one which best represents the epistemic [15] relations of the objects to one another.

In the third chapter (C), we shall treat of the problem of the *basis* of the constructional system, i.e., of basic objects of two essentially different kinds, namely, the basic elements and the basic relations, where the latter expression refers to the order which is initially established between the basic elements. We choose as basic elements of the system

13 Stufenformen
14 Kennzeichen
15 erkenntnismässig

"my experiences" (more precisely, entities which initially have neither names nor properties, and which can be called terms of relations only after certain constructions have been carried out). Thus, we choose a system form with an "autopsychological basis". It will then be shown how it is possible to envisage these basic elements as unanalyzable units and nevertheless to construct those objects which are later on called the "properties" or "constituents" of these experiences through a procedure which is actually synthetic, but takes on the linguistic forms of an analysis. (We shall call this procedure "quasi analysis".)

The actual basic concepts of the constructional system, i.e., those concepts to which all other concepts of science are to be reduced, are not the basic elements, but the basic relations. This corresponds to a fundamental assumption of construction theory, namely, that *a system of relations is primary relative to its members*. We will choose the basic relations after certain nonformal [16] considerations. These considerations will already prepare the lower levels of the system by dealing with the question as to how and in what sequence the objects of the lower levels can be constructed, and what basic relations are required for the purpose. As it turns out, a very small number of basic relations, perhaps even only one, suffices.

In the fourth chapter (D), we shall discuss why and in what manner the constructions in the system outline (which constitutes Part IV) are given in four languages: namely, in the language of logistics, which is the proper language of the system, and in three translations which are to facilitate both the understanding of the individual constructions and the investigation into whether these constructions fulfill certain formal requirements. These three translations are: paraphrase of the constructional definitions in word language, the transformation of each definition into a statement indicating a state of affairs [17] in realistic language, and the transformation of each definition into a rule of operation on the basis of certain fictions which serve as an aid to intuition ("language of fictitious constructive operations").

8. *The Outline of a Constructional System* (Part IV)

In Part IV, some of the results of the preceding investigations are applied in practice; an outline of a constructional system is attempted. The lower levels of the system are given in great detail (Chapter A) by representing

16 sachlich
17 Sachverhaftsangabe

the individual constructions in symbolic form and translating them into three auxiliary languages (cf. § 7). We give this part in such great detail, not because its content is absolutely secure, but in order to give a very clear example of the point of the whole investigation and, in addition, to do some spade work on the problem of achieving a reasonable formulation of the lower levels. Using only one basic relation, we shall construct in this part, among other things, the sense qualities, the sense modalities, the visual sense, the spatial order of the visual field, the qualitative order of the color solid, and a preliminary time order.

In the second chapter (B), the constructions are given only in the word language, and no longer with the previous precision, but the individual steps are still clearly described. Here, the space-time world and the visual things in it, including "my body", as one of these visual things, the other senses (besides vision), and the other "autopsychological" entities, components, and states are constructed. The visual world is supplemented by the other senses until it becomes the sensory world, and this world is contrasted with the world of physics," which is no longer concerned with sensory qualities.

In the third chapter (C), constructions are given in rough outline and only to the extent necessary to show that they can be carried out. In particular, we shall indicate the construction of the "heteropsychological" on the basis of "other persons" (as physical things) with the aid of the expression relation; the construction of the "world of the other person" and the "intersubjective world". Finally, the construction of cultural objects and values is also briefly indicated.

9. The Clarification of Some Philosophical Problems (Part V)

In Part V, we shall consider some of the traditional philosophical problems and show how construction theory can be used in order to clarify the problem situations to the extent to which they are part of (rational) science. The problems which are treated there are to serve only as examples of the method, and we shall not discuss them in great detail. To begin with (Chapter A), some problems of essence [18] are discussed, especially the problems of identity, of psychophysical dualism, of intentionality, and of causality.
In Chapter B, we shall try to clarify the problem of psychophysical parallelism.

18 physikalische Welt
19 Wesensprobleme

Subsequently (C, D), the problem of reality is discussed. It is shown that construction theory is the common basis of the various philosophical positions which attempt an answer to this problem, namely, realism, idealism, and phenomenalism; it will also be shown that these positions differ from one another only where they go beyond construction theory; that is, in the field of metaphysics.

In the last chapter (E), the aims and limits of science are discussed, and their clear separation from metaphysics is demanded.

Summary

(The numbers given in parentheses refer to the sections of the book.)

I. INTRODUCTION: OBJECTIVE AND PLAN OF THE INVESTIGATION (1-9)

A. The Objective (1-5)

Construction theory engages in formal (logical) and substantive (epistemological) investigations which lead to the formulation of a constructional system. A constructional system is a system which (in principle) comprises all concepts (or objects) of science, not indeed as a classificatory, but as a derivational, system (genealogy): each concept is constructed from those that precede it in the system (1). A concept is said to be reducible to others, if all statements about it can be transformed into statements about these other concepts; the general rule for this transformation of statements for a given concept is called the construction of the concept (2). Logistics, in particular its most important branch, namely the theory of relations, serves as a methodological aid (3). Consequence of the possibility of a constructional system: all concepts are elements of one structure; hence, there is only one science (4). We take the constructional system to be, at the same time, the system of all objects; the only distinction between "concepts" and "objects" is a difference in modes of speech (5).

B. *The Plan of the Investigation* (6-9)

(A preliminary indication of the contents of the individual chapters)

PART TWO

PRELIMINARY DISCUSSIONS

CHAPTER

A

THE FORM OF SCIENTIFIC STATEMENTS

10. Property Description and Relation Description

In the following, we shall maintain and seek to establish the thesis that *science deals only with the description of structural properties of objects*. At the outset we shall define the concept of a structure. Afterward, in order to establish the thesis, we shall undertake an investigation concerning the possibility and meaning of structural descriptions. However, an actual proof for the thesis can be given only by demonstrating the possibility of a constructional system which is formal, but which nevertheless contains (in principle, if not in practice) all objects. We shall attempt this demonstration by formulating a constructional system in outline (Part IV).

In order to develop the concept of a structure, which is fundamental for construction theory, we make a distinction between two types of description of the objects of any domain; these we call property description and relation description. A *property description* indicates the properties which the individual objects of a given domain have, while a *relation description* indicates the relations which hold between these objects, but does not make any assertion about the objects as individuals. Thus, a property description makes individual or, in a sense, absolute, assertions while a relation description makes relative assertions.

EXAMPLES. A property description looks something like this: the domain is formed by objects *a*, *b*, *c*; a, 6, care persons, *a* is 20 years old and tall; *b* 21 years old, short, and thin; *c* is fat. A relation description looks something like this: the domain is formed by objects *a*, *b*, *c*; *a* is father of *b*, *b* the mother of *c*, *c* is the son of *b*, *a* is 60 years older than *c*.

No matter how many different forms both of the two types of description may assume, they are nevertheless fundamentally different from one another. From property descriptions, one can frequently draw conclusions concerning relations (in the first example, *b* is one year older than *a*), conversely, from relation descriptions, one can frequently infer something about properties (in the second example, *a* and *c* are male, *b* is female); however, the conclusion is then not equivalent to the premises, but contains less: the inference cannot be reversed. Thus, the fundamental difference remains. Frequently, both kinds of description are found together.

EXAMPLES. *Property descriptions*: Description of the set of conic sections through an account of the characteristics of the individual sections. Description of a curve through its coordinate equation, i.e., by giving the ordinate for each point on the abscissa. List of historical persons with a statement of the dates of birth and death for each of them.

Relation descriptions: Description of a geometrical figure which consists of points and straight lines through an indication of the relations of incidence. Description of a curve through its natural equation, i.e., through an indication of the position of each element of the curve relative to the preceding ones. Description of a group of persons by means of a genealogy, i.e., by giving their kinship relations.

We place such strong emphasis upon the difference between these two types of description because we shall maintain that they are not of equal value. Relation descriptions form the starting-point of the whole constructional system and hence constitute the basis of unified science.

Furthermore, it is the goal of each scientific theory to become, as far as its content is concerned, a pure relation description. It can, of course, take on the linguistic form of a property description; this will sometimes even be an advantage; but it differs from a genuine property description in the fact that it can be transformed, if necessary, without loss into a relation description. In science, any property description either plays the role of a relation description except that it is in more convenient form, or else, if transformation is not yet possible, it indicates the provisional character of the theory in question.

EXAMPLE. In physics, we apparently have a property description when the color names ("blue", "red", etc.) aroused. In present-day physics, descriptions of this kind are nothing but abbreviations, since they presuppose wave theory and since the color names can be translated into expressions of this theory (i.e., rates of oscillation). However, formerly, these property descriptions revealed the incomplete character of the theory of light, since they were not transformable into relation descriptions.

11. *The Concept of Structure*

There is a certain type of relation description which we shall call *structure description*. Unlike relation descriptions, these not only leave the properties of the individual elements of the range unmentioned, they do not even specify the relations themselves which hold between these elements. In a structure description, only the *structure* of the relation is indicated, i.e., the totality of its formal properties. (A more precise definition of structure will be given later.) By formal properties of a relation, we mean those that can be formulated without reference to the meaning [20] of the relation and the type of objects between which it holds. They are the subject of the theory of relations. The formal properties of relations can be defined exclusively with the aid of logistic symbols, i.e., ultimately with the aid of the few fundamental symbols which form the basis of logistics (symbolic logic). (Thus these symbols do not specifically belong to the theory of relations, but form the basis for the entire system of logic—propositional logic, the theory of propositional functions (concepts), the theory of classes, and the theory of relations.)

Let us now consider some of the most important of these formal properties.

A relation is called *symmetrical* when it is identical with its converse (e.g,, contemporaneousness); otherwise, it is called *nonsymmetrical* (e.g., brother); a nonsymmetrical relation is called *asymmetrical* when it excludes its converse (e.g., father). A relation is called *reflexive* if, in the case of identity (within its field), it is always fulfilled (e.g., contemporaneousness); otherwise, it is called *nonreflexive* (e.g., teacher). A nonreflexive relation is called *irreflexive* if it excludes identity (e.g.,father). A relation is called *transitive* when it always holds also for the next member but one (e.g., ancestor); otherwise, *nontransitive* (e.g., friend). A nontransitive relation is called *intransitive* if it never holds for the next member but one (e.g., father). A relation is called con-

---
20. inhaltlicher Sinn

nected if, between any two different members of the field, either it or its converse always holds (e.g., for a table group of six persons, the relation "one, two. Or three seats to the left of'). A relation is called a *sequence* if it is irreflexive and transitive (and hence asymmetrical) and connected (e.g., "smaller than" for real numbers). A relation is called a *similarity relation* [21] if it is symmetrical and reflexive, and an *equivalence* if it is also transitive (cf. §§ 71, 73).

Other formal properties of relations are one-many-ness, many-one-ness, one-one-ness; specific number of elements in the field, of elements in the domain, of elements of the converse domain, of initial elements, last elements, etc.

In order to understand what is meant by the structure of a relation, let us think of the following arrow diagram: Let all members of the relation be represented by points. From each point, an arrow runs to those other points which stand to the former in the relation in question. A double arrow designates a pair of members for which the relation holds in both directions. An arrow that returns to its origin designates a member which has the relation to itself. If two relations have the same arrow diagram, then they are called *structurally equivalent*, or *isomorphic*. The arrow diagram is, as it were, the symbolic representation of the structure. Of course, the arrow diagrams of two isomorphic relations do not have to be congruent. We call two such diagrams equivalent if one of them can be transformed into the other by distorting it, as long as no connections are disrupted (topological equivalence).

12. *Structure Descriptions*

One can give a verbal description which is equivalent to an arrow diagram (where this diagram does not name the individual members) by listing all pairs for which the given relation holds, without, however, using any descriptions which have meaning outside of this list. For example, one can number the members arbitrarily and only for the purpose of producing the list. Such a list can be inferred from the diagram, i.e., it contains no more than the diagram; conversely, the list of pairs allows us to construct the diagram. Thus, the list of pairs, as well as the arrow diagram, gives the complete structure description.

If two relations have the same structure, then they are equivalent in all formal properties. Thus, all formal properties of a relation are determined if its structure is described. On the other hand, there is no general

21 Ähnlichkeit

rule as to which formal properties suffice to determine the structure of a relation; it is the task of the theory of relations to investigate this question in detail. The graphic rendition of the structure of a relation by means of an arrow diagram is, of course, possible only if the number of members is finite. It must be possible to give an exact definition of the concept of structure and to indicate the structure of a given relation without the aid of diagrams. But, in this context, it is quite permissible to use the arrow diagram for the purposes of illustration, since, whenever such a diagram can be drawn, it precisely reflects the structure, and since it exhibits all the fundamental aspects of the general concept of structure.

We saw earlier that it was possible to draw conclusions concerning properties of individuals from relation descriptions. In the case of structure descriptions, this no longer holds true. They form the highest level of formalization and dematerialization. If we are given an arrow diagram which contains nothing but double arrows, then we know that it represents the structure of a symmetrical relation, but it is no longer evident whether it represents persons under the relation of acquaintance, or towns under the relation of direct telephone connection, etc. Thus, our thesis, namely that scientific statements relate only to structural properties, amounts to the assertion that scientific statements speak only of forms without slating what the elements and the relations of these forms are. Superficially, this seems to be a paradoxical assertion. Whitehead and Russell, by deriving the mathematical disciplines from logistics, have given a strict demonstration that mathematics (viz., not only arithmetic and analysis, but also geometry) is concerned with nothing but structure statements. However, the empirical sciences seem to be of an entirely different sort: in an empirical science, one ought to know whether one speaks of persons or villages. This is the decisive point: empirical science must be in a position to distinguish these various entities; initially, it does this mostly through definite descriptions utilizing other entities. But ultimately the definite descriptions are carried out with the aid of structure descriptions only. We shall give a detailed discussion of this in the sequel.

REFERENCES. The derivation of the concept of structure (or “relation number”) is found in Russell [Princ. Math.] II, 303 ff. Russell also comments on the subject ([Math. Phil.] 53 ff.) and gives an indication of the importance of this concept for philosophy and science in general ([Math. Phil.] 61 ff.). Cf. also Camap [Logistik] § 22.

Recently (in connection with ideas of Dilthey, Windelband, Rickert), a “logic” of individually has repeatedly been demanded; what is desired here is a method which allows a conceptual comprehension of, and does justice to, the peculiarity of individual entities, and which does not attempt to grasp this peculiarity through inclusion in narrower and narrower classes. Such a method would be of great importance for individual psychology and for all cultural sciences, especially history. (Cf,, for example, Freyer [Obj. Geist] 108 f.). I merely wish to mention in passing that the concept of structure as it occurs in the theory of relations would form a suitable basis for such a method. The method would have to be developed through adaptation of the tools of relation theory to the specific area in question. Cf. also Cassirer's theory of relational concepts [Substanzbegr.] esp. 299, and the application of the theory of relations (but not yet to cultural objects) in Camap [Logistik] Part II.

13. *About Definite Descriptions*

A scientific statement makes sense only if the meaning of the object names which it contains can be indicated. There are two ways of doing this. The first of these is through *ostensive definitions*; the object which is meant is brought within the range of perception and is then indicated by an appropriate gesture, e.g., "That is Mont Blanc." The second consists of an unequivocal circumscription which we call *definite description*. A definite description does not indicate all properties of the object and thus replace concrete perception; on the contrary, it actually appeals to perception. Also, definite descriptions do not even list all essential characteristics, but only as many characterizing properties as are required to recognize unequivocally the object which is meant within the object domain under discussion. To give an example: the name "Mont Blanc" is used to indicate the highest mountain in the Alps, or the mountain so many kilometers east of Geneva. In order for the definite description to be successful, it is not sufficient that the describing sentence be meaningful. Rather, in the given object domain, there must be at least one object with the indicated properties and, secondly, there must be at most one such object. Thus, questions whether and what a definite description describes cannot be answered a priori, but only by reference to the object domain in question.

In most cases, as also in the examples given, a definite description indicates the relation of the object in question to other objects. Thus, it seems that the problem of the determination of objects is only pushed back one more step with each definite description, and that it can be finally resolved only through ostensive definitions. However, we shall presently see that, within any object domain, a unique system of definite descriptions is in principle possible, even without the aid of ostensive definitions. However, in a given case such a system may not be obtainable, and for a given object domain one cannot decide a priori whether or not it can be devised. It is of especial importance to consider the possibility of such a system for the totality of all objects of knowledge. Even in this case it is not possible to make an a priori decision. But we shall see later that any intersubjective, rational science presupposes this possibility.

REFERENCES. About definite descriptions, see Russell [Princ. Math.] I, 31ff,, 69ff,, 181 ff,, [Math. Phil.] 168ff.; Camap [Logistik] §§ 7,14.

14. *Example of a Definite Description Which is Purely Structural*

How can it be possible to give a definite description of all objects within a given object domain without indicating any one of them through an ostensive definition and without making any reference to an object outside of the given object domain? That there is such a possibility can be seen most easily by way of a concrete example which we shall give in great detail because of the importance of the general principle which it illustrates.

EXAMPLE. Let us look at a railroad map of, say, the Eurasian railroad network. We assume that this map is not a precise projection, but that it is distorted as much or more than the customary maps found in ticket offices. It does not then represent the distances, but only the connections within the network; (in the terminology of geometry): it indicates only the topological, not the metrical, properties of the network. The example of the railroad map has previously been used to clarify the concept of topological properties. It is equally well suited to clarify the closely related, but more general, logical concept of structural properties. We assume now that all stations are marked as points, but the map is not to contain any names nor any entries other than rail lines. The question now is: can we determine the names of the points on the map through an inspection of the actual railroad network? Since it is difficult to observe an actual railroad network, let us use in its stead a second map which contains all the names. Since our (first) map may be distorted more than the customary railroad maps, we will gain little by looking for characteristic shapes, for example, the long Siberian railroad. But there is a more promising way: we look up the intersections of highest order, i.e., those in which the largest number of lines meet. We will find only a small number of these. Assume that we find twenty intersections in which eight lines meet. We then count, for each such point, the number of stations between it and the next intersection on each of the eight lines, and we will hardly find two of the eight to coincide in all eight numbers. Thus, we have identified all twenty points. But if there are still two, or even all twenty, which have the same numbers, then all we have to do is to consider the connections between each of the eight neighboring intersections: whether or not they have direct connections, how many stations there are between them, how many lines meet in these neighboring intersections, etc. Given the network as it actually exists today: if we do all this, we will certainly not find any further coincidences. But if we are confronted with a network where even these characteristics do not allow us to differentiate, we would have to proceed, step by step, from the neighboring intersections to their neighbors, etc., in order to find still further characteristics for the main intersections. We proceed in this way until we find characteristics which no longer coincide, even if we have to survey our entire net. But once we have discovered the name for even one point on the map, the others are easily found, since only very few names qualify for the neighboring points.

But what happens if there are two intersections for which we cannot find any difference even after surveying the entire system? This simply means that there are two points with identical structural characteristics (homotopic points) as far as the relation to neighboring railroad stations is concerned. We would gather that this relation does not suffice to give a definite description of the objects of the given object domain. We would have to take recourse to ostensive definitions or to one or more other relations. To begin with, we would choose relations of similar kind: next to one another on the highway, on the telephone line, etc. However, in order to stay within the limits of purely structural statements, we must not mention these relations by name, but must represent them only through the arrow diagram of their total network. We must presuppose that by inspection of the geographic facts one can determine unequivocally whether a given network map represents the Eurasian highways or the telephone connections, etc. Through each of these further relations, we would then seek to describe first a few and then all the points of the network, analogously to the procedure employed with respect to the railroad connections. No one will suppose that there can still be two points which are homotopic under all of the relations we have introduced. However, such a case merely contradicts our notion of what actually exists, but is not altogether unimaginable. Thus, in order to solve the problem in principle, we must still pose the further problem: how can we produce a definite description if all of these relations do not suffice? So far we have utilized only spatial relations, since their schematic spatial representation on a map is both customary and easily understood. But we can also employ all other geographic relations and establish a connection between the various locations through relations between the numbers of inhabitants (not the numbers of inhabitants themselves), through economic processes, relations of climate, etc. If we are still left with two homotopic elements of the object domain, then we simply have two locations that are geographically indistinguishable. If we then move on to a new type of relation and take into account all historical relations between the locations, etc., we shall ultimately have used up all the concepts of the cultural as well as the physical sciences. If there should still be two locations for which we have found no difference even after exhausting all available scientific relations, then they are indistinguishable, not only for geography, but for science in general. They may be subjectively different: I could be in one of these locations, but not in the other. But this would not amount to an objective difference, since there would be in the other place a man just like myself who says, as I do: I am here and not there.

15. *The General Possibility of Structural Definite Descriptions*

From the preceding example, we can see the following: on the basis of a structural description, through one or more only structurally described relations within a given object domain, we can frequently provide a definite description of individual objects merely through structure statements and without ostensive definitions, provided only that the object domain is not too narrow and that the relation or relations have a sufficiently variegated structure. Where such a definite description is not unequivocally possible, the object domain must be enlarged or one must have recourse to other relations. If all relations available to science have been used, and no difference between two given objects of an object domain has been discovered, then, as far as science is concerned, these objects are completely alike, even if they appear subjectively different. (If the given assumptions are all fulfilled, then the two objects are not only to be envisaged as alike, but as identical in the strictest sense. This is not the place to give a justification for this apparently paradoxical assertion.) Thus, the result is that a definite description through pure structure statements is generally possible to the extent in which scientific discrimination is possible at all; such a description is unsuccessful for two objects only if these objects are not distinguishable at all by scientific methods.

Through the method of structural definite descriptions, it now becomes possible to assign unique symbols to empirical objects and thus to make them accessible to conceptual analysis.[22] On the other hand, it is precisely this assignment of symbols which allows the characterization of empirical objects as individuals. Thus, in this method lies the explanation for the "strange fact that, in cognition,[23] we correlate two sets, the elements of one of which are defined only through this correlation" (Reichenbach [Erk.] 38).

> The purely structural definite descriptions which I have here discussed are closely related to the *implicit definitions* which Hilbert has used for his axiomatic geometry [Grundlagen] and whose general methodology and scientific importance have been discussed by Schlick [Erkenntnisl.] 29 ff. An implicit definition or definition through axioms consists in the following: one or more concepts are precisely determined by laying down that certain axioms are to hold for them. Of the axioms we require nothing but consistency, a formal-logical property which can be ascertained through purely logical considerations. Statements which can then be made about an object that has in this way been implicitly defined follow deductively from the axioms, i.e., through another purely logical procedure. Strictly speaking, it is not a definite object (concept) which is implicitly defined through the axioms, but a class of them or, what amounts to the same, an "indefinite object" or "improper concept"; cf. Carnap [Uneigentl.].
>
> A structural definite description, in contradistinction to an implicit definition, characterizes (or defines) only a single object, to wit, an object belonging to an empirical, extralogical domain. (In the example of §14, it was an individual railroad station in the object domain which consisted of the Eurasian railroad stations.) Thus, for the validity of such a description, it is not only required that the describing structure statements be consistent, but, in addition, the following empirical requirements must also be fulfilled: in the object domain in question, at least one object must exist which answers the description, and at most one such object must exist. Further statements about the object which has thus been described are then not all of them analytic, that is, deducible from the defining statements, as is the case with implicitly defined objects, but some of them are synthetic, namely, empirical findings within the object domain in question.

16. *All Scientific Statements are Structure Statements*

It becomes clear from the preceding investigations about structural definite descriptions that each object name which appears in a scientific

22 Bearbeitung
23 Erkenntnis

statement can in principle (if enough information is available) be replaced by a structural definite description of the object, together with an indication of the object domain to which the description refers. This holds, not only for the names of individual objects, but also for general names, that is, for names of concepts, classes, relations (as we have seen in the example of §14, for the relation of road connections and so forth). Thus, each scientific statement can in principle be transformed into a statement which contains only structural properties and the indication of one or more object domains. Now, the fundamental thesis of construction theory (cf. §4), which we will attempt to demonstrate in the following investigation, asserts that fundamentally there is only one object domain and that each scientific statement is about the objects in this domain. Thus, it becomes unnecessary to indicate for each statement the object domain, and the result is that *each scientific statement can in principle be so transformed that it is nothing but a structure statement*. But this transformation is not only possible, it is imperative. For science wants to speak about what is objective, and whatever does not belong to the structure but to the material (i.e., anything that can be pointed out in a concrete ostensive definition) is, in the final analysis, subjective. One can easily see that physics is almost altogether desubjectivized, since almost all physical concepts have been transformed into purely structural concepts.

> To begin with, all mathematical concepts are reducible to concepts which stem from the theory of relations: four-dimensional tensor and vector fields are structural schemata; the network of world lines with the relations of coincidence and local time order is a structural schema in which only two relations are still named; and even these are uniquely determined through the character of the schema.

From the point of view of construction theory, this state of affairs is to be described in the following way. The series of experiences is different for each subject. If we want to achieve, in spite of this, agreement in the names for the entities which are constructed on the basis of these experiences, then this cannot be done by reference to the completely divergent content, but only through the formal description of the structure of these entities. However, it is still a problem how, through the application of uniform formal construction rules, entities result which have a structure which is the same for all subjects, even though they are based on such immensely different series of experiences. This is the problem of inter-subjective reality. We shall return to it later. Let it suffice for the moment to say that, *for science, it is possible and at the same time necessary to restrict itself to structure statements*. This is what we asserted in our thesis. It is nevertheless evident from what has been said in §10 that scientific statements may have the linguistic form of a material relation description or even the form of a property description.

> REFERENCES. Considerations similar to the preceding ones have sometimes led to the standpoint that not the given itself (viz., sensations), but "only the relations between the sensations have an objective value" (Poincaré [Wert] 198). This obviously is a move in the right direction, but does not go far enough. From the relations, we must go on to the structures of relations if we want to reach totally formalized entities. Relations themselves, in their qualitative peculiarity, are not intersubjectively communicable. It was not until Russell ([Math. Phil.] 62 f.) that the importance of structure for the achievement of objectivity was pointed out

CHAPTER
B

SURVEY OF THE OBJECT TYPES AND THEIR RELATIONS

17. *The Significance of Object Types for Construction Theory*

In the present chapter (II, B), we do not undertake any new investigations, but merely give a survey of the different independent object types according to their familiar characteristic properties. We shall also discuss those relations between these types which have given rise to metaphysical problems (as, for example. the psychophysical relation), or are important for the logical-epistemic relation between the object types and therefore also for the problems of construction (as, for example, the expression relation).

The problem of object types and their mutual relations is of great importance for construction theory since its aim is a system of objects. The various differences and relations which can be indicated, and especially the differences between the various "object spheres", must somehow be reflected in the system that we are about to develop. This is an especially important test for our form of construction theory, since we subscribe to the thesis that the concepts of all objects can be derived from a single common basis.

When, later on, we give a presentation of construction theory, we shall not presuppose any of the factual results and problems of the present chapter, but will undertake the entire construction from the very beginning. There are only a few stages in the development of the system where we shall pay any attention to some of these facts. They will become the most important test when we judge our final result. On the other hand, the theory will lead to the conclusion that the *problems* which are discussed in the present chapter do not even occur in the newly developed system of objects; the obscurity and confusion which is the source of these problems did not arise because the facts themselves are complicated but because of certain traditional conceptual mistakes, which must be explained historically rather than by reference to the facts in question. (Objections against the assertions of this chapter should therefore be postponed until these assertions are later on employed in the formulation of the system.)

Thus, this chapter, even more than the preceding one (II, A) has a preparatory character, and can therefore be omitted without disturbing the context of construction theory which will be presented in the subsequent chapters. The only exceptions are the more fundamental discussions in §§ 20, 22, and 25.

18. *The Physical and Psychological Objects*

The concepts of the physical and the psychological are here to be taken in their customary sense, and therefore we will not give any explicit explanation, much less a definition, especially since both of them are in certain respects vague and, moreover, “logically impure” concepts (§29).

As examples of physical objects, we consider their most important type, namely, *physical bodies*. These are characterized especially by the fact that, at a given time, they occupy a given space (i.e., an extended piece of space). Thus, place, shape, size, and position belong to the determining characteristics of any physical body. Furthermore, at least one sensory quality belongs to these determining characteristics, e.g., color, weight, temperature, etc. Since we take the word “object” here always in its widest sense (i.e., as something about which a statement can be made), we make no distinction between events and objects. To the psychological objects belong, to begin with, the acts of consciousness: perceptions, representations,[24] feelings, thoughts, acts of will, and so on. We count among them also unconscious processes to the extent to which they can be con-

24 Vorstellungen

sidered analogous to acts of consciousness, for example, unconscious representations.

The psychological objects have in common with the physical ones that they can be temporally determined. In other respects, a sharp distinction must be drawn between the two types. A psychological object does not have color or any other sensory quality and, furthermore, no spatial determination. Outside of these negative characteristics, psychological objects have the positive characteristic that each of them belongs to some individual subject.

19. *Psychophysical Relation, Expression Relation, and Designation Relation*

The *psychophysical relation* holds between a psychological process and the "corresponding" or "parallel" process of the central nervous system. The theory which is advanced most frequently holds that all psychological objects belong to the domain of this relation, while the converse domain is formed by only a very small segment of the physical objects, namely, the processes in the nervous system of the living animal (or, perhaps, only the human) body.

Through voice, facial expressions, and other gestures, we can understand "what goes on within" a person. Thus, physical processes allow us to draw conclusions concerning psychological ones. The relation between a gesture, etc., and the psychological process we call the *expression relation*. To its domain belong almost all motions of the body and its members, in particular also the involuntary ones. To its converse domain belong part of the psychological objects, especially the emotions.

> Many physical objects, which we utilize to understand other people and of which we say that they "express" something psychological, do not stand in the direct expression relation, as we have explained it, to that which they express, but rather in a more complex relation. This holds for all physical objects which are not processes of the body of another person. For example, it holds for pieces of writing and other artifacts, spoken words (i.e., the sound waves in the air), etc. There is a causal relation between these physical objects and the member of the domain of the expression relation proper, that is, the motions of the body. This causal relation is of such a nature that it preserves the characteristic features which carry the expression. Only because handwriting coincides in certain characteristic features with the motion of the hand can we use it in graphology as a sign of psychological facts. Thus, even

in these cases, we have to go back to the actual expression relation which holds between the motions of the hand (but not the marks on the paper) and the psychological events.

The expression relation must be carefully distinguished from the *designation relation*. This relation holds between those physical objects which "designate" and that which they designate, for example, between the sign "Rome" and the city of Rome. All objects, inasmuch as they are objects of conceptual knowledge, are somehow designated or, at least, can in principle be designated. Thus, to the converse domain of the designation relation belong the objects of all object types.

In some cases, the same physical object stands at the same time in an expression relation and in a designation relation to something psychological. In these cases, the relations must and can be well distinguished. For example, spoken words are, in any case, expression for something psychological, no matter what their content. For the sound of the voice, the speed, or the rhythm, but also the choice of the individual words and the style, betray something about the momentary psychological condition of the speaker. But, in addition to this, the words have a meaning. The difference between their expressive content and their meaning content is easily recognized, especially when the meaning concerns something other than psychological processes within the speaker.

20. *Correlation Problem and Essence of a Relation*

With each relation, there are connected two problems of a different kind; the difference between them is of special importance when the relation holds between objects of different object types. We call the *correlation problem* the question: between which pairs of objects does the relation hold? More precisely, what is the general law of correlation of the relation in question? The answer, then, has the following form: If the referent is of such and such a nature, then the corresponding relatum has such and such a nature (or vice versa).

EXAMPLE. Let us consider the *designation relation* as it holds between written words and their meanings. Since natural languages do not have general rules which allow us to deduce the meaning of a word from its form, there is no way of indicating the extension of this relation except by enumeration of all its member pairs. If a basic language is already known, then this is done through a dictionary; otherwise, the answer takes on the form, for example, of a botanical garden, that is, a collection of objects, each of which has its name written on it.

> If the meanings of the *words* are known, then the answer to the correlation problem of the designation relation for *sentences* can be solved through a general function, which, however, is usually very complicated. It is the syntax of the language in question cast in the form of a meaning rule. A meaning rule may (in an elementary case) have the following form: if a sentence consists of three words, a noun in the nominative case; a verb in the third person singular, present tense, active mood; and a noun in the accusative case, then it designates the state of affairs that the object of which the first word is the sign stands to the object of which the third word is the sign in the relation of which the verb is the sign.

From the correlation problem, we distinguish the *essence problem*. Here we do not simply ask between what objects the relation obtains, but what it is between the correlated objects, by virtue of which they are connected. The question does not ask for the constitution of the related object, but asks for the essence of the relation itself. Later on, on the basis of construction theory, we shall indicate the difference between science and metaphysics (§ 182), and we shall see that the essence problems belong to *metaphysics* (§§ 161, 165, 169).

> EXAMPLE. The *causal relation* (i.e., the relation between cause and effect, as it occurs within physics) gives us a very clear example of the meaning of the essence problem in contradistinction to the correlation problem and the resulting division of labor between the special sciences and metaphysics. The question which cause is causally related to which effect (that is, the correlation problem), is the concern of physics. Its task is to find an answer to that question in the form of a general functional law (i.e., in the form: if the cause is of such and such a nature, then the effect is of such and such a nature). The answers which physics gives to this question are the natural laws. On the other hand, physics does not answer the question of what kind is the relation which obtains between two events that are related to one another as cause and effect. It does not tell us the nature of their connection, of "causal efficacy".

The problems of causality will be more precisely formulated and discussed subsequent to construction theory (§ 165).

The nature of the essence problem is closely connected with the concept of an *essential relation*.[25] By this is meant that which connects the members of a relation "essentially" or "really" or "actually", in contradistinction to the relation as a mere correlation which only points out the members that are so correlated. Later on it will be shown (§ 161)

---

25 Wesensbeziebung

that the problem of essential relations as well as the essence problem of a relation can, within (rational) science, neither be resolved nor even posed. It belongs to metaphysics.

> EXAMPLE. Especially in connection with the *problem of causality*, the concept of essential relations plays an important role. In discussions about the foundations of physics, one frequently finds the (erroneous) assumption, which is directed against certain positivistic or mathematicizing theories, that causality as a central concept of physics means not only correlation (i.e., mathematical function), but also an essential relation between the correlated processes, namely, the "effect", in the narrow sense, of one process upon the other.

21. *Problems of Correlation and Essence of the Afore-mentioned Relations*

The example of causality shows that the investigation of the correlation problem is the task of the special sciences. The same holds for the correlation problems of the earlier mentioned relations. Brain physiology, psychology, and psychopathology concern themselves with the correlation problem of the psychophysical relations. They attempt to ascertain what kind of physiological process in the central nervous system corresponds to a given psychological process, and vice versa. Very little has been done to solve this problem. The technical difficulties of such an investigation are patent; on the other hand, it is certainly not the case that there are fundamental obstacles, i.e., absolute limits to our knowledge of these matters. There has not been much research into the expression relation, even though it is very important for practical life, since our understanding of other persons depends upon it. However, we possess and utilize this knowledge, not in a theoretically explicit manner, but only intuitively ("empathy"). This is the reason why there is no satisfactory solution of the correlation problem of this relation. On the other hand, there are today promising beginnings to theories of physiognomies, graphology, and characterology. The correlation problem of the vast and variegated designation relation can hardly be resolved within a single theoretical system. In spite of the immense extension of the designation relation (written signs, signals, badges, etc.), there are fewer difficulties to be expected in this case than with the other discussed relations; at least, there will be no fundamental difficulties.

Thus we see that the correlation problems of the indicated relations will have to be solved within certain special sciences, and that no fundamental difficulties stand in the way of these solutions. On the other hand, the *essence problems* of these relations are a different matter. Since we are here concerned not with the *ascertainment*, but with the *interpretation* of facts, these questions cannot be empirically answered. Thus, their treatment is not among the tasks of the special sciences.

If, in connection with correlation problems, we encounter several competing hypotheses, between which we cannot decide, we can at least indicate which empirical data would be required to decide in favor of one hypothesis or another. On the other hand, no decisions have been made between various fundamentally opposed answers to essence problems, and apparently it is impossible to make such decisions: a depressing aspect for the impartial observer, since, even with the boldest hopes for future progress in knowledge, he cannot expect to find out which empirical or other sort of knowledge could bring about such a decision.

The question about the essence of the expression relation has received different, diverging, and even in part contradicting answers. The expressive act has sometimes been interpreted as the effect of the psychological facts that are expressed (thus, the problem has been pushed back to the essence problem of the causal relation), or else as its cause, or the two have been identified with one another. Occasionally, the expressed emotion is said to "inhere" in a special, unanalyzable way in the physical expression. Thus, the most divergent essential relations have been envisaged. The problem of the designation relation is somewhat simpler, since the connection between sign and signified object always contains a conventional component; that is, it is somehow brought about voluntarily. Only rarely has a special essential relation of "symbolizing" been assumed.

22. *The Psychophysical Problem as the Central Problem of Metaphysics*

The essence problem of the psychophysical relation can be called simply the psychophysical problem. Among the traditional problems of philosophy, it is the one which is most closely connected with the psychophysical relation, and, in addition, it has gradually become the main problem of metaphysics.

The question is this: provided that to all or some types of psychological processes there correspond simultaneous processes in the central nervous system, what connects the processes in question with one another? Very little has been done toward a solution of the correlation problem of the psychophysical relation, but, even if this problem were solved (i.e., if we could infer the characteristics of a brain process from the characteristics of a psychological process, and vice versa), nothing would have been achieved to further the solution of the essence problem (i.e., the "psychophysical problem"). For this problem is not concerned with the correlation, but with the essential relation; that is, with that which "essentially" or "fundamentally" leads from one process to the other or which brings forth both from a common root.

> The attempted solutions and also their irreconcilable divergences are well known. The theories of occasionalism and of preëstablished harmony have perhaps only historical interest. Thus, there still remain, in the main, three hypotheses: mutual influence, parallelism, and identity in the sense of the two-aspect theory. The hypothesis of *mutual influence* assumes an essential relation between the two terms (i.e., a causal efficacy in both directions). The hypothesis of *parallelism* (in the narrowest sense, i.e., excluding the identity theory) denies the existence of an essential relation and assumes that there is only a functional correlation between the two types of objects (types of processes). Finally, the *identity theory* does not even admit that there are two types of objects, but assumes that the psychological and the physical are the two "aspects" ("the outer" and "the inner") of the same fundamental process. The counterarguments which are brought forth against each of these hypotheses by its adversaries seem to be conclusive: science generally assumes an uninterrupted causal nexus of all spatial processes; but this is not consistent with psychophysical mutual influence. On the other hand, one cannot see how a merely functional correlation, that is, a logical and not a real relation, can result in an experience which corresponds to the stimuli that impinge upon the senses. And the identity of two such different types of objects as the psychological and the physical remains an empty word as long as we are not told what is meant by the figurative expression "fundamental process" and "inner and outer aspects." (We do not wish to say anything against parallelism or the hypothesis of mutual influence as long as they are merely used heuristically, as working hypotheses for psychology. We are here concerned with metaphysical opinions.)

Three contradicting and equally unsatisfactory answers and no possibility of finding or even imagining an empirical fact that could here make the difference: a more hopeless situation can hardly be imagined. It could lead us to wonder whether the questions concerning problems of essence, especially the psychophysical problem, are not perhaps posed in a fallacious way. Construction theory will in fact lead to the conclusion that this is so. Once the constructional forms of the objects and the object types are found and their logical locations in the constructional system are known, and if furthermore the correlation problem of one of the above relations has been resolved, then we have found everything (rational) science can say about that relation. An additional question concerning the "essence" of the relation would lack any sense. It cannot even be formulated in scientific terms. The discussions of Part V will show this in more detail (§157 ff.).

23. *The Cultural Objects*

For philosophy, the most important types of objects, outside of the physical and the psychological ones, are the *cultural* (historical, sociological) *objects*.[26] They belong in the object domain of the cultural sciences.[27] Among the cultural objects, we count individual incidents and large scale occurrences, sociological groups, institutions, movements in all areas of culture, and also properties and relations of such processes and entities.

The philosophy of the nineteenth century did not pay sufficient attention to the fact that the cultural objects form an autonomous type. The reason for this is that epistemological and logical investigations tended to confine their attention predominately to physics and psychology as paradigmatic subject matter areas. Only the more recent history of philosophy (since Dilthey) has called attention to the methodological and object-theoretical peculiarity of the area of the cultural sciences.

The cultural objects have in common with the psychological ones the fact that they, too, are subject bound; their "bearers" are always the persons of a certain group. But, in contrast to the psychological objects, their bearers may change: a state or a custom can persist even though the bearing subjects perish and others take their place. Moreover, the cultural objects are not composed of psychological (much less physical) objects. They are of a completely different object type; the cultural objects belong to other object spheres (in a sense to be explained later on, §29) than the physical and the psychological objects. This means that no cultural object may be meaningfully inserted into a proposition about a physical or a psychological object.

Later on, in the context of construction theory, we shall show in what way the assertion of the unity of the entire domain of objects of knowl-

26 geistige Gegenstände
27 Geisteswissenschaften

edge refers to the derivation ("construction") of all objects starting from one and the same basis, and that the assertion that the various spheres of objects are different means that there are different constructional levels and forms. Thus the two apparently opposing positions are reconciled (cf. §41).

24. *The Manifestations and Documentations of Cultural Objects*

I wish to discuss here only the two most important relations between cultural and other objects, since knowledge of cultural objects, and thus their construction, depends entirely upon these relations. We call these two relations "manifestation" and "documentation".

A cultural object, which exists during a certain time, does not have to be actual (i.e., manifested) at all points during this span. The psychological processes in which it appears or "manifests" itself, we shall call its (psychological) *manifestation*. The relation of the (psychological) manifestation of a cultural object to the object itself, we shall call the *manifestation relation* (more precisely: the psychological-cultural or, more briefly, the psychological manifestation relation).

> EXAMPLE. This relation holds, for example, between the present resolve of a man to lift his hat before another man, and the custom of hat-lifting. This custom does not exist merely during those moments in which somebody somewhere manifests it, but also during the times in between, as long as there are any persons who have the psychological disposition to react to certain impressions by greeting somebody through lifting their hats. During the times in between, the custom is "latent".

A physical object can also be the manifestation of a cultural one. Thus, the custom of hat-lifting manifests itself, for example, in the appropriate bodily motions of a certain man. But closer scrutiny shows that, even here, the psychological manifestation relation is fundamental. Thus we shall always mean the latter when we simply speak of the manifestation relation.

We call *documentations* of a cultural object those permanent physical objects in which the cultural life is, as it were, solidified: products, artifacts, and documents of a culture.

> EXAMPLES. The documentations or representations of an art style consist of the buildings, paintings, statues, etc. which belong to this style. The documentation of the present railroad system consists of all stationary and rolling material and the written documents of the railroad business.

It is the task of the cultural sciences to deal with the correlation problems of the manifestation and documentation relation. These sciences have to ascertain in which acts (in the physical and psychological sense) the individual cultural objects become overt and manifest themselves. In so doing they form, as it were, definitions for all the names of cultural objects. On the other hand, the documentation relation is of special importance for the cultural sciences, because the research into no longer existing cultural objects (and these, after all, form the larger part of the domain) rests almost exclusively upon conclusions drawn from documentation, namely, from written records, illustrations, things that have been built or formed, etc. But these conclusions presuppose that the documentation correlation (that is, the answer to the correlation problem of the documentation relation) is known. Thus, for the cultural sciences, the tasks of providing definitions, and of finding criteria for the recognition of their objects will be fulfilled by resolving these two correlation problems.

As with the relations which we considered earlier (§§21, 22), here, too, examination of the correlation problems is part of the task of the special sciences. The study of the essence problems, on the other hand, belongs to metaphysics. I do not wish to discuss at this time the attempted solutions of the essence problems (e.g., emanation theory, incarnation theory, psychologistic and materialistic interpretation). Here we find a situation very similar to that which held for the earlier essence problems: a struggle between divergent opinions, where there seems to be no possibility that a decision can be made through empirically obtained information.

25. *The Multiplicity of Autonomous Object Types*

After the physical, the psychological, and the cultural object types, I wish to give some examples of further autonomous object types. In the sequel, we shall reformulate the statement that each of these object types is "autonomous" by saying, more precisely, that they belong to different "object spheres" (§29). Later on, after we have given an account of construction theory, we shall have to ascertain whether the conceptual system which is based on this theory, namely, the "constructional system" (Part IV), provides a place for each of the object types which we have just mentioned.

Later (§41), we shall show that the assertion of the multiplicity of independent object types only apparently contradicts the thesis of the unity of the object domain.

EXAMPLES. *Logical objects*: Negation, implication, indirect proof. These are logical objects in the narrower sense, i.e., excluding the mathematical objects which are closely connected with them, but which, in accordance with the customary separation of the sciences, we shall not mention at this time. However, the boundary line is somewhat arbitrary. (The logical objects will later on be incorporated into the "constructional system" of concepts [they will be "constructed"] § 107.)

Mathematical objects: The number 3, the class of all algebraic numbers, the equilateral triangle. The triangle is here to be understood, not in the concrete-spatial, but in the mathematical-abstract, sense (construction of mathematical objects: § 107).

The object type of spatial configurations: The sphere, the equilateral triangle. Here, these expressions are not meant as expressions of abstract, nonspatial geometry, but in their ordinary, concrete-spatial sense (cf. mathematical objects). Physical objects are to be distinguished sharply from spatial configurations, since the latter lack the determinations of time, space, color, weight, etc. (construction of the spatial configurations: § 125).

The object type of the colors: grey, red, green. The colors do not have any determination of time or space (they are meant in the purely phenomenal sense); also, they are not, strictly speaking, determined as to color, weight, or other sensory qualities; this distinguishes them from physical objects. The difference between the colors and the psychological objects consists in the difference between the contents of a representation and the representation. (Construction of colors: §118; in order to construct them as intersubjective objects, we would still have to apply to them the procedures of §149. The same holds also for the constructions that are given below.)

The object type of *pitches*: c, e, the chord c-e-g. The object types of odors and tastes are also to be mentioned as independent object types, just as colors and pitches (construction of the sensory qualities: §§ 131, 133).

Biological objects: the oak, the horse. (Both of them are to be understood as species and not as individuals.) Such a biological object is not a sum of physical objects, but a complex of them; that is to say, a class; about the difference between a complex and a collection, cf. § 36, specifically between class and collection, § 37 (construction of biological objects: § 137).

Ethical objects: duty, obedience, ethical value (of an act). About the difference between these and psychological objects, cf. what has been said in connection with the colors (construction: § 152).

It can easily be seen that this list of object types can be continued, but it should suffice for our purposes. It shows that there is a multiplicity of object types, and it can be used to test the adequacy of a system of objects, in our case, of the constructional system.

Summary

II. PRELIMINARY DISCUSSIONS (10-25)

A. *The Form of Scientific Statements (10-16)*

A *property description* of a domain indicates the properties of the individual objects of that domain; a *relation description* indicates merely the relations between the objects. Construction theory views the latter as more fundamental (10). Two relations are said to be "isomorphic" or "of the same structure" if they agree in their formal properties, more precisely, if there is a one-to-one correspondence between them (to help visualize this: two relations are isomorphic if they have the same arrow diagram). That which is common to isomorphic relations (in the terminology of logistics: the class of these relations) is called their *structure* (II). A relation description is called a *structure description*, if the relations which occur are not themselves mentioned but only their structure is indicated. A structure description is given either through an (unnamed) arrow diagram or through a list of number-pairs. The structure description forms the highest level of formalization in the representation of a domain. Thesis: the representation of the world in science is fundamentally a structure description (12). By the *definite description* of an object is meant a unique characterization of that object, i.e., a characterization which allows an unequivocal identification of that object in the object domain in question (13). Thesis: every object of science can be uniquely characterized within its object domain through mere structure statements (14, 15). Hence it is in principle possible to transform all statements of science into *structure statements*; indeed, this transformation is necessary if science is to advance from the subjective to the objective: all genuine science is structural science [28] (16).

B. *Survey of the Object Types and Their Relations (17-25)*

In order to obtain a preliminary, very rough, division, we distinguish *physical*, *psychological*, and *cultural*, objects. The expressions "physical" and "psychological" are here taken in their customary sense; by "cultural" objects we mean objects of the cultural sciences (or *Geisteswissenschaften*): cultural or sociological events, states and entities (18,23). The *psychophysical relation* is the relation between a psychological process and the parallel process in the nervous system. The *expression relation* is the relation between

28 Strukturwissenschaft

a motion, a facial expression, or a vocal utterance of a person, and the psychological process which can be recognized in this utterance. The *designation relation* is the relation between a physical sign (written symbol, sound, badge, etc.) and that which is designated (19). For each relation there arises a *correlation problem* (which objects have this relation to one another?) and an *essence problem* (what is the nature of the relation? what is it that connects the correlated objects? (20). To investigate the correlation problems of the relations mentioned above is part of the task of science (namely of psychology and physiology; psychology and characterology; and various branches of semiotics, respectively). On the other hand, the solution of the essence problems of those relations does not consist in the ascertainment of facts, but in their interpretation; it is not a task of science. This is already indicated in the fact that various contradictory solutions have been proffered between which no (conceivable) experience could decide. Hence essence problems must be transferred from science to metaphysics; this is particularly obvious with the psychophysical problem (21, 22).

The psychological events in which a cultural object (cultural event) appears are called the latter's *manifestations*; the physical objects in which a cultural object is reflected are called its *documentations*. The correlation problem of these two relations is investigated in the cultural sciences, while the essence problem is again to be referred to metaphysics (24). The three indicated object types are merely the most important examples; there is a large number of other autonomous object types (25).

PART THREE

THE FORMAL PROBLEMS OF THE CONSTRUCTIONAL SYSTEM

CHAPTER
A

THE ASCENSION FORMS

26. *The Four Main Problems of Construction Theory*

The aim of construction theory consists in formulating a constructional system, i.e., a stepwise ordered system of objects (or concepts). The stepwise ordering is a result of the fact that the objects on each level are “constructed” from the objects of the lower levels in a sense to be made precise later. In the formation of such a system, the following four main problems are encountered. To begin with, a basis must be chosen, a lowest level upon which all others are founded. Secondly, we must determine the recurrent forms through which we ascend from one level to the next. Thirdly, we must investigate how the objects of various types can be constructed through repeated applications of the ascension forms. The fourth question concerns the over-all form of the system as it results from the stratified arrangement of the object types. We call these four problems the problems of *basis*, *ascension form*, *object form*, and *system form*. The problems of basis, object form, and system form are closely connected with one another. Their solutions are dependent upon one another, since the construction of the objects, and thus the form of the system, depends upon the choice of the basis, while the basis is chosen so as to allow the construction of all object types from it. On the other hand, the problem of the ascension forms depends less upon the efficacy which we require of the system as a whole and is also less complicated. While the basis of the system consists of extralogical entities which must be chosen from an unlimited number of candidates, the ascension forms must be chosen from a small number of logical forms as simple and as few in number (namely, two) as we maintain. This result will follow from later considerations concerning definition as a form of construction (§§ 38-40). A confirmation of this view can only be found in the actual formation of the constructional system itself (Part IV).

The problems of the basis, the object forms, and the system form will be dealt with in the later chapters of this part (B-D), where we must take into account empirical facts, namely, the properties and relations of objects which are investigated in the special sciences. Subsequently we shall consider the symbolic and linguistic forms which will be used to represent the constructional system (Chapter E). Here, (in Chapter A), the formal-logical problem of the ascension forms is to be resolved.

27. *The Quasi Objects*

We can divide (linguistic) signs into those which have independent meaning and those which have meaning only in connection with other signs. Strictly speaking, only those (mostly complex) signs which designate a proposition, i.e., *sentences*, have independent meaning. Among the signs which are not themselves sentences and which occur in science only as parts of sentences, we wish to distinguish the so-called proper names, i.e., signs which designate a definite concrete individual object (e.g., “Napoleon”, “moon”) from the other parts of sentences. The traditional view is that the proper names have a relatively independent meaning and are thereby distinguished from the other signs. These other signs we call, after Frege, *incomplete symbols*.[29]

It should be noted that this distinction is not logically precise. We make it to follow an established tradition and shall not attempt to give a more precise definition of the concept of “proper name”. Perhaps there is only a difference in degree and the choice of a boundary line is arbitrary; at least, this seems to be the upshot of the later discussions on individual and general objects (§ 158).

---

29 ungesättigte Zeichen

In the original usage of signs, the subject position of a sentence must always be occupied by a proper name. However, it proved advantageous to admit into the subject position also signs for general objects and, finally, also other incomplete symbols. This improper use, however, is permissible only when a transformation into proper use is possible, i.e., if the sentence can be translated into one or more sentences which have only proper names in their subject positions. More about this later. Thus, in improper use, incomplete symbols are used if they designated an object in the same way as an object name. One even speaks of "their designata", consciously or unconsciously introducing the fiction that there are such things. We wish to retain this fiction for reasons of utility. But, in order to remain perfectly aware of this fictional character, we will not say that an incomplete symbol designates an "object", but that it designates a *quasi* object. (In our view, even the so-called "general objects", e.g., "a dog" or "dogs" are already quasi objects.)

> EXAMPLES. If, for example, "Fido" and "Caro" are proper names of dogs, then the sentences "Fido is a dog" and "Caro is a dog" have the common constituent "... is a dog". This is an incomplete symbol (a propositional function, cf. § 28). Analogously, one finds as the common constituent of other sentences "... is a cat". This shares with the previous one the constituent "... is ...", while the remainders "... a dog" and "... a cat" remain as incomplete symbols of a different sort. Let us now try to express the fact that all dogs are mammals. If we wanted to retain the sentence form "... is ...", where the subject position is properly occupied by an object name, we would have to form the following complicated sentence: "it holds for all values of variable *x* that "*x* is a dog" implies "*x* is a mammal". Instead, we form a new sentence form by allowing ourselves to introduce an incomplete symbol into the subject position as if it were an object name. We say, "A dog is a mammal." In this sentence, no object name occurs, but we say about the incomplete symbol "a dog" that, while it does not designate an object, it designates a quasi object (since it occupies a position in the sentence as if it designated an object).
>
> If we want to get a more precise grasp of the indicated relations, then we will have to replace by logical symbols all those parts of the sentences which designate not extralogical entities, but logical relations. The meaning of these symbols becomes apparent through a comparison with the above-mentioned sentences ("logistic formulation of the logical skeleton," § 46). To begin with, we have the sentences, "Fido ε dog", "Caro ε dog", then the incomplete symbols "... ε dog" and "... ε cat" (or "*x* ε dog", and "*x* ε cat"); these designate propositional func-

tions. Furthermore, we have the incomplete symbols "dog" and "cat", which designate classes. In the sentence "dog $\subset$ mammal", the class symbol is used like an object name. (About $\subset$, cf. § 33.) Since all class symbols are introduced for precisely this purpose, it follows that all classes are quasi objects (§ 33).

The form of the sentence "dog $\subset$ mammal", as containing no object symbols, only class symbols, can be justified only through the fact that it can be transformed into a sentence in which only proper names occur in the subject position, namely, into the above-mentioned sentence with variable x. Further investigation would show that the classes "dog" and "mammal" are complexes of individual animals (§ 36).

The "objects" of science are almost without exception quasi objects. Present-day nominalism would find this quite acceptable if it held merely for general concepts (cf. § 5), but it also holds for most individual objects of scientific investigation, as construction theory will show (cf. § 158 about individual and general objects). The two *ascension forms* of construction which will be used in our system and which will be discussed in the sequel *are forms of quasi objects*.

REFERENCES. The theory of the incomplete symbols originated with Frege [Funktion], [Grundges.] 1, 5; Russell gives extensive comments [Princ. Math.] I, 69ff., [Math. Phil.] 182ff. As indicated, our position is even more radical, but we cannot give a full account of the matter at this time.

The position which treats general objects as quasi objects is closely related to *nominalism*. It must be emphasized, however, that this position concerns only the problem of the *logical function* of symbols (words) which designate general objects. The question whether these designata have reality (in the metaphysical sense) is not thereby answered in the negative, but is not even posed (cf. **VD**).

28. *Propositional Functions*

If we delete from a sentence one or more object names (i.e., at first proper names, but then also names of quasi objects), then we say of the remaining incomplete symbol that it designates a *propositional function*. By introducing the deleted names as *arguments* into the blanks (the argument positions), we regain the original sentence. But, in order to produce some sentence, either true or false, we do not have to introduce precisely the deleted names, but can take others as long as they make sense together with the incomplete symbol. We call them *permissible arguments* of the propositional function. Instead of leaving the argument positions empty, it is better to mark them with the sign of a *variable*.

If the introduction of an object results in a true sentence, then we say that this object *satisfies* the propositional function. All other objects, as long as they are permissible arguments, result in a false sentence. A propositional function with precisely one argument position we call a *property* or *property concept*. All objects which satisfy this function "have" the property or "fall under" the (property) concept. A propositional function with two or more argument positions we call a (two-place or many-place) *relation* or a *relational concept*. Of the pairs, triples, etc., which satisfy this function, we say that the relation "holds for them" or "obtains" between them or the objects "stand in this relation" to one another. Thus every propositional function represents a concept, viz., either a property or a relation.

> EXAMPLES: Propositional functions: a. *Property*. From the sentence "Berlin is a city in Germany" results, by deletion of the subject term "Berlin", a propositional function with one argument position, namely, "... is a city in Germany" or "x is a city in Germany". It represents the property of being a city in Germany, or, more briefly, the concept "city in Germany". This incomplete symbol is turned into a true sentence by substituting the name "Hamburg", into a false sentence through substitution of the name "Paris", while substituting the word "moon" produces a meaningless string of words. Thus we say that Hamburg, but not Paris, falls under the concept "city in Germany"; while the object moon neither does nor does not fall under this concept, for the moon is not, unlike Berlin and Paris, a permissible argument of the function.
>
> b. *Relation*. From the sentence "Berlin is a city in Germany" results, by deletion of the two object names "Berlin" and "Germany", a propositional function with two argument positions, namely, "... is a city in..." or "x is a city in y". It represents the two-place relation between a city and the country in which this city lies. This incomplete symbol is turned into a true sentence by substituting the pair of names "Munich, Germany", into a false sentence through substitution of the words "Munich, England", and into a meaningless string of words by substituting the words "moon, Germany". Thus, Munich stands to Germany, but not to England, in the indicated relation, while one may not assert either that the relation holds, or that it does not hold, for the pair "moon, Germany".

29. *Isogeny; Object Spheres*

Two objects (and this always includes quasi objects) are said to be *isogenous* if there is an argument position in any propositional function for which the two object names are permissible arguments. If this is the case, then it holds for any argument position of any propositional function either that both names are permissible arguments, or that neither of them is. This is a consequence of the logical theory of types, which we cannot here discuss in detail. If two objects are not isogenous, then they are termed *allogeneous*.

> EXAMPLES. In Example a) of the preceding section, Hamburg and Paris turned out to be isogenous; the moon, on the other hand, was allogeneous relative to both Hamburg and Paris. In Example b), Berlin and Munich showed themselves to be isogenous, and also Germany and England. "Moon, Germany" was not a permissible pair of arguments. From this it does not follow that neither of them is a permissible argument for the position in question, but that at least one of them is not. Since Germany was a permissible argument for its position, it follows that the moon is not a permissible argument. Thus the moon is allogeneous relative to both Berlin and Munich.

By the *sphere* of an object we mean the class of all objects which are isogenous with the given object. (Since isogeny is transitive, the object spheres are mutually exclusive.) If every object of a given object type is isogenous with every object of another object type, then we call the object types themselves "isogenous". Correspondingly, we also speak of "allogeneous" object types. For *pure* object types, these are the only possible cases; that is to say, we call an object type pure, if all its objects are isogenous with one another, i.e., if the type is a subclass of an object sphere. All other types we call *impure*. Only the pure types are logically unobjectionable concepts; only they have classes as extensions (cf. § 32f.). However, in the practical pursuit of science the impure types play an important role. Thus the main object types, namely, the physical, the psychological, and the cultural are impure types, as we shall see.

30. *"Confusion of Spheres" as a Source of Error*

If we wish to test whether or not two objects are isogenous, and if the statements about these objects are expressed in a word language, then we have to ascertain ultimately whether or not a string of words forms a meaningful sentence. This test frequently becomes quite complicated through a special sort of ambiguity of language. This ambiguity is frequently overlooked and has thereby caused considerable philosophical difficulties; in particular, it has very markedly retarded progress in the task with which we are presently concerned, namely, the formation of a conceptual system, and it complicates this task even now. We are here not concerned with straightforward ambiguity (homonymy) as it occurs, e.g., in such words as “cock”, “spring”, etc., nor with somewhat more subtle ambiguities as they occur in many expressions of ordinary life, of science and of philosophy, as, for example, in the words “representation”, “value”, “objective”, “idea”, etc. In our daily lives, we are well aware of the first type of ambiguity, while in philosophy we concern ourselves with the second, and we can thus avoid at least the more obvious mistakes. Let me explain, by way of example, the third type of ambiguity, the one which concerns us here. The expression “thankful” seems unambiguous when it is taken in its root sense (i.e., setting aside any use of the term in a metaphorical sense; this would fall under the second type of ambiguity considered above, e.g., when “thankful” is used relative to a task or work). However, we not only say of a person that he is thankful, but also of his character, of a look, of a letter, of a people. Now each of these five objects belongs to a different sphere. It follows from the theory of types that the properties of objects which belong to different spheres themselves belong to different spheres. Thus, there are five concepts, “thankful”, which belong to different spheres, the confusion of which would lead to contradictions. However, generally speaking, there is no danger that we might draw an invalid conclusion since precisely the fact that these objects are of different spheres keeps us from misunderstanding which of the five concepts is meant. In general, using only one word for these different objects is innocuous, and therefore useful and justifiable. This ambiguity must be noted only if finer distinctions between concepts are to be made, distinctions which are important for epistemological and metaphysical problems. Neglect of the difference between concepts of different spheres, we call confusion of spheres.

REFERENCES. There has been no explicit recognition of the indicated type of ambiguity in logic. But it bears a certain resemblance to the multiplicity of “suppositions” of a word which the Schoolmen used to distinguish; cf. Erdmann [Bedeutung] 66 ff. It is more closely related to the *theory of types* which Russell has developed in order to overcome the logical paradoxes and which he has utilized in his logistic system [Types], [Princ. Math.], 1, 39 ff., 168 ff., [Math. Phil.] 133 ff., cf. Carnap [Logistik] § 9. However, Russell has applied this theory only to formal-logical structures, not to a system of concrete concepts (more precisely: only to variables and logical constants, not to nonlogical constants). Our object spheres are Russell's “types” applied to extralogical concepts. Thus, the justification for making a distinction between the various object spheres and for claiming that there were five concepts, "thankful", in the preceding example is derived from the theory of types, even though the examples may not have sounded very persuasive since they were given in a word language. Although the theory of types is not generally accepted, none of its opponents has been able to produce a logical system which could avoid the contradictions (the so-called paradoxes) from which the older logic suffers, without using a theory of types.

That the indicated ambiguity can become a source of error in the investigation of isogeny is obvious from an inspection of the five objects of which thankfulness can be asserted and which one could erroneously presume to be isogenous on the basis of the criterion of § 29. The following example will make the matter more clear.

31. *An Application*

EXAMPLE. Let us investigate, to begin with, which objects are isogenous with a (definite, particular) stone. Sentences about this stone would be, for example, "The stone is red", "The stone weighs 5 kg.", "The stone lies in Switzerland", "The stone is hard". These sentences are unquestionably meaningful; it makes no difference whether they are true or false. Now we must substitute in these sentences the names of the objects we want to test, and must ascertain whether or not the sentences still have a meaning. It is of no concern whether the sentences become true or false. If we wish to apply the test to another stone or to a chicken, we will find that meaningful sentences result. Thus these objects are isogenous with the first stone (if we were to continue our investigation, we should find that they all belong to the sphere of physical bodies). On the other hand, the following list of objects which begins with the stone, does not contain any further object which is isogenous with the stone, for in no case do we obtain a meaningful sentence when we substitute the appropriate name for the name of the stone.

List of representative objects: (physical [30] objects): a particular stone, aluminum; (psychological objects): a (certain, particular) worry, the vivacity of Mr. N.; (cultural objects): the constitution of the Reich, expressionism; (biological objects): the Mongolian race, heredity of acquired traits; (mathematical and logical objects): the Pythagorean theorem, the number 3; (phenomenal [31] objects): the color green, a certain melody; (objects of physics [32]): the electrical elementary quan-

30 physisch
31 sinnesphänomenologisch
32 physikalische Gegenstände

tum, the melting point of ice; (ethical object): the categorical imperative; (temporal object): the present day.

The following cases will show how the above-mentioned ambiguity (confusion of spheres) renders more difficult the test for isogeny and increases the likelihood of error: the sentences, “The stone is hard” and “The stone is red” seem to be meaningful also for aluminum, i.e., the first sentence true and the second false. Only the realization that the other two sentences about the stone (“It weighs 5 kg.”, “It lies in Switzerland”) are meaningless for aluminum shows that the two objects belong to different spheres. This leads to a more detailed investigation of the matter and leads to the recognition that the properties “red” and “hard” relative to a thing are not the same as the properties “red” and “hard” relative to a substance.

The example shows that it is frequently necessary to consider several different sentences in testing isogeny. Otherwise, one may be misled by the fact that words are frequently impure as far as spheres are concerned.

A more detailed investigation of the above list of objects would show that the indicated objects all belong to different spheres. One can show this for the first object, the stone, by means of the above-stated four sentences. We have already seen that some of these sentences seem to indicate isogeny with other objects on the list. Taken together, however, they show that the stone does not belong to the same sphere with any of the objects mentioned subsequently. There is no other object name on the list which would result even in apparently meaningful sentences in all four cases. The test for every other object on the list could be carried out accordingly.

The fact that the objects on the list belong to different object spheres means that each of them represents a different object sphere. Now the list can easily be extended at will in such a way that all further objects are also all from different spheres; thus we see that the number of different object spheres is large. At the moment, there is no way of telling whether this number is finite. In other words, not only is the number of object types which are coordinated with one another (e.g., as the species in a classification) very large, but also the number of object types which are *toto coelo* different from one another. (They are *toto coelo* different from one another in that each of them has its own *coelum*, its own object sphere.)

In the list of objects given above, several object types are represented by more than one object. Since these objects are not isogenous, this means that these object types are impure. It holds almost without exception that the traditional object types which are found in the sciences are almost always impure, i.e., they are not logically permissible concepts (e.g., physical, psychological, etc.).

32. *The Extension of a Propositional Function*

If two propositional functions stand to one another in such a relation that every object (or couple, triple, etc.) which satisfies the first also satisfies the second, then we say that the first *universally implies* the second. If two propositional functions stand to one another in a relation of mutual universal implication, then they are called *universally equivalent* or *coextensive*. Hence, coextensive propositional functions are satisfied by exactly the same arguments. If we assign the same symbol to propositional functions that are coextensive and if we from then on use only these new symbols, then we obviously disregard all points of difference between coextensive propositional functions and express only those factors in which they agree. Such a procedure we call an *extensional procedure*; the symbols which are the same for all coextensive propositional functions we call *extension symbols*. They have no independent meaning and they may be used only if we indicate for all sentence forms in which they are to be used how such sentences may be transformed into sentences in which extension symbols no longer occur; thus, in translating back, we replace these symbols by the appropriate propositional functions (more precisely, each extension symbol is replaced by any one of the coextensive propositional functions to which it was assigned). The extension symbols have no independent meaning, i.e., they are incomplete symbols (to an even higher degree than propositional functions). Nevertheless, in conformity with the customary usage, we speak of them as if there were objects which they designate. These objects we call extensions. Thus, extensions, too, are quasi objects. For example, we say of two coextensive propositional functions that they have the same extension (hence the word "coextensive"), since they have assigned to them the same extension symbol. Furthermore, if there are two propositional functions which are so related that each object (couple, triple, etc.) which satisfies the first also satisfies the second, then it is easily seen that the relation of universal implication is also fulfilled if each of these two propositional functions is replaced by another, co-extensive, one. It is for this reason that we can express this relation with the aid of extension symbols; the symbol $\subset$ between two extension symbols is defined as indicating the universal implication between the corresponding propositional functions. If we now speak again as if there were objects which are designated by extension symbols, we may say the sentence "$a \subset b$" means "(the extension) a is contained in (the extension) b." We call this relation between two extensions *inclusion* or *subsumption*.

Given a propositional function, we form a *symbol for its extension* by enclosing the propositional function in parentheses and by writing in front of it the appropriate variables with accents: $\hat{x}\hat{y}$ (... x ... y...). We shall give examples during the following discussion of the two lands of extensions, namely, classes and relation extensions.

33. *Classes*

The extension of a propositional function with only one argument position, i.e., the extension of a property, is called a *class*. Thus, coextensive properties have the same class. An object *o* which satisfies a given propositional function is called an *element* of the corresponding class, call it *a*. (In symbols, $o \in a$)', *o* "belongs to" class *a* (not "is included in"!). If a class a is included in class b (in the above-defined sense of subsumption), then a is called a *subclass* of b (in symbols: $a \subset b$).

Let me briefly discuss some main concepts of the theory of classes. The class of objects which do not belong to a certain class *a* is called the "negate" or "complement" of *a* (in symbols, —*a*). —*a*, of course, does not comprise all remaining objects, but only the permissible but not satisfying arguments. To the "intersection" of two classes ($a \cap b$) belong all those objects which are elements of both *a* and *b*. To the "union" of two classes ($a \cup b$) belong those objects which are elements of at least one of them. The union of a class with its complement forms the object sphere of elements of this class, for it comprises all permissible arguments of the corresponding propositional function.

Classes, since they are extensions, are quasi objects. Thus the class symbols do not have independent meaning; they are merely aids for making statements about all the objects which satisfy a given propositional function without having to enumerate them one by one. Thus the class symbol represents, as it were, that which these objects, i.e., the elements of that class, have in common.

EXAMPLE. Let us assume, for example, that the propositional function "x is a man" is satisfied by the same objects as the propositional function "x is a rational animal" and "x is a featherless biped." Thus these three propositional functions are coextensive, and we assign to them the same extension symbol, e.g., *ma* (i.e., we define: *ma* $=_{df} \hat{x}$ (x is a man), cf. §32). Since this is a propositional function with only one argument position, *ma* is a class symbol. Moreover, *ma* is an incomplete symbol; by itself it means nothing, but the sentences in which it occurs have meaning, since it is clear how this class symbol can be eliminated from them. For example, the sentence "d $\in$ *ma*" can be transformed into the sentence "*d* is a man" or "*d* is a featherless biped". Even though *ma* itself does not designate anything, one speaks of "the designatum of *ma*" as if it were an object. We want to be somewhat more cautious and call it a quasi object. It is "the class of all men", i.e., the extension of the propositional function "*x* is a man".

We must emphasize the fact that classes are quasi objects in relation to their elements, and that they belong to different spheres. This is important because a class is frequently confounded with the whole that consists of the elements of that class. These wholes, however, are not quasi objects relative to their parts, but are isogenous with them. We shall discuss the difference between classes and wholes, and the fact that elements belong to different spheres from their classes, more thoroughly in the sequel (§ 37).

REFERENCES. The theory of propositional functions and their extensions originated with Frege [Funktion] [Grundges.] (Frege calls them *Wertverläufe*) and was utilized by Whitehead and Russell in their system of logistics ([Princ. Math.], cf. also [Math. Phil.], 157ff.). A good exposition is also found in Keyser [Math. Phil.] 49ff.; Keyser gives an interesting elaboration of the concept of a propositional function in the form of his "doctrinal function" ("Theoriefunktion", 58 ff.). Cf. Camap [Logistik] § 8.

Frege has already shown that extension symbols, and thus the class symbols, are *incomplete symbols* (cf. the quotations in § 27). According to Russell, it is irrelevant for logic whether or not there are actual objects which are designated by class symbols, since classes are not defined by themselves, but only in the context of total sentences ("no class theory"). More recently, Russell has expressed himself even more strongly and has called classes logical fictions or symbolic fictions [External W.] 206 ff., [Math. Phil.] 182ff.This corresponds to our notion of classes as quasi objects. Furthermore, according to Russell, classes are sharply distinguished from their elements in that no statement can be meaningful for a class (i.e., either true or false), if it is meaningful for one of its elements (theory of types). This corresponds to our notion that classes and their elements belong to different spheres (§37).

34. *Relation Extensions*

The extension of a propositional function with several argument positions, i.e., of a relation [33] is called a *relation extension*.[34] Thus relation extensions stand in exact formal analogy to classes which are the extensions of propositional functions with only one argument position, i.e., properties. Hence we can here be somewhat briefer, since by virtue of the analogy some points will be clear without further explanation. Like classes, relation extensions are *quasi objects*.

Coextensive relations correspond to the same relation extension. A pair of objects *x*, *y* (the same holds for triples, quadruples, etc.) which satisfies a given propositional function and thus all propositional functions coextensive with it is called an *ordered pair* (or triple, etc.) of the relation extension which corresponds to the propositional function (xQy, where Q designates the relation extension). Since it is not generally permissible to interchange the argument positions of a propositional function, the different members of an ordered pair (or triple, etc.) must be differentiated. In an ordered pair (i.e., in the case of a two-place relation extension), we call them referent and relatum. Relation extensions have the capacity to produce order, and this capacity derives from the differentiation between their various argument positions. Hence the importance of the theory of relations for the exhibition of order in any subject area whatever.

Relation extensions are quasi objects. Nevertheless, in order to aid intuition, language treats them as if they were a third thing which is suspended between the two members. Through this reification, the linguistic expression becomes more graphic, and it is not often dangerous since we are for the most part conscious of it as a figurative and improper mode of speech. For the sake of simplicity, we follow common usage in this case as well and use symbols of relation extensions as if they were names of objects, but, in order to emphasize the improper mode of expression, we call them quasi objects.

Let us briefly indicate some main concepts of the elementary theory of relations. The class of the possible referents of a relation extension Q is called the "domain" of Q (in symbols: D'Q). The class of possible relata is called the "converse domain" ('Q). If domain and converse domain are isogenous with one another, then the relation extension is

---

33 Beziehung
34 Relation

called *homogeneous*, m this case there exists a union of domain and converse domain, called the *field* of Q (C'Q). The relation extension which holds for all Q pairs in opposite direction is called the converse of Q ($\breve{Q}$). If *a*P*b* and *b*Q*c* both hold, then there is a relation extension for *a* and c, called the *relative product* of P and Q (P|Q). *Powers of relations*: R^2 means R|R, R^3 means R^2|R, etc. R_{po} means the union of the powers (power relation or chain), R^0 means identity in the field of R.

The concepts of symmetry, reflexivity, transitivity, and connectedness have been explained above (§ 11). A relation extension is called *one-many* if, for each relatum, there exists only one referent; it is called *many-one* if, for each referent, there exists only one relatum. If both conditions are fulfilled, it is called *one-one*.

A relation extension R is called a *correlator* between two relation extensions P and Q if it establishes a one-one correspondence between the elements of P and the elements of Q such that to each pair of P there corresponds a pair of Q and vice versa. If such a correlator exists for two relation extensions P and Q, then P and Q are called *isomorphic* or *of the same structure*. This corresponds to our earlier graphic definition of structural equivalence with the aid of the arrow diagram. We now can give an exact definition of the structure or relation number [35] of a relation extension P: it is the class of relation extensions which are isomorphic to P. (Cf. the analogous definition of cardinal number, § 40.)

35. *Reducibility and Construction*

Above (§2), we have explained the concept of reducibility with the aid of the imprecise concept of the “transformation” of a statement. We must now indicate more precisely what is to be meant by a “transformation”. To this end we can now utilize the concept of coextensiveness (or universal equivalence) of prepositional functions (§ 32). We say that a proposition or prepositional function is “exclusively about objects *a*, *b*,...”if, in its written expression, there appear as extralogical symbols only “*a*”, “*b*”, ...; logical constants (§ 107) and general variables may also occur. If for each propositional function which is exclusively about objects *a*, *b*, *c* (where *b*, *c*, ... may be absent) there exists a coextensive propositional function exclusively about *b*, *c*, . . . then *a* is said to be reducible to *b*, *c*, . . . Thus we can say more briefly but with less precision that *an object is said to be “reducible” to others, if all statements about it can be translated into statements which speak only about these other objects*.

35 Relationszahl

In the simplest and most important case the object to be reduced occurs alone and without the other objects in the propositional function in question.

> EXAMPLE, "*x* is a prime number" is coextensive with "*x* is a natural number whose only divisors are 1 and *x* itself". The object (or concept) prime number is thereby reduced to these objects: natural number, 1, divisor.

The previously explained concept of a construction (§2) must now likewise be more precisely determined. By *constructing* a concept from other concepts, we shall mean the indication of its "constructional definition" on the basis of these other concepts. By a *constructional definition* of the concept *a* on the basis of concepts *b* and *c*, we mean a rule of translation which gives a general indication how any propositional function in which *a* occurs may be transformed into a coextensive propositional function in which *a* no longer occurs, but only *b* and *c*. In the simplest case, such a translation rule will consist in the prescription to replace *a* in all its occurrences by a certain expression in which only *b* and *c* occur ("explicit" definition).

If a concept is reducible to others, then it must indeed be possible in principle to construct it from them. However, if one knows that a concept is so reducible, one does not thereby know how it is constructed, since the formulation of a general transformation rule for all statements about this concept is a separate problem.

> EXAMPLE. The reducibility of fractions to natural numbers is easily understood, and a given statement about certain fractions can easily be transformed into a statement about natural numbers (cf. §2). On the other hand, the construction, for example, of the fraction 2/7, i.e., the indication of a rule through which all statements about 2/7 can be transformed into statements about 2 and 7, is much more complicated (cf. § 40). Whitehead and Russell have solved this problem for all mathematical concepts [Princ. Math.]; thus they have produced a "constructional system" of the mathematical concepts.

36. *Complex and Whole*

If an object is logically reducible to others, then we call it a logical complex or, in brief, a complex of these other objects, which we shall call its elements. According to what we have said above (§§ 33,34), classes and relation extensions are examples of complexes.

If an object stands to other objects in a relation such that they are its parts relative to an extensive medium, e.g., space or time, then we call the first object the *extensive whole* or, in brief, the *whole* of the other objects. The whole consists of its parts.

The difference between a complex and a whole must not be confounded with the difference between a “true whole” (“organic whole”, “Gestalt”) and “(mere) collection” (or “sum”); the second distinction is important for psychology and biology, but is not of the same fundamental importance for construction theory as the first one, since it is merely a differentiation between two types of wholes. Moreover, it is doubtful whether this second distinction is not actually a mere difference in degree, i.e., whether it is not perhaps the case that all wholes have, to more or less high degree, all properties which are generally attributed only to “true wholes”. Perhaps there are no mere collections at all. However, no final decision can be made in this matter, since so far no sufficiently precise definition of true whole and Gestalt is available.

> REFERENCES. Driesch ([Ordnungsl.] [Ganze] esp. 4) differentiates the whole (in the sense of the true or organic whole) from the sum by saying that it loses essential properties if a part is taken away from it. Gestalt theory is concerned with entities which are characterized by the fact that “properties and functions of a part depend upon its position in the whole to which it belongs” (Kohler [Gestaltprobl.] 514; cf. also Wertheimer [Gestaltth.]). The close connection between the two definitions is obvious; examples which fall under both definitions are: an organism as the whole of its members, a melody as the whole of its tones, a house as the whole of its stones. On the other hand, it is very difficult to find an example of a mere collection; even a stone as the collection of its molecules and a heap of stones as the collection of its stones are true wholes. It is questionable whether, for example, the totality of all the iron on earth could be called a mere collection.

To be sure, the concepts whole and complex are not mutually exclusive; but construction theory is especially concerned with those complexes which do not consist of their elements, as a whole consists of its parts. Such complexes we call *autonomous complexes*. Thus we differentiate a whole from an autonomous complex by the fact that in the former the elements are parts in the extensive sense; in the latter, they are not.

From the definition of construction and complex, it follows that, if an object is constructed from other objects, then it is a complex of them. *Thus all objects of a constructional system are complexes of the basic objects of the system*.

If we are concerned with a statement about a quasi object, i.e., a statement which is expressed in the form of a sentence in which an incomplete symbol occurs in a position where the sentence structure originally allows only an object name, then this use of the incomplete symbol must be defined; it must be possible to translate this sentence into another sentence, where we find only proper object names in argument (e.g., subject) positions. From this it follows that a quasi object which belongs to a certain object domain is always a complex of the objects of this domain; i.e., it is an autonomous complex and not the whole of its elements. For a whole is an object of the same object type as its elements. Since classes are quasi objects relative to their elements, it follows that they are autonomous complexes of these elements (cf. § 37); relation extensions are likewise autonomous complexes of their members.

37. *A Class Does Not Consist of Its Elements*

We say of a class and of a whole that they "correspond" to one another when the parts of the whole are the elements of the class. Since a whole can be divided into parts in various ways, there are always many classes which correspond to one whole. On the other hand, to each class there corresponds at most one whole, for the elements are uniquely determined through the class, and two objects which consist of the same parts are identical. Now, if a class were to consist of its elements (i.e., if it were identical with the whole that corresponds to it), then all those many classes which correspond to the same whole would be identical with one another. But, as we have seen, they are different from one another. *Thus, classes cannot consist of their elements as a whole consists of its parts*. Classes are quasi objects relative to their elements; they are complexes of their elements, and, since they do not consist of these elements, they are autonomous complexes of their elements.

The same holds for the mathematical concept of a *set*, which corresponds to the logical concept of a class. A set, too, does not consist of its elements. This is important to notice, since the character of a whole or a collection (or of an "aggregate") has erroneously been connected with the concept of a set ever since its inception (i.e., ever since Cantor's definition). In set theory itself, this notion does not generally have any consequences, but it seems to be the reason that the methodologically most advantageous and logically unobjectionable form of definition for the concept of power (or cardinal number), one of the central concepts of set theory, is frequently opposed (cf. § 41).

EXAMPLE. One can envisage the organs or the cells or the atoms as parts of the whole *dog*. On the other hand, the class of the organs of the dog, the class of its cells, the class of its atoms, are three different classes, each with different elements. Each of them has a different cardinality; consequently, they cannot be identical. All these different classes correspond to the whole which is the dog. Since these classes are not identical with one another, they can also not be identical with the whole which is the dog. They are of the same logical status because the various different partitions are of the same logical status; thus, it also cannot be that at least one of the classes is identical with the whole.

REFERENCES. Frege has already expressed very clearly the thesis of this section. “The extension of a concept does not consist of the objects which fall under the concept.” [Krit.] 455. Russell has made the same point by calling attention to unit classes and null classes [Math. Phil.] 184. Cf. also the very pertinent remarks of Weyl [Handb.] 11.

Thus, not only is it not the case that a class is identical with the whole corresponding to it; it even belongs to a different sphere. As we have seen, extensions are quasi objects relative to their elements. “Thus we see that it is part of the logical doctrine that an extension cannot be a permissible argument for the same argument position of the propositional function for which its elements are permissible arguments. Nothing can be asserted of a class that can be asserted of its elements, and nothing can be asserted of a relation extension that can be asserted of its members. (The well-known theorem of logistics, that one cannot say of a class either that it does, or that it does not, belong to itself, is only a special case of this.)

Since a whole is isogenous with its parts while a class does not belong to the same sphere as its elements, it follows that a class is allogeneous to the whole that corresponds to it.

EXAMPLE. The difference between a wall as the whole of its bricks and the class of these bricks becomes especially clear through the fact that the wall is, but the class is not, isogenous with the bricks. This follows from an application of the criterion which employs propositional functions (§ 29). The propositional functions “*x* is made of burnt clay”, “*x* is rectangular”, “*x* is hard”, are satisfied by a brick as well as by the wall; the propositional functions “*x* is of one color”, “*x* is (spatially) small” are satisfied by a brick, and either they or their negations are satisfied by the wall. At any rate, for all five propositional functions, brick and wall are permissible arguments. On the other hand, the class of bricks is not a permissible argument for any of these propositional functions. It is a permissible argument for the propositional function “x has the cardinal number 100”, “*x* is a subclass of the class of bricks in general”; for these, neither the wall nor a brick is a permissible argument.

38. *Construction Takes Place Through Definition*

If, in the course of the formation of the constructional system, a new object is “constructed”, then this means, according to our definition of construction, that it is shown how statements about it can be transformed into statements about the basic objects of the system or the objects which have been constructed prior to the object in question. Thus a rule must be given which enables us to eliminate the name of the new object in all sentences in which it could occur; in other words, a *definition* of the name of the object must be given.

Now, two different cases must be distinguished. In the simpler case, a symbol can be introduced which is composed out of already known symbols (i.e., out of the basic symbols and other already defined ones) such that this symbol can always be put in the place of a new object symbol if this is to be eliminated. Here, the construction takes place by way of an *explicit definition*: the new symbol is declared to have the same meaning as the compound one. In this case, the new object is not a quasi object relative to certain of the older objects, since what it is can be explicitly indicated. Thus, it remains within one of the already formed object spheres, even if we should consider it as a representative of a new object type. We have already seen that the differentiation of types, unlike the opposition between spheres, is not logically precise but depends upon practical purposes of classification.

The second case arises when no explicit definition is possible. In this case, a special kind of definition is required, namely, the so-called “definition in use”.

39. *Definitions in Use*

If no explicit definition is possible for an object, then its Object name, given in isolation, does not designate anything in the manner of already constructed objects; in this case, we are confronted with a quasi object relative to the already constructed objects. However, if an object is to be called “constructed on the basis of the previous objects,” then it must nevertheless be possible to transform the propositions about it into propositions in which only the previous objects occur, even though there is no symbol for this object which is composed of the symbols of the already constructed objects. Thus we must have a translation rule which generally determines the transformation operation for the statement form in which the new object name is to occur. In contrast to an explicit definition, such an introduction of a new symbol is called a definition in use (*definitio in usu*), since it does not explain the new symbol itself—which, after all, does not have any meaning by itself—but only its use in complete sentences.

REFERENCES. Cf. Russell [Princ. Math.] 1, 25, 69. The expression "implicit definition" is customary for an entirely different determination of objects through axiomatic systems and should be reserved for this purpose. (Cf. §15). Occasionally, when one is concerned with the contrast between implicit and explicit definitions, both, definitions in use and explicit definitions proper, are called "explicit definitions in the wider sense."

In order for a translation rule to be applicable to all sentences of a certain sentence form, it must refer to propositional functions. It must equate with one another the expressions for two propositional functions, one of which contains the new object name, while the other contains only old ones, and both of which must contain the same variables. Under these conditions, the second expression is to be considered a translation of the first. A simple consideration shows that we have to proceed in this way. If the expression which contains the new symbol were not to contain any variables (i.e., if it were not the expression for a propositional function, but for a proposition, i.e., a sentence), then the rule would not hold for different sentences, but only for this one. And if this expression contains variables, then the translation which is prescribed by the rule must contain the same variables, since otherwise it would not tell us how, in the application to a sentence which is to be translated, the object names which occur in the argument positions are to be transferred to the new sentence.

EXAMPLES. The form of an explicit definition is probably well enough known, but it is important that its distinction from a definition in use should become as clear as possible. If the number 1 and the operation + are known, then the other numbers can be defined explicitly: "$2 =_{df} 1 + 1$", "$3 =_{df} 2+ 1$", etc. ("=df" is to be read as "equals by definition" or "is always replaceable by").

Definition in use. Let us assume that the concepts of a natural number and of multiplication are known. The concept of prime number is to be introduced. The expression "prime number" cannot be defined explicitly in the way in which we have previously defined the symbols “2” and “3”. Thus it might seem as if a definition of the following form would be permissible: “prime numbers $=_{df}$ those numbers which . . .” or “A prime number $=_{df}$ a number which ...” But a definition of this form only appears explicit; this deception is brought about by the linguistic forms which make it appear as if such expressions as “the prime numbers” or “a prime number” designate objects, since they use such expressions as subjects of sentences. Expressions such as “those which . ..”or “a. ..”are already (very useful) abbreviations for *definitions in use*; they correspond to the class symbols of logistics. The prime number concept is not a proper object relative to the numbers 1, 2, 3, . . . Thus it can be defined only in use by indicating which meaning a sentence of the form “a is a prime number” is to have, where a is a number. This meaning must be indicated by giving a propositional function which means the same as the propositional function “*x* is a prime number,” and contains nothing but already known symbols, and which could thus serve as a translation rule for sentences of the form “n is a prime number”. Thus, we could define: “*x* is a prime number” $=_{df}$, “x is a natural number and has only 1 and x as divisors.”

40. *The Ascension Forms: Class and Relation Extension*

We have seen that the construction of an object has to take the form of a definition. Now a constructional definition is either explicit or it is a definition in use. In the first case, the object to be constructed is isogenous with some of the preceding objects (i.e., no new constructional level is reached through it). Thus, the ascension to a new constructional level takes place always through a definition in use. Now, every definition in use indicates that a propositional function which is expressed with the aid of a new symbol means the same as a propositional function which is expressed only with the older symbols. By “same meaning”, we mean that both propositional functions are satisfied by the same objects. A propositional function which is coextensive with another one (§ 32) is satisfied by the same objects as the latter; hence, in a contextual definition we can always replace the second propositional function by any other propositional function which is coextensive with it. Thus, a propositional function which is expressed with the aid of the new symbol is not associated with just a single, determinate, previously introduced propositional function *f*, but with all propositional functions that are co-extensive with *f*, in other words, the new propositional function is associated with the extension of *f*. Thus, we can interpret the new propositional function purely extensionally: we introduce the new symbol as an extension symbol. Thus, through a constructional definition which leads to a new constructional level, we always define either a class or a relation extension, depending upon whether the defining propositional function has only one argument position or whether it has several of them. Let us illustrate both forms with examples from arithmetic.

EXAMPLE. 1. *Class*. The cardinal numbers (or powers) are defined in logistics as classes of equinumerous classes (or "sets"). Two classes are said to be equinumerous if there exists a one-to-one correspondence between them. Thus, for example, all classes which have five elements are equinumerous; the class on the second level which has all these classes as elements is then called the "cardinal number 5". The development of arithmetic on the basis of this definition shows that this definition is formally unobjectionable and satisfactory, since it allows us to derive all arithmetical properties of cardinal numbers and does not lead to contradictions. Nevertheless, there has been much opposition to this definition, not from logical, but from easily understandable intuitive motives. For example, the class to which supposedly belong all the classes of five objects in the world seems to be so boundless and comprehensive that its identity with the clearly delineated arithmetical entity, the cardinal number 5, appears to be absurd. However, this illusion rests solely upon the intuitive substitution of the class by the corresponding whole, which we have discussed above (cf. § 37). While this substitution is frequently useful, in this case it leads to errors. Let us return to our example: the class of the fingers of my right hand is not the whole "my right hand", and the class of all such classes of five objects does not consist of all hands, feet, piles of five stones, etc. This boundless collection would of course be quite useless as an arithmetical entity. Rather, we cannot say what the class of the fingers of my right hand is, because this class is only a quasi object (i.e., an autonomous complex). A symbol introduced for it would not have any meaning by itself, but would only serve to make statements about the fingers of my right hand, without having to enumerate these five objects one by one (i.e., statements about that which they have in common, for example, the properties of form, color, and matter, which these five fingers share). Likewise, one cannot say what the class of all classes of five objects itself is (i.e., the class of those classes, the elements of which can be brought into a one-to-one correspondence to the elements of the class of the fingers of my right hand). It, too, is only a quasi object (i.e., an autonomous complex). If we introduce a symbol for it—for example, cl_5—this symbol does not designate a proper object, but merely serves to make statements about the elements of this class (i.e.

about all classes of five objects) without having to enumerate them one by one, which, in this case, would not be practicable anyway, because their number is infinite. Now, if cl_5 is a symbol which allows us to make statements about all those properties which all classes of five objects have in common, then what could be the difference between it and the arithmetical sign “5” (for the cardinal number)? The cardinal number 5 is a quasi object, just as the class cl_5 is; the symbol “5” does not designate a proper object, but only serves to make statements about those properties which all possible classes of five objects have in common. Thus, we see that the indicated definition of cardinal number does not replace the cardinal numbers by other schematically constructed entities, which have a certain formal analogy with cardinal numbers, but that this definition meets precisely the arithmetical concept itself. It is only the rarely articulated, but frequently tacit, conception of classes as wholes or collections which has obscured this fact.

REFERENCES. Frege was the first to give the above-mentioned definition of cardinal number: [Grundig.] 79 ff., [Grundges.] 1, 57. Russell has independently rediscovered it in 1901, and has applied it to the foundations of mathematics: [Principles] 114, [External W.] 199 ff., [Math. Phil.] 11, [Princ. Math.] 1.

Objections of the indicated kind were made against this definition, for example, by: Hausdorff [Mengenl.] 46, J. Koenig [Logik] 226 note, cf. Fraenkel [Mengenl.] [44]. The earlier Russell, in his attempt to stay as close as possible to common usage, has been guilty, in spite of his “no-class theory”, of being not decisive enough in rejecting the interpretation of classes as wholes [Princ. Math.], [External W.] 126. Lately, he has definitely emphasized the differences between a class and “a pile or collection,” in our terminology, wholes or collections [Math. Phil.] 184. Nevertheless, he believes that, with this definition of cardinal numbers, he has to accept an oddity, only in order to gain a definite, unambiguous concept [Math. Phil.] 18. Our conception agrees with that of Weyl [Handb.] 11.

EXAMPLE. 2. *Relation extension*. We have seen above that fractions can be reduced to natural numbers and that they therefore must be envisaged as complexes of natural numbers (§2). This means that fractions are independent complexes, namely, quasi objects, for they can be defined as relation extensions of natural numbers. For example, “$2/3 =_{df} \hat{x}\hat{y}$ (x and y are natural numbers, and it holds that $3x = 2y$)”.

41. *The Constructional Levels*

If, in a constructional system of any kind, we carry out a step-by-step construction of more and more object domains by proceeding from any set of basic objects by applying in any order the class and relation construction, then these domains, which are all in different spheres and of which each forms a domain of quasi objects relative to the preceding domain, are called *constructional levels*. Hence, constructional levels are object spheres which are brought into a stratified order within the constructional system by constructing some of these objects on the basis of others. Here, the relativity of the concept "quasi object", which holds for any object on any constructional level relative to the object on the preceding level, is especially obvious.

It is now clear how the two seemingly contradictory theses of the unity of the object domain (§ 4) and the multiplicity of independent object types (§25) are to be reconciled. In a constructional system, all objects are constructed from certain basic objects, but in step-by-step formulation. It follows from the construction on the basis of the same basic objects that statements about all objects are transformable into statements about these basic objects so that, as far as the logical *meaning* of its statements is concerned, science is concerned with only one domain. This is the sense of the first thesis. On the other hand, in its practical procedures, science does not always make use of this transformability by actually transforming all its statements. Most of the statements of science are made in the form of statements about constructed entities, not about basic objects. And these constructed entities belong to different constructional levels which are all allogeneous to one another. As far as the logical *form* of its statements is concerned, science therefore is concerned with many autonomous object types. This is the sense of the second thesis. The compatibility of these two theses rests on the fact that it is possible to construct different allogeneous levels from the same basic objects.

42. *Being and Holding* [36] (May be omitted)

Following an occasionally used terminology, one could speak of the different "modes of being" [37] of the objects of different object spheres. This expression is particularly apt to make clear that allogeneous objects are completely dissimilar and cannot be compared. Fundamentally, the difference between being and holding, of which so much has been made in recent philosophy, goes back to the difference between object spheres, more precisely, to the difference between proper objects and quasi objects. For, if a quasi object is constructed on the basis of certain

36 Sein nnd Gelten
37 Seinsarten

elements, then it “holds” for these elements; thus it is distinguished as something that holds from the elements which have being. That a relation (extension) “holds” between its members is familiar terminology; we are less accustomed to saying that a class “holds” for its elements, though the expression could here be used with the same justification, since the relationship is the same in both cases. Construction theory goes beyond the customary conception of being and holding by claiming that this contrast does not arise only once, that there is only one boundary between being and holding, but that this relationship, constantly repeated, leads from level to level: what holds for objects of the first level has a second mode of being, and can in turn become the object of something that holds of it (on a third level) etc. So far as construction theory is concerned, this is the logically strict form of the dialectic of the conceptual process. Hence the concepts being and holding are relative and express the relation between each constructional level and the succeeding one.

> EXAMPLE. Stepwise progress of construction, in which the relationship between being and holding recurs several times: Classes are constructed from things. These classes do not consist of the things. They do not have being in the same sense as the things; rather, they hold for the things. These classes, even though they hold of things, can now be envisaged as having a second mode of being. From them we can proceed, for example, to the cardinal numbers, which hold for these classes. (For the construction of cardinal numbers as classes of classes, cf. § 40.) Cardinal numbers belong to a third mode of being and allow us to construct the fractions as relation extensions which hold for certain cardinal numbers (cf. § 40). These fractions can also be reified [38] that is, they can be envisaged as belonging to a fourth mode of being, and can be made elements of certain classes which hold for them, namely the real numbers. The latter belong to a fifth mode of being, while the complex numbers, being relation extensions that hold for certain real numbers, belong to a sixth mode of being, etc.

This example involves only six steps, but it gives an idea that construction will lead to completely different kinds of objects if many such steps are taken. Eventually we shall arrive at objects which do not disclose, at first sight, nay for which it seems impossible, that they are constructed from the basic objects. Hence the appearance of paradox in Kronecker's saying that all of mathematics treats of nothing but natural numbers, and even more in the thesis of construction theory that

38 vergegenständlicht

the objects of all sciences are constructed from the same basic objects through nothing but the application of the ascension forms of class and relation extension.

43. *An Objection Against the Extensional Method of Construction*

We have seen earlier that a constructional definition in the form of a definition in use (§39) consists in declaring that two propositional functions have the same meaning. We have furthermore considered (§40) that the propositional function so introduced can be determined only as far as its extension is concerned and that it is therefore sufficient to introduce, by way of a constructional definition, merely the extension symbol of the propositional function rather than the propositional function itself. Through this procedure, concepts are defined only extensionally. We therefore speak of an *extensional method* of construction. It is based upon the *thesis of extensionality*: in every statement about a concept, this concept may be taken extensionally (i.e., it may be represented by its extension [class or relation extension]). More precisely: in every statement about a propositional function, the latter may be replaced by its extension symbol.

One could now object that difficulties might result from the extensional method when one proceeds from an extensionally defined concept to statements about it and then to other concepts. For traditional logic does not subscribe to the thesis of extensionality: it claims that not all statements about a concept can be brought into the form of an extension statement.

REFERENCES. The above objection is related to the old distinction between *extensional* and *intensional* logic. However, there is no exact criterion as to when a statement concerns the extension or the intension of a concept. This distinction became important when the first systems of logistics or symbolic logic (Boole, Venn, and Schroder) not only developed logic as merely extensional, but confined the field even more by taking subsumption to be the only statement form. Proceeding from Frege's theories, Russell then went beyond this narrow limitation: his system combines intensional with extensional logic. Frege was the first who made precise the much-discussed and age-old distinction between intension and extension of a concept by differentiating the concept as a function, the values of which are truth values, from its course of values [39] (in our terminology, “propositional function” and “extension”). Utiliz-

---

39 Wertverlauf

ing this distinction, Russell developed intensional logic as the theory of propositional functions and extensional logic as the theory of extensions (classes and relation extensions). In this system, even extensional logic contains not only subsumptive statements but a large number of statements which differ from one another by the kind of predicate they have, while intensional logic is not bound to any definite statement form. According to Russell's earlier opinions, statements of intensional logic are not all translatable into statements about extensions: [Princ. Math.] I, 76ff., [Math. Phil.] 187f. Wittgenstein has attacked this position [Abhandig.] 243 f., and subsequently Russell has been inclined to abandon it: Preface to Wittgenstein [Abhandig.] 194 ff., [Princ. Math.] P pp. xiv and 659 ff.

From a position which is closely related to that of Wittgenstein, we shall show that the indicated conception is in fact not tenable. We shall show the validity of the thesis of extensionality, so that any objection against the extensional method loses all force.

The *objection to the extensional method* is aimed not only at the constructional system with which we are here concerned, but philosophers who stand aloof from mathematics have made it a principal objection to any formal method which uses predominantly extensions, especially when it is concerned, as we are here, not with purely logical, but with epistemological, problems. Now, Russell's formulation of the difference between “extensional” and “intensional” statements has so far been the only attempt to give a clear account of the extension-intension problem. Thus, in spite of Russell's own scruples, it is the strongest weapon which we can hand our opponents so that a valid decision may be achieved.

A statement is called *extensional* if it can be transformed into an extension statement (class or relation (extension) statement); otherwise, it is called *intensional*. It is a necessary and sufficient condition for the extensionality of a statement about a propositional function *f* that we can replace *f* by any coextensive prepositional function without changing the truth value of the statement. The thesis of extensionality states that all statements about any propositional function are extensional (i.e., that there are no intensional statements).

REFERENCES. Russell [Princ. Math.] 1, 72 ff. and [Math. Phil.] 187 f.; both give examples for (apparently) intensional statements.

EXAMPLE. Let us consider the coextensive prepositional functions, “x is a man” and “x is a rational animal”. The following statement about the first of these propositional functions is to be evaluated with respect to its extensionality: “ “x is a man” universally implies (i.e., for all values of its argument) “x is mortal” “. We do not have to investigate whether this statement is true or false. It retains, in any case, its truth value (i.e., it remains true or false) if, in place of “x is a man”, we put the coextensive prepositional function “x is a rational animal” or any other coextensive propositional function whatever. Thus the criterion is fulfilled and the implication statement in question is shown to be extensional. That it can in fact be transformed into an extension statement, more precisely, into a class statement, can easily be shown: “The class of all men is contained in the class of all mortals”. (Here, the second propositional function has been transformed at the same time.)

As a counter-example, let us consider the following statement about the same propositional function: “I believe that “x is a man” universally implies “x is mortal” “. Here, we may not simply replace “x is a man” by some other coextensive propositional function. For one cannot conclude, from the given statement, whether my thinking and believing was at all concerned with other coextensive concepts, for example, with the concept, “rational animal”. The above statement, “I believe that...” thus seems to be a nonextensional, i.e., intensional, statement about the propositional function, “x is a man”. We shall later on return to this example and to the thesis of extensionality, but, at this point, I wish to introduce some new concepts which are required for the solution of this problem.

44. *The Distinction Between Sign Statements, Sense Statements, and Nominatum Statements*

In order to give a foundation for the thesis of extensionality and thus to justify the extensional method of construction, we have to introduce another, more general, classification of statements than the above-discussed distinction between extensional and intensional statements about propositional functions. This classification will be concerned not only with statements about propositional functions, but with statements about any objects whatever, as well as about statements and functions. We distinguish sign statements, sense statements, and nominatum statements.

This distinction is connected with the three different ways in which a sign may be used. We distinguish from the sign itself, on the one hand the sense which it “expresses” and on the other hand the nominatum which it “designates”. (This distinction stems from Frege [Sinn], [Grundges.] 1,7.) If a sign is placed in the argument position of a propositional function, then it is not in itself clear what is meant as the argument for the propositional function, even if the sign and its nominatum are known. Though one can generally guess it from the context, we shall introduce some auxiliary symbols (only for §§44, 45) in order to make the distinction clearer and to indicate which of the three kinds is meant. If the *sign itself* is the argument of the propositional function, then we enclose it in quotation marks, e.g., " "7" is an Arabic number", " "5 + 2" consists of three constituents". We enclose the argument sign in brackets if its *nominatum*, i.e., that which is designated by it, is meant, as is usually the case. For example: "[7] is an odd number". But there is a third thing that could be meant by the sign 7. We shall distinguish it from the nominatum by calling it the sense of the sign and shall indicate it by pointed brackets, e.g., "I just had the representation [40] <7>". What is meant by this becomes still more clear when we compare the substitutions which are possible in the three cases, if the truth value is to be preserved. In the case of the sign statement, very little variation is permissible. The above statement about "7" allows neither the substitution of "VII" nor of "5 +2". On the other hand, in the sentence which contains the sign "(7)", we may replace it by "<VII>", for the sense statement asserts that I have a representation of the number seven, and this fact can be expressed equally well with any of the three signs {seven), (7), (VII). On the other hand, the statement, "I just had the representation <5 + 2>" does not necessarily have the same truth value; it is not required that I should have had a representation of the sum of five and two. The *nominatum expression* shows itself least susceptible to change. In the sentences, "[7] is an odd number" or "[7] > 6", I may substitute [VII] as well as [5+2]. Consequently, we give the following definitions: by the *sign itself* we mean the written (or linguistic, etc.) figure; 7, VII, 5 + 2 are different from one another as far as the signs themselves are concerned; hence, in our nomenclature: "7", "VII", and "5 + 2" are different objects. By the *sense* of a sign we mean that which the intensional objects, i.e., representations, thoughts, etc., which the sign is to evoke, have in common. 7 and VII have the same sense, namely, the number seven as the content of a representation or thought; 5 + 2 has a different sense. Hence, <7> is the same as <VII>, but <5+2> is something different. Likewise, <the evening star> is the same as <der Abendstern>, but <the morning star> is something different; <Scott> is something different from <the author of *Waverley*>. By the nominatum of a sign we mean the object which it designates; 7, VII, and 5 + 2 have the same nominatum, namely, the number seven (arithmeti-

40 Vorstellung

cal equality is logical identity, as Frege [Grundges.] I, p. ix has shown); [7], [Vll], and [5+2] are the same; further, [the morning star] and [the evening star] are identical, likewise, [Scott] and [the author of Waverley].

The difference between the sign itself, its sense, and its nominatum, which has here been explained for signs which designate *objects* in the narrower sense, also holds for sentences as signs for propositions and for signs for propositional functions. We can be very brief because of the analogy to what has gone before. Let us first consider *sentences*. The sense of a sentence is the thought which it expresses. The nominatum of a sentence is, according to Frege, its truth value, i.e., either truth or falsity.

> EXAMPLE. Consider the following three sentences: A. Socrates is a man; B. *Socrates homo est*; C. 2+2= 4; call them A, B, C. A, B, and C are different from one another as signs (sentences); A and B have the same sense; A, B, and C have the same nominatum, i.e., the same truth value: truth. Statements about these sentences can be classified as above: " "A" consists of four words" is a sign statement; neither B nor C may be substituted for A. "<A> is a historical fact" is a sense statement. We may put <B>, but not <C>, in the place of <A>. "[A] is equivalent to (i.e., has the same truth value as) [1+1 = 2]" is a nominatum statement. In this case we may substitute [B] as well as [C] for [A].

45. *Justification of the Extensional Method*

The most important case of this tripartition occurs with statements about propositional functions. Let us choose the following as examples of propositional functions: 1. *x* is a man, 2. *x homo est*, 3. *x* is a rational animal. These three prepositional functions are coextensive since they are satisfied by the same values for x, thus, they have the same nominatum. However, the sense of the first is identical only with that of the second, not with that of the third. In a sign statement about the first one, e.g., " "x is a man" consists of 7 letters", we can substitute neither the second nor the third. "I believe that there are things which satisfy <x is a man>" is a sense statement; here we may substitute the second, but not the third, propositional function, since my thinking and believing does not necessarily have to be concerned with the concept of a rational animal too. "[x is a man] universally implies [x is mortal]" is a nominatum statement. In this case, we may substitute the second as well as the third propositional function, or any other coextensive one. According to the previously stated criteria (§43), this nominatum statement is an extensional statement, while the sense statement is an intensional statement, about the propositional function: *x* is a man. The indicated sign statement does not deal with the propositional function at all, but only with its sign, i.e., a group of letters. Our considerations now show us that the nominatum statement and the sense statement are not really concerned with the same thing, for <*x* is a man> is not the same as [x is a man]. The difference is analogous to that between <5 + 2> and [5 + 2], i.e., between that which I represent to myself in connection with the sum of 5 and 2, and the number seven.

Thus our considerations have led to the following result: the distinction between extensional and intensional statements about a propositional function is not valid, for the statements in question are not about the same object. Only those statements which we have called extensional are concerned with the propositional function itself. The so-called intensional statements deal with something altogether different (e.g., a concept as the content of a representation or thought). Thus the thesis of extensionality is valid: there are no intensional statements about propositional functions; what were taken to be such were actually not statements about propositional functions, but statements about their sense. Every statement that does not concern the sense of a propositional function, but the function itself, retains its truth value if any coextensive propositional function whatever is substituted; i.e., it can be stated in the form of an extensional statement.

> Without giving further reasons, let me here indicate that this result can be extended. For the above argument holds not only for statements about propositional functions, but, according to our previous considerations, in an analogous way also for statements about statements and for statements about objects in the narrower sense. Thus we obtain the general result: there are no intensional statements. All statements are extensional. In every sentence, the sign which represents the object that is to be judged, whether it is an object in the narrower sense, or a statement, or a propositional function, or whatever, may be replaced by any sign which has the same nominatum, even if it has a different sense.

Since every statement about a propositional function can be brought into the form of an extensional statement, the possibility of making statements about propositional functions is not restricted in any way if we introduce for them merely their extensions. *Thus the extensional method of construction is justified*.

CHAPTER
B

THE SYSTEM FORM

1. FORMAL INVESTIGATIONS

46. *The System Form Depends Upon Reducibility*

After having discussed the problem of the ascension forms and having found that the individual levels of the constructional system are to be erected by means of definitions using classes and relation extensions, we are now confronted with a second problem, namely, that of the "system form" (i.e., the over-all form of the constructional system). How are we to proceed with the step-by-step construction of our system, so that all the objects of science find a place in it? In the preliminary Chapter II B, we have already considered several object types. Now, the objects of the various types are to be brought into a system. The order in the constructional system is determined by the fact that an object *a* can be constructed on the basis of the objects *b*, *c*, ... which precede it. In other words, *a* must be reducible to *b*, *c*, ... (i.e., propositional functions about *a* must be transformable into coextensive propositional functions about *b*, *c*, . . .)

For a precise application of this criterion, it would be required that the propositional functions with which we are concerned are given a logistic rendition, either in their entirety, or as far as their logical skeleton is concerned, or at least that they are given in logical form. We say that a statement or a propositional function has been given a *logistic rendition*, if it is expressed in logistic symbols. By the *logical skeleton* of a statement or a propositional function, we mean its logical structure.[41]Thus, we shall say of a statement that its logical skeleton has been given a logistic rendition, if all extralogical concepts are expressed by the customary words, while the logical relations between these extra-logical concepts, which constitute the skeleton, are expressed by logistic signs. We shall say that a statement is given in *logical form* if it is expressed entirely in words of the natural language, but in such words that there is a unique way, on the basis of either explicit or tacit agreements, of giving the skeleton in logistic rendition.

> EXAMPLE. Statement in natural language: "If somebody is a Negro, then he is also a man." Logical form: "If somebody belongs to the class of Negroes, then he also always belongs to the class of men". Logistic rendition of the logical skeleton: "(x): x ∈ Negro. ⊃ . x ∈ man"; logistic expression of the entire statement: "(x): x ∈ ne. ⊃ . x ∈ ma".

> REFERENCE. About the logical skeleton: Carnap [Logistik] § 42 ff., with examples for the logistic rendition of statements.

47. *The Criterion for Reducibility in Realistic Language*

The purpose of construction theory is to order the objects of all sciences into a system according to their reducibility to one another. Thus, later on, we shall have to inquire into the reducibility of the various object types. There will then arise the difficulty that we have to apply the criterion of reducibility to statements and statement forms which are given only in word language. In view of this task, it is advisable to express the criterion in still another form so that we no longer speak of propositional functions and their logical relations, but of states of affairs and their factual [42] relations. Thus, we translate it from the formal-logical, in this case the *constructional language*, into the language of facts, or *realistic language*. (About the difference between these two languages, see § 52.)
We now arrive at a *factual criterion* [43] *of reducibility* which is wanting

41 formal-logische Form
42 sachlich
43 Sachverhaltskriterium

in logical strictness, but allows easier application to the empirical findings of the individual sciences. It is the following: We call an object *a* "reducible to the objects *b*, *c*, ..." if, for any state of affairs whatever, relative to the objects *a*, *b*, *c*,..., a *necessary and sufficient condition* can be indicated which depends only upon objects b, c, . . .

We must now show that this criterion coincides with the one we gave earlier (§35). The coextensiveness of two propositional functions A, B, means: A universally implies B and vice versa (§ 32). Now, if A universally implies B, then this means that, in each case in which A is satisfied, B is also satisfied; in other words, that A is a sufficient condition for B; and if B universally implies A, then this means that B is never satisfied in any case in which A is not satisfied, so that A is a necessary condition for B. Thus, if A and B are coextensive, then A is a necessary and sufficient condition for B (and, at the same time, B is a necessary and sufficient condition for A, a point with which we are not here concerned). However, there seems to be a deviation in one point: the new criterion speaks of "states of affairs",[44] while the earlier one speaks of propositional functions. The question is whether a state of affairs is indicated through a propositional function or through a statement. Here we must make the following distinction: *individual* states of affairs are to be indicated through statements; *general states of affairs*, through propositional functions. Linguistic usage does not make a precise distinction between these two types. In the case of the reducibility criterion, we are concerned with general states of affairs, since only they allow us to speak of conditions. (The same holds for states of affairs which occur in natural laws.) Thus, the two criteria agree in this point also.

48. The *Basic State of Affairs Relative to an Object*

The factual criterion of reducibility offers still another difficulty which arises from the expression "any state of affairs whatever". Thus, strictly speaking, we would have to test the frequently very large number of possible states of affairs in which the objects might occur, in order to decide upon the reducibility of one object to another. However, it turns out that for each object there is a basic state of affairs. It occurs in any other state of affairs only in connection with this basic one. To put it more precisely and in constructional language: for every object, there is a fundamental propositional function such that all occurrences of the object can be expressed with the aid of this fundamental propositional

44 Sachverhalt

function. For a property concept, the basic state of affairs is the occurrence of this property (fundamental prepositional function: "*x* has the property . . ." or "*x* is a. . .");for a relational concept, the basic state of affairs is the fact that the relation holds (fundamental propositional function: "x stands to y in the relation . . .").

In conformity with the extensional method of construction (§43) let a class symbol stand for a property concept, for example, say, *c*, and a relation (extension) symbol for a relational concept, say, Q; then that fundamental propositional functions are "$x \in c$" and "*x* Q *y*". It is in fact the case that every sentence in which the class symbol *c* occurs can be transformed in such a way that *c* occurs only in the context "$x \in$ c", and every sentence in which the relation symbol Q occurs can be transformed so that Q occurs only in the context "*x* Q *y*".

Any definition, through which an object in the constructional system is constructed (i.e., its "constructional definition"), has to make use of the basic state of affairs for this object. The propositional function of the basic state of affairs is the definiendum; the propositional function which designates the necessary and sufficient condition for this basic state of affairs is the definiens, for two propositional functions are coextensive if one of them designates a necessary and sufficient condition for the other (§47). The juxtaposition of two coextensive propositional functions the first of which contains, besides the variables, only one symbol which does not occur in the other one, can be viewed as a definition of this symbol (i.e., as a contextual definition § 39).

> EXAMPLE. Construction of an object with the aid of its basic state of affairs. The basic state of affairs of temperature equilibrium is: "*x* stands to *y* in the relation of temperature equilibrium." A necessary and sufficient condition for this is the state of affairs: "If bodies *x* and *y* are brought into spatial contact (either directly or through the mediation of other bodies), then they show neither increase nor decrease in temperature." Thus, these two propositional functions are coextensive. Hence, we could use them for the formulation of a definition for the object of the first propositional function, namely, temperature equilibrium: "We call "temperature equilibrium" that relation between *x* and *y* which is characterized by the fact that bodies *x* and *y*, if they are brought into (direct or indirect) spatial contact with one another, neither increase nor decrease in temperature." In the formulation of the constructional system, the object "temperature equilibrium" can be introduced (i.e., "constructed") in this way, provided that the other objects which are referred to in the definition have been constructed previously.

49. *Indicators* [45] *and Conditions*

According to the preceding considerations, the proof of the reducibility of an object is to be based on the determination of a necessary and sufficient condition for the basic state of affairs for that object. The question arises whether such a condition can be established for every basic state of affairs. To solve this problem, we introduce the concept of *scientific indicator*. The indicator of a state of affairs is a sufficient condition for the state of affairs, but not every sufficient condition can be called an indicator. We shall use the term "indicator" only for such conditions as are ordinarily used to identify the state of affairs (i.e., which are usually recognized *before* the state of affairs).

> EXAMPLE. High air pressure and a high barometer reading are conditions for one another: if the air pressure is high, then the barometer reading is high; if the barometer reading is high, then the air pressure is high. But only in the second case do we call the condition an indicator.

Science usually gives indicators for many states of affairs of which it treats, especially for the elementary ones, of which the others are composed, that is, especially for those that are suitable as basic states of affairs as, for example, "This thing is an oak tree", "This thing is a cooperative purchasing corporation." True, the process of recognizing such a state of affairs (i.e., the presence of a certain concept) is frequently not based on these indicators, but is carried out intuitively, even in science. But even this intuitively recognized concept can be considered a fully determined scientific object only because such indicators can be given. In many cases, especially in the cultural sciences, when we are concerned, for example, with the stylistic character of a work of art: etc., the indicators are given either very vaguely or not at all. In such a case the decision as to whether a certain state of affairs obtains is not made on the basis of rational criteria but by empathy. Such empathy decisions are justly considered *scientific* decisions. The justification for this rests upon the fact that either it is already possible, even though very complicated in the individual case, to produce indicators whose application does not require empathy or else that the task of finding such indicators has been recognized as a scientific task and is considered as solvable in principle. A decision, which has been made through empathy or otherwise, which cannot *in principle* be subjected to a rational test

45 Kennzeichen

through conceptual criteria would forfeit every claim to scientific status. Even the cultural sciences observe this limit to the admissibility of empathy decisions, if not explicitly, then at least in their practical procedures.

Thus we say that *in principle there are indicators for all scientific states of affairs*. That is to say, we have the task of determining an indicator for every scientific state of affairs, and in principle this task can be fulfilled. A more detailed analysis, which we must omit for lack of space, would show furthermore that, in principle, there is an *infallible and at the same time always present* indicator for any scientific state of affairs (i.e., an indicator which is present when and only when the state of affairs is also present). Such an indicator can always be produced by conjoining the various indicators for the individual cases; it is a necessary and sufficient condition for the state of affairs. *Thus, the construction of any scientific object can be carried out by producing such an indicator for its basic state of affairs*.

> EXAMPLE. The indicator from which the rattlesnake has derived its name is an infallible and always present indicator for the fact that an animal is a rattlesnake. Thus the following propositional functions are coextensive: "*x* is a rattlesnake" and "*x* is an animal which carries a number of rattles at the end of its body". With these propositional functions, the first of which expresses the basic state of affairs of the object rattlesnake, we can produce a constructional definition for the rattlesnake which, in the customary formulation, would read: "By "rattlesnake", we mean an animal which carries rattles at the end of its body."

50. *Logical and Epistemic Value*

If we transform a sentence about an object by replacing the object name by its constructional definition, then the intuitive meaning [48] of the sentence, and thus its epistemic value, is frequently changed. This could lead to weighty objections against the method of construction which I am here suggesting; therefore, I want to concern myself with the question, in what respects the transformed sentence agrees with the original one and in what respects it does not.

If *a* is reducible to *b*, *c*, then the propositional functions K, L, ... about *a* are coextensive with the propositional functions K', L', which are exclusively about b, c. The *constructional transformation* (i.e., the

46 der vorstelinngsmässige Sinn

elimination of the object *a* with the aid of its constructional definition) consists in the transformation of the propositional functions K, L, ... into K', L', ... Since the former are coextensive with the latter, the constructional transformation of a propositional function leaves the extension unchanged (§32); in the case of a statement, the truth value remains unchanged (i.e., it remains either true or false). Let us summarize these two cases in the following way: a constructional transformation leaves the *logical value* of a propositional function, as well as of a statement, untouched. We contrast this logical value with the "epistemic value". A constructional transformation may, for example, turn a true, epistemically valuable statement into a triviality; in such a case, we say that the "epistemic value" has been changed. But, since the trivial statement is also true, the logical value has not changed. A *constructional transformation of a statement* (or propositional function) *always leaves the logical value, but not necessarily the epistemic value, unchanged*. (In contradistinction to translations from one natural language to another, these transformations do not have to preserve the intuitive content.) This is an essential characteristic of the constructional method: as regards object names, statements, and prepositional functions, *it is concerned exclusively with logical, not with epistemic, value; it is purely logical, not psychological*.

EXAMPLE. In § 49, we have given a constructional definition for the rattlesnake. Let us use this definition in order to carry out the constructional transformation of the following sentence: "This animal, which carries rattles at the end of its body, is a rattlesnake." The result is the following tautology: "This animal, which carries rattles ..., is an animal which carries rattles ...". The epistemic value of the original sentence has been lost in the transformation. On the other hand, the logical value has been retained: the truth value of the tautology is truth, just as was the case with the original sentence.

REFERENCES. Our theory of indicators and definite descriptions is, on the whole, based on Russell's theory of descriptions; ([Princ. Math.] 1, 181 ff., [Math. Phil.] 168 ff., [Description]). There is, however, a deviation which follows from our distinction between logical and epistemic value: we consider a definite description as equivalent with (of the same logical value as) the proper name of the object which is described; Russell's argument from triviality ([Princ. Math.] 1, 70, [Math. Phil.] 175 f.) is no objection, since a triviality may have the same logical value as a statement with a positive epistemic value. This conception is related to the thesis of extensionality (§43 ff.).

51. *Logical Translation and Translation of Sense*

Constructional theory constructs an object by seeking an infallible and always present indicator for it (more precisely, for its basic state of affairs). This is claimed to be a definition of the object, but it does not seem to achieve what we generally require of a definition in the sense of a conceptual definition.” As such, it would have to indicate the essential characteristics of a concept, but these are frequently not contained in the indicator.

We can view a definition as a rule of substitution or replacement; it states that a certain sign (the definiendum) may be replaced in all statements by another (generally complex) sign (the definiens). We can require different kinds of invariance from such a translation. If we require that the translated statements have the same logical value as the original ones, but not necessarily the same epistemic value, then we speak of a logical translation. On the other hand, if we make the more comprehensive requirement that the translation leave invariant also the epistemic value that is, the sense of the statements (as, for example, in the translation of a text from one natural language to another), then we speak of a translation of sense; (in this case the logical value, too, remains necessarily unchanged). Since the construction of an object in the constructional system has always to do only with logical value and not with epistemic value (§50), a constructional definition which employs the indicator of an object and which thus produces a logical translation achieves exactly what we demand of it.

> REFERENCES. The concern with nothing but the logical value (truth value) for a constructional derivation agrees with Leibniz' definition of identity: “*Eadum sunt, quorum unum potest substitui alteri salva veritate*.”

52. *Realistic and Constructional Language*

One could raise still another objection against the use of an indicator in a constructional definition. There seems to be a fundamental opposition between construction theory and the empirical sciences as concerns the conception of reality. For example, we construct heteropsychological objects[48] (i.e., the psychological occurrences in another person) on the

47 Begriffserklärung
48 das Fremdpsychische

basis of physical indicators, namely, expressive motions and bodily reactions, including linguistic utterances, of the other person. To this, one could object from a realistic viewpoint, that heteropsychological occurrences are in reality something different from the reaction behavior, which plays only the role of an indicator.

> EXAMPLE. Let us consider anger (here taken as something heteropsychological, i.e., as the anger of another person, in contrast to one's own anger, which we assume to be already constructed). The constructional definition of somebody else's anger would be something like this: “anger of person A” means “state of the body of A characterized through such and such physical processes of this body or through a certain disposition to react to stimuli of such and such a kind through physical processes of such and such a kind” (where the type of process is characterized with the aid of processes of my own body when I am angry). Here, the realistic objection would run somewhat like this: “The physical behavior of the other person's body is not itself the anger, but only an indicator of the anger.”

Let K stand for the physical reaction behavior which is the indicator of a certain heteropsychological process. The objection amounts, then, to the following: the concept of this heteropsychological process is not itself identical with K, and therefore requires its own symbol, for example, F. To this objection, we make the following reply: all scientific (though not all metaphysical) statements about F, especially all statements which are made within psychology itself, can be transformed into statements about K that have the same logical value. Now, since K and F satisfy the same propositional functions, they are to be considered as identical (as far as logical value is concerned). No meaning for F, which is not identical with K, can be given in scientific (i.e., constructable) expressions. (This question is connected with Leibniz' thesis of the identity of indiscemibles, cf. §51; and also with the problem of introjection and with the metaphysical component of the problem of reality, § 175 f.).

The realistic language, which the empirical sciences generally use, and the constructional language have actually the same meaning: they are both neutral as far as the decision of the metaphysical problem of reality between realism and idealism is concerned. It must be admitted that, in practice, linguistic realism, which is very useful in the empirical sciences, is frequently extended to a metaphysical realism; but this is a transgression of the boundary of science (cf. §178). There can be no objection against such a transgression, as long as it influences only the mental representations which accompany the scientific statements; this transgression is objectionable only if it influences the content of the statements of science.

Let us emphasize again the neutrality especially of the constructional language. This language is not intended to express any of the so-called epistemological, but in reality metaphysical, doctrines (for example, realism, idealism, solipsism), but only epistemic-logical relations. In the same sense, the expression “quasi object” designates only a certain logical relationship and is not meant as the denial of a metaphysical reality. It must be noted that all real objects (and construction theory considers them as real to the same degree as do the empirical sciences, cf. § 170) are quasi objects.

Once it is acknowledged that the realistic and the constructional languages have the same meaning, it follows that constructional definitions and the statements of the constructional system can be formed by translating indicator-statements and other statements which are found in the realistic language of the empirical sciences. Once realistic and constructional languages are recognized as nothing but two different languages which express the same state of affairs, several, perhaps even most, epistemological disputes become pointless.

53. *Summary. Method for Solving the Problem of the System Form*

The problem of the system form is expressed in the question: how can the different object types be brought into a system such that the higher ones can always be constructed from the lower ones (i.e., such that the former are reducible to the latter)? To solve this problem, we must inquire into the mutual reducibility of the various object types. To accomplish this end, we take into account the information available in the special sciences, and, with its aid, attempt to find for each object under investigation the various possibilities for necessary and sufficient conditions for the basic state of affairs of that object. We can proceed by asking the special science in question for an (infallible and always present) indicator of the basic state of affairs. But through this method we cannot find all necessary and sufficient conditions. For this method looks only in one direction; it proceeds from a given object to those other objects which are already known. In the system form which we shall later on choose for the constructional system, construction will generally proceed in this direction, since this system is intended to reflect the epistemological hierarchy of objects. Thus, we can frequently use the method of indicators. However, in order to see the possibilities of other system forms, we shall have to pay some attention to conditions other than indicators.

After having here developed a method for testing reducibility, this test is to be applied in the following, second, half of this chapter, to the most important object types. This will allow us to discern the various possible system forms.

REFERENCES. The investigation of the reducibility of one object to other objects corresponds to what has been called in realistic language, "determination" of real objects from other real objects or from the given.[49] The methods and particular criteria which are to be used in such determinations have been discussed in great detail by Külpe ([Realis.] esp. Vol. III).

Construction theory can accept and utilize all the results of investigations about "realization",[50] for example, those of Külpe; but we must be careful not to substitute the metaphysical concept of reality for the purely constructional concept (cf. § 175 f.). In construction theory, we must exercise a methodological abstinence as far as the postulating of reality [51] is concerned (cf. § 64); thus it is advisable to use a neutral language: in construction theory, we translate the findings of the empirical sciences from the "realistic" language into the "constructional" language (cf. § 52).

2. MATERIAL INVESTIGATIONS

54. *Epistemic Primacy*

Using the method which has been developed in the first half of this chapter, we now have to investigate the relations of reducibility which obtain between the objects of knowledge. Frequently, these relations hold in different directions, so that they alone do not uniquely determine the order of the system.

The system form which we want to give to our outline of the constructional system is characterized by the fact that it not only attempts to exhibit, as any system form, the order of the objects relative to their reducibility, but that it also attempts to show their order relative to epistemic primacy. An object (or an object type) is called epistemically primary relative to another one, which we call epistemically secondary,

49 "Bestimmung" der Realitäten aus anderen Realitaten oder aus dem Gegebenen
50 Realisierung
51 Realsetzung

if the second one is recognized through the mediation of the first and thus presupposes, for its recognition, the recognition of the first. Fortunately, the sequence of constructions which is required for the expression of epistemic primacy is maintained when the method of indicators is applied, since an indicator is epistemically primary relative to its object. However, we also wish to investigate here other directions which reducibility relations may take, so as to ascertain the various possible system forms.

The fact that we take into consideration the epistemic relations does not mean that the syntheses or formations of cognition, as they occur in the actual process of cognition, are to be represented in the constructional system with all their concrete characteristics. In the constructional system, we shall merely reconstruct these manifestations in a rationalizing or schematizing fashion; intuitive understanding is replaced by discursive reasoning.

55. *Cultural Objects are Reducible to Psychological Objects*

We have seen earlier that the manifestation relation holds between psychological and cultural objects, and the documentation relation between physical and cultural objects (§24). It is these two relations which mediate the recognition of cultural objects. Admittedly, not every cultural object must necessarily be immediately manifested or documented. There may be some which are based upon other cultural objects and whose recognition is mediated through the latter. But, even then, they are indirectly recognized through manifestation and documentation.

> EXAMPLE. We ascertain the religion of a given society through the representations, emotions, thoughts, volitions of a religious sort which occur with the members of this society; also, documents in the form of writings, pictures, and buildings are considered. Thus, the recognition depends upon the manifestation and the documentation of the object in question.

It is occasionally claimed that it is possible to recognize cultural objects without having to take a detour via psychological processes in which they manifest themselves or via physical documentation. But so far, such methods are not known to science and have not yet been applied. The cultural sciences recognize their objects, whether a custom, a language, a state, an economy, art, or whatever, not through discursive reasoning, but through "empathy" or *verstehen*. But this intuitive procedure, without exception, begins with manifestations and documentations. Furthermore, it is not merely the case that intuitive understanding, or empathy, is occasioned by the recognition of the mediating psychological or physical objects, but its content is completely determined through the character of the mediating objects.

> EXAMPLE. The awareness of the aesthetic content of a work of art, for example a marble statue, is indeed not identical with the recognition of the sensible characteristics of the piece of marble, its shape, size, color, and material. But this awareness is not something outside of the perception, since for it no content other than the content of perception is given; more precisely: this awareness is uniquely determined through what is perceived by the senses. Thus, there exists a unique functional relation between the physical properties of the piece of marble and the aesthetic content of the work of art which is represented in this piece of marble.

Our considerations show that all cultural objects are reducible to their manifestations and documentations, either directly or through the mediation of other cultural objects. However, the documentation of a cultural object necessarily takes place with the aid of a manifestation. For, if a physical object is to be formed or transformed in such a way that it becomes a document, a bearer of expression for the cultural object, then this requires an act of creation or transformation on the part of one or several individuals, and thus psychological occurrences in which the cultural object comes alive; these psychological occurrences are the manifestations of the cultural object.

From this it follows that the domain of objects to which the cultural objects are reducible can be narrowed down: *every cultural object is reducible to its manifestations, that is, to psychological objects*.

56. *The Construction of Cultural Objects from Psychological Objects*

The recognition that all cultural objects are reducible to psychological objects does not in itself determine whether or not we shall construct the former from the latter within the constructional system. It is imaginable that there are certain persuasions (for example, the theory which interprets the entire world process dialectically as the emanation of a spirit) which lead to the assumption that all psychological objects are reducible to cultural ones. Such a supposition would indicate the possibility of a construction in the opposite direction. We shall not investigate the correctness of this assumption at this time.

In the system form which we shall use for our outline of a constructional system, we shall construct the cultural objects from the psychological ones and not vice versa. The reason for this lies in the epistemic relation between the two object types as it is expressed in the method of science. We have seen earlier that manifestations of cultural objects (and, furthermore, also documentations, which, however, lead to manifestations) have the role of indicators; more precisely, the role of objects which mediate recognition, from whose characteristics alone science ascertains the characteristics of the cultural objects themselves. This establishes the epistemic primacy of the psychological objects over the cultural objects. Since we have previously laid down a principle, according to which we shall choose a system form for which the direction of construction is determined by epistemic primacy, it is now determined that, in our constructional system, the cultural objects are constructed from the others, and especially from the psychological objects, and not vice versa.

Natural science tends to the opinion that a state, a custom, a religion consists of the psychological processes in which the entity in question manifests itself, just as a piece of iron consists of its molecules. In opposition to this, the cultural sciences tend to consider such entities as entities of a special type, not just as a sum of psychological processes.

Construction theory claims indeed that cultural objects are reducible to psychological ones and constructs the former from the latter in one of its system forms. Nevertheless, it considers the position of the cultural sciences justified. Cultural objects are not compounded out of psychological objects. We have already emphasized their peculiar character and have shown, not only that they are widely different from psychological objects, but that they belong to another “object sphere” (§§23,31).

Thus, construction theory agrees with the cultural sciences as far as the independence of the cultural object type is concerned. On the other hand, it fulfills a requirement which is emphasized especially in the natural sciences, namely, the requirement of an analysis of cultural objects (i.e., their reduction to other objects). However, by analysis, we do not mean decomposition into constituents. “Reducibility” and “construction” have the previously defined meaning of translatibility of statements (§§2, 35). In principle, all statements about cultural objects can be transformed into statements about psychological objects. But this is to be taken in a very modest sense. We cannot reproduce the *sense* of a statement about cultural objects in statements about psychological objects. (It can be done sometimes, but not always.) When we claim that a transformation in the constructional sense is possible, we mean that a transformation rule is possible, the application of which will leave the logical value, though not always the epistemic value, unchanged. This has been discussed earlier (§ 50f.).

> REFERENCES. The question whether or not cultural objects can be resolved into psychological processes is a matter of dispute (cf., for example, Freyer [Obj. Geist] 53). According to our considerations, this question must be answered in the negative, if by resolution is meant the proof of composition out of constituent parts, but it is to be answered in the affirmative if by resolution is meant the proof of logical reducibility.

57. *Physical Objects are Reducible to Psychological Objects and Vice Versa*

Statements about physical objects can be transformed into statements about perceptions (i.e., about psychological objects). For example, the statement that a certain body is red is transformed into a very complicated statement which says roughly that, under certain circumstances, a certain sensation of the visual sense ("red") occurs.

Statements about physical objects which are not immediately about sensory qualities can be reduced to statements that are. If a physical object were irreducible to sensory qualities and thus to psychological objects, this would mean that there are no perceptible indicators for it. Statements about it would be suspended in the void; in science, at least, there would be no room for it. Thus, all physical objects are reducible to psychological ones. For every psychological process, there is a corresponding "parallel process" in the brain, i.e., a physical process. There is a univocal correspondence between each property of the psychological process and some (even though entirely different) property of the brain process. *Thus, every statement about a psychological object is translatable into a statement about physical objects*.

Since the correlation problem of the psychophysical relation (cf. §21) has not yet been solved, the present state of science does not allow us to indicate a general rule of translation. However, for our present purposes, the logical existence of this rule (i.e., the fact that a correlation of this kind holds) allows us to draw the conclusion that *it is in principle possible to reduce all psychological objects to physical objects*.

REFERENCES. The indicated position of a thoroughgoing and univocal psychophysical correlation is maintained, for example, by Wundt [Phys. Psychol.] 111,752; opponents of this position are, for example, Becher [Gehirn] and Bergson [Materie]. A comprehensive bibliography about this problem is given in Busse [Geist]. Cf. also §§ 58, 59.

An entirely different kind of reduction of psychological to physical objects is based, not upon the almost altogether unknown psychophysical relation, but upon the expression relation. To the expression relation in the narrower sense (§ 19), we must here add another relation which one could call, for example, reporting relation. By this we mean the relation between a bodily motion and a psychological process, provided that this motion indicates through speech, writing, or other sign-giving the presence and the nature of the psychological process. An example is the relation between the speech motions of a man which form the sentence: "I am glad about the beautiful weather" and his gladness about the beautiful weather. Expressive motions,[52] including reports of this type, are the only indicators by which we can recognize the psychological processes in other persons, the heteropsychological processes. Now, every heteropsychological process is in principle recognizable, that is, it can either be inferred from expressive motions or else questions can be asked about it. (It can be reported.) Thus, every statement about a psychological object can be transformed into a statement about those indicators. Thus it follows that all psychological objects can be reduced to expressive motions (in the wider sense), i.e., to physical objects. From the recognizability in principle of every kind of heteropsychological process and from the uninterrupted causal nexus [53] among physical processes, it follows that all types of psychological processes have physical parallels (in the central nervous system). (This runs counter to the position of Bergson and others; see below.) We shall not here concern ourselves with a proof for our contention; it is not as important for the system form which we are here using as it would be for a form with physical basis (§59).

58. *The Autopsychological and the Heteropsychological*

We now have to decide whether our system form requires a construction of the psychological objects from the physical objects or vice versa. Because of their mutual reducibility, it is logically possible to do either.

52 Ausdrucksbewegungen
53 geschlossene Gesetzmässigkeit

Hence, we have to investigate the epistemic relation between these two object types. It turns out that psychological processes of other subjects can be recognized only through the mediation of physical objects, namely, through the mediation of expressive motions (in the wider sense) or, if we assume a state of brain physiology which has not yet been reached, through the mediation of brain processes. On the other hand, the recognition of our own psychological processes does not need to be mediated through the recognition of physical objects, but takes place directly.

Thus, in order to arrange psychological and physical objects in the constructional system according to their epistemic relation, we have to split the domain of psychological objects into two parts: we separate the *heteropsychological* objects from the *autopsychological* objects. The autopsychological objects are epistemically primary relative to the physical objects, while the heteropsychological objects are secondary. Thus, we shall construct the physical objects from the autopsychological and the heteropsychological from the physical objects.

Thus, the sequence with respect to epistemic primacy of the four most important object domains is: the autopsychological, the physical, the heteropsychological, the cultural. Thus, our system form requires an arrangement within the constructional system which corresponds to this sequence. For the moment, this gives us a rough indication of the overall form. We shall later discuss the arrangement of the individual object types within these major domains.

> REFERENCES. Especially Dingier [Naturphil.] has clearly shown the necessity for treating separately the autopsychological and the heteropsychological, especially when we are concerned with epistemological investigations ("autopsychology"—"allopsychology"). Becher [Geisteswiss.] 285 ff. has shown against Scheler that the heteropsychological can be recognized only through a mediation of the physical. For a detailed proof that the heteropsychological is reducible to the physical and indeed that it is epistemically secondary, see Carnap [Realismus].

59. A *System Form with Physical Basis*

If it is not required that the order of construction reflect the epistemic order of objects, other system forms are also possible. The possibility of placing the basis of the system in the domain of the cultural objects is quite problematic. While it is in fact possible to envisage all psychological processes as manifestations of cultural entities, the difficulty, if not impossibility, of such a system form lies in the fact that one cannot suppose all properties of psychological processes to be determined through the nature of the cultural entities which are manifested in them. Thus there is no thoroughgoing reducibility of psychological objects to cultural ones.

Since all cultural objects are reducible to psychological, and all psychological to physical objects, the basis of the system can be placed within the domain of physical objects. Such a system form could be called materialistic, since a system of this form would seem the most appropriate from the standpoint of materialism. However, it is important to separate clearly the logico-constructional aspect of the theory from its metaphysical aspect. From the logical viewpoint of construction theory, no objection can be made against scientific materialism. Its claim, namely, that all psychological (and other) objects are reducible to physical objects is justified. Construction theory and, more generally, (rational) science neither maintain nor deny the additional claim of metaphysical materialism that all psychological processes are essentially physical, and that nothing but the physical exists. The expressions “essence” and “exists” (as they are used here) have no place in the constructional system, and this alone shows them to be metaphysical; cf. §§ 176, 161.

A materialistic constructional system has the advantage that it uses as its basic domain the only domain (namely, the physical) which is characterized by a clear regularity of its processes. In this system form, psychological and cultural events become dependent upon the physical objects because of the way they are constructed. Thus they are placed within the one law-governed total process.[54] Since the task of empirical science (natural science, psychology, cultural science) consists, on the one hand, in the discovery of general laws, and, on the other hand, in the explanation of individual events through their subsumption under general laws, it follows that from the standpoint of empirical science the constructional system with physical basis constitutes a more appropriate arrangement of concepts than any other. (For the basis problem of this system, cf §62.) We cannot, at this time, give an explicit characterization of this system and its importance for science.

From an epistemological viewpoint (in contradistinction to the viewpoint of empirical science), we are led to another arrangement of concepts, namely, to a constructional system with autopsychological basis (§ 60).

---

54 in das eine gesetzmässige Gesarntgeschehen

REFERENCES. The so-called *behavioral psychology* (the "behaviorism" of Watson, Dewey, and others; see the bibliography in Russell [Mind]) reduces all psychological phenomena to what can be perceived through the senses, i.e., to the physical. Thus a constructional system which is based upon this position would choose a physical basis. According to what we have said above, such a system would be quite possible and practicable. However, the further claim of behaviorism, namely, that this ordering of objects is also a correct reflection of the epistemic relations, would still remain problematic.

It could seem to be an open question whether in a constructional system with physical basis there is room for the domain of values. This doubt, however, has been removed by Ostwald [Werte] with his derivation of values of several types upon a basis of energetics (based upon the second principle of energetics with the aid of the concept of dissipation). From a philosophical-standpoint, it must be admitted that there is a methodological justification and fruitfulness, not only for the experiential, "phenomenological", but also for the energistic derivation of values. (We shall employ the phenomenological method in the outline of our constructional system, cf. § 152.) The decision between the two is not a question of validity, but one of system form; the difference lies merely in the way in which the problems are posed and the concepts constructed. Science as a whole needs both theories to exhibit both directions of logical reducibility, just as it needs a behavioristic as well as an introspective psychology; in general, it needs both an experiential and a materialistic derivation of all concepts.

(regardless of its logical possibility) does not always allow a construction in the order of the epistemic relation. Thus, because of our intention to express the epistemic order of the objects, we must use only the system form with the autopsychological basis for our outline of the constructional system.

> REFERENCES. Gatschenberger ([Symbola] 437ff., esp. 451) shows the possibility of two "sublanguages", which correspond to (in our terminology) the system forms with psychological and physical basis respectively: the scientific "language of the postulated"[55] and the psychological "language of the given". Gatschenberger is of the opinion that a *pure* language of the given cannot be accomplished; however, by using such a language in our constructional system, we shall show that a system form with psychological basis can be achieved.

---

55 des Geforderten

CHAPTER
C

THE BASIS

1. THE BASIC ELEMENTS

61. *The Two Parts of the Basis Problem: Basic Elements and Basic Relations*

The problem of the basis of the constructional system falls into two parts. At first we must decide which objects to take as basic elements (i.e., as objects of the lowest constructional level). However, if further construction is to be possible, still other objects must be placed at the beginning of the constructional system, namely, either classes ("basic classes") or relation extensions ("basic relations" [56]). For, if the basic elements were given as coexisting without properties and without relations, then no constructional step, through which we could advance beyond them would be possible. We shall proceed, as is explained later, by placing, not classes, but relation extensions, the *basic relations*, at the beginning of the constructional system. These, and not the basic elements, form the undefined basic objects (basic concepts) of the system, and all other objects of the system are constructed from them. As far as construction is concerned, the basic relations take precedence over the basic

56 Grundrelationen

elements which are their members; generally speaking, construction theory considers individual objects as secondary, relative to the network of relations in which they stand. Thus, we divide the basis problem into the quest for basic elements and the quest for basic relations.

62. Possible Physical Bases

As was shown above, there seem to be two possibilities for the over-all form of the constructional system, namely, a system form with a physical basis or one with a psychological basis (a system form with a cultural basis appeared unworkable). In order to gain a general view of the various possibilities for constructional systems, we shall deal with the basis problem as it occurs in these different system forms and not only as we find it in the form which we shall ultimately adopt. Concerning the choice of a physical basis, we shall briefly indicate, by way of example, three possibilities without thereby excluding others.

> EXAMPLES. 1. One could choose as basic elements the electrons (including the protons, with positive elementary charges) and, as basic relations, the spatial and temporal relations between them. Properties of the electromagnetic field can then be defined through implication statements about the acceleration of electrons. The atoms of all chemical elements are constructed as certain constellations of electrons, and gravitation is constructed through implication statements about the acceleration of atoms. The derivation of the remaining constants and other concepts of physical science then no longer offers any principal difficulties since, in physics, they can all be reduced to magnetic fields, electrons, and gravitation. The perceptible physical things and properties can then easily be constructed from the things and properties of physical science, since they are uniquely determined by them.
>
> 2. We may choose as basic elements the space-time points of the four-dimensional space-time continuum and, as basic relations, their relative location in the continuum and the one-many relations between real numbers and space-time points which correspond to the individual components of the potential functions: the electromagnetic four-dimensional vector field and the tensor field of gravitation. According to the general theory of relativity in Weyl's formulation, all concepts of physical science can in principle be derived from these data. The electrons are constructed as locations of peculiar distributions of potentials [57] or as topological individuals through their relative locations; all other derivations take place as in (1).

57 Potentialverteilung

3. We may choose as basic elements the world points, in the sense of elements of the "world lines" of physical points (on the basis of Minkowski's formulation). They are not identical with the space-time points of the second example, but stand in a many-one relation to them. As basic relations, we may here choose coincidence and local time-order. From this we have to construct at first all topological, but then also the metrical, determinations of the space-time world (cf. Carnap [Abhäng.], [Logistik] § 37; Reichenbach [Axiomatik]), and finally the vector and tensor field of the above-mentioned theory of Weyl; after this, the construction proceeds as above.

After we have constructed the physical objects by proceeding from such a physical basis, we can construct the other object types according to our earlier considerations concerning the reducibility of psychological objects to physical ones and of cultural objects to psychological ones (§ 55 ff.).

63. *Possible Psychological Bases*

In selecting a psychological basis, either of the following alternatives is possible: the autopsychological (or "solipsistic") or the general psychological [58] basis. With the autopsychological basis, the available basic elements are restricted to those psychological objects which belong to only one subject. As we have seen above, in this case the psychological domain must be divided into two constructionally different parts: from the autopsychological objects we first construct the physical ones, and only then can we construct the heteropsychological objects. If we choose the general psychological basis, then the psychological objects of all psychological subjects are taken as basic elements. This method has the advantage that the construction of the totality of psychological objects is easier; it is carried out in precisely the same way in which the autopsychological objects are constructed if we choose the autopsychological basis. If we select the general psychological basis, this construction completes the task of constructing all psychological objects, while, if we choose the autopsychological basis, we still have, after the construction of the physical, the entirely different and quite difficult task of constructing the heteropsychological. In both cases we have, in addition, the choice of different *types* of psychological objects as basic elements, for example, the undivided experiences (of all subjects or of the one subject) or the parts of these experiences, or certain kinds of parts of

58 allgemeinpsychisch

experiences, for example, the sensations. We shall consider these possibilities when we discuss the autopsychological basis (§67), which we shall choose.

64. *The Choice of the Autopsychological Basis*

In spite of the indicated advantages of the general psychological basis, we choose the autopsychological basis for our constructional system. The most important reason for this lies in our intention to have the constructional system reflect not only the logical-constructional order of the objects, but also their epistemic order (§54). It is for the same reason that we excluded the system form with physical basis, various versions of which were logically possible. Occasionally, one encounters the opinion that, not autopsychological, but general psychological, objects form the basis even in the epistemic order of objects, but this position cannot be maintained in view of the fact that it is impossible to recognize heteropsychological objects without the mediating recognition of physical ones (§ 58).

The second reason for preferring a system form with an autopsychological basis is a formal-logical one. For, even if a constructional system with a general psychological basis reflected the epistemic order of objects, a system with an autopsychological basis still has the advantage that the totality of all objects is constructed from a considerably smaller basis.

The autopsychological basis is also called *solipsistic*. We do not thereby subscribe to the solipsistic view that only one subject and its experiences are real, while the other subjects are nonreal. The differentiation between real and nonreal objects does not stand at the beginning of the constructional system. As far as the basis is concerned, we do not make a distinction between experiences which subsequent constructions allow us to differentiate into perceptions, hallucinations, dreams, etc. This differentiation and thus the distinction between real and nonreal objects occurs only at a relatively advanced constructional level (cf. § 170ff.). At the beginning of the system, the experiences must simply be taken as they occur. We shall not claim reality or nonreality in connection with these experiences; rather, these claims will be "bracketed" (i.e., we will exercise the phenomenological "withholding of judgment", ἐποχή in Husserl's sense ([Phänomenol.] §§31, 32).

Within the autopsychological realm, the basis must be still more precisely delimited. The term "psychological" could perhaps be thought of as comprehending unconscious occurrences, but the basis consists only in conscious appearances (in the widest sense): all experiences belong to it, no matter whether or not we presently or afterward reflect upon them. Thus, we prefer to speak of the *stream of experience*. The basis could also be described as the given, but we must realize that this does not presuppose somebody or something to whom the given is given (cf. § 65). The expression "the given" has the advantage of a certain neutrality over the expressions "the autopsychological" and "stream of experience". Strictly speaking, the expressions "autopsychological" and "stream of experience" should be written in the symbolism introduced in § 75 as ^{p}autopsychological^{p} and ^{p}stream of experience^{p}.

REFERENCES. Since the choice of an autopsychological basis amounts merely to an application of the form and method of solipsism, but not to an acknowledgment of its central thesis, we may describe our position as *methodological solipsism*. This viewpoint has been maintained and expounded in detail, especially by Driesch, as the necessary starting point of epistemology ([Ordnungsl.] esp. 23). I mention here some further adherents of this theory, some of whom apply the solipsistic method only in the initial stages of their systems and eventually make an abrupt jump to the heteropsychological. Since they do not, for the most part, employ any precise forms of construction, it is not always clear whether this transition amounts to a construction on the solipsistic basis, as is the case in our constructional system, or whether it is a desertion of that basis.

Von Schubert-Soldem ([Erkth.] 65 ff.) explicitly wants his solipsism to be taken, not in a metaphysical, but only in a "methodological" sense, ([Solipsismus] 49, 53), a fact that is frequently overlooked by his critics (Gomperz [Ereignis] 236 ff., Ziehen [Erkth.] 37, 39, 277 ff., Husserl [Phänomenol.] e.g. 316; necessity to intersubjectivize: 317. Dingier [Naturphil.] 121 f., Reininger [Psychophys.] 51, Jacoby [Ontol.]. Volkelt ([Gewissheit] 55 ff.) chooses a "monological" (that is to say, an autopsychological) starting point for the theory of knowledge and gives a good criticism of the not (or not purely) autopsychological starting point of Avenarius, Cornelius, Petzold, and Rehmke. However, the method which Volkelt uses to break through the limits of individual subjectivity differs considerably from our own. Russell ([External W.] 96 f., [Sense-Data] 157 f.) considers the construction of the physical from an autopsychological basis very desirable, but also very difficult and presently altogether unattainable.

In opposition to the systems mentioned above, many others do not apply methodological solipsism, and some oppose it explicitly. Mach ([Anal.] 19) is especially conspicuous for his non-autopsychological basis, since it does not seem to be in harmony with the rest of his views. I do not here wish to enumerate the opponents of an autopsychological basis, but wish to mention only Frischeisen-Köhler ([Wissensch.]). He takes as the epistemological subject, not the self, but "consciousness in general", to which the individual selves are phenomena. It must all the more be noted that even this opponent cannot escape placing the fundamental phenomenon [59] of cognition in the autopsychological domain: "To find a starting point for methodical reflection, we have to go back to personal experience" (p. 244); "The limitation of the given to the sphere of my own self cannot be denied" (p. 254); "Thus, from the beginning of my reflection, I have to rely upon my own, and only my own, self-consciousness." (p. 265). He especially emphasizes the independence of this fact from one's attitude vis-à-vis the realism problem: "There are no objects of experience which are common to a number of experiencing subjects. Even this sentence—no matter how paradoxical it sounds—is not based upon any hypothesis concerning the reality or nonreality of the outside world. In order to grasp it, we do not have to leave the basis of naïve realism." We are even in a position to save ourselves a discussion of the antisolipsistic position of Mach, Schuppe, and Cassirer by pointing to Frischeisen-Köhler's refutation of these views. It is all the more difficult to understand how Frischeisen-Köhler can still think, in spite of these admissions, that he cannot use an autopsychological basis for his theory of knowledge. The explanation probably lies in the fact that it seems almost impossible to proceed from an autopsychological basis to the cognition and construction of other subjects, of the heteropsychological, and of an intersubjective external world. It can be assumed that this was the main reason why some other philosophers, too (for example, Natorp, Rickert [System] 184ff., and others) have chosen a non-autopsychological basis. Since construction theory removes the obstacles, indicating and clearing the way from an autopsychological basis to the beteropsychological and to an intersubjective world (cf. §§ 66, 140, 145-149), there should be no reason left for adopting any other basis.

65. *The Given Does Not Have a Subject*

The expressions "autopsychological basis" and "methodological solipsism" are not to be interpreted as if we wanted to separate, to begin with, the "*ipse*", or the "self", from the other subjects, or as if we wanted to single out one of the empirical subjects and declare it to be the epistemological subject. At the outset, we can speak neither of other

---

59 Urphänomen

subjects nor of the self. Both of them are constructed simultaneously on a higher level. The choice of these expressions merely means that, *after* the formulation of the entire constructional system, we shall find various domains which we call, in conformity with the customary usage, the domain of the physical, of the psychological (i.e., of the auto- and heteropsychological), and of the cultural. Any complete constructional system, no matter what its system form, must contain these domains. In order to characterize the differences between the system forms, we shall indicate in which of the object domains the basic elements are located after the formation of the system is completed. Before the formulation of the system, the fundamental elements are without properties and do not fall into specific domains; at this point, we cannot even speak of these domains and especially not of a differentiation between different subjects. In our system form, the basic elements are to be called experiences of the self *after* the construction has been carried out; hence, we say: in our constructional system, "my experiences" are the basic elements. (More precisely, in the terminology of § 75: "my experiences".)

> This state of affairs can be explained through an analogy: if we construct from the numbers 1,2, 3,... at first zero and then the corresponding negative numbers, then step by step the rational numbers, the real numbers, the complex numbers, then we shall finally characterize our starting point within the entire system of numbers by saying that we have chosen the real, positive integers as the initial elements. At the beginning of the construction, the designation of the elements as "real", "positive", and "integral" is meaningless. It makes sense only after the construction of the domains of the complex, negative, and fractional numbers, since it indicates the boundary toward these other domains. Likewise, the characterizations of the basic elements of our constructional system as "autopsychological", i.e., as "psychological" and as "mine", becomes meaningful only after the domains of the nonpsychological (to begin with, the physical) and of the "you" have been constructed. Then, however, they are quite meaningful and indicate how this system differs from other system forms with general psychological or physical basis. These other basis descriptions are also meaningful, not for the basic elements as such, but only in view of the system as a whole. Before the formation of the system, the *basis is neutral* in any system form; that is, in itself, it is neither psychological nor physical. *Egocentricity* [60] *is not an original property of the basic elements*, of the

60 Ich-Bezogenheit

given. To say that an experience is egocentric does not make sense until we speak of the experiences of others which are constructed from "my" experiences. We even must deny the presence of any kind of duality in the basic experience, as it is often assumed (for example, as "correlation between object and subject" or otherwise). Frischeisen-Köhler writes, "Since the beginning of modem philosophy, it is a common feature of all theories that, in the data which must be considered presuppositions of all thought ... two components may be separated." [Wissensch.] 190. These theories are the victims of a prejudice, the main reason for which is the subject-predicate form of the sentences of our language.

This egocentricity does not seem equally fundamental in the different sense modalities. To begin with, it seems to hold only for visual perceptions and seems to be connected with spatial disposition and the resulting awareness of distance. One can conclude this from the fact that the blind, on the basis of their tactile impressions, do not arrive at a subject-object dualism, a fact which is oftentimes obscured because the blind adopt the language of the sighted. Furthermore, the behavior of a blind man to whom sight has been restored shows that, to begin with, "optical impressions are not given to them in depth," since these blind are "still all impression." From this it follows that the experiences of all the sense modalities, even of sight, are originally simple, undivided experiences and that the self-object division is the result of a synthesis which is carried out in analogy to the spatial ordering in the synthesis of visual impressions.

REFERENCES. About the indicated experiences of blind persons: Wittman [Raum] 15 f., based upon Ahlmann [Opt. Vorst.].

Volkelt ([Gewissheit] 59 ff.) gives an especially clear account of the "neutral character" of experiences as basic elements: that they are "my" experiences and that they are "psychological" can, strictly speaking, be said about them only after the "you" and the "physical" have been recognized.

The following philosophers agree that the self is not implicit in the original data of cognition: Mach [Anal.] 19ff,, v. Schubert-Soldem [Erkth.] 65 ff., Nietzsche [Wille] §§ 276, 309, 367 ff.: "It is merely a formulation of our grammatical habits that there must always be something that thinks when there is thinking and that there must always be a doer when there is a deed." Aster ([Erkenntnisl.] 33), too, refers to the misleading influence of linguistic forms. Likewise Gomperz [Ereignis], following Wahle. Ziehen [Erkth.] 50 ff., 279, 445 ff., explicit rebuttal of Schuppe in [Schuppe]. Dingier [Naturphil.] 120 ff. Schlick [Erkenntnisl.] 147 f. Gatschenberger [Symbola] 151.

On the other hand, with our notion of the subjectless given, we deviate from various systems with which we agree in other important aspects: Schuppe (cf. Ziehen [Schuppe]); Natorp [Psychol.] 26ff.; Driesch [Ordnungsl.] 19; Husserl [Phänomenol.] 65, 160; Jacoby [Ontol.] 169; Russell [Description] 210. We have already referred to Frischeisen-Köhler [Wissensch.]. The weakness of his position becomes especially apparent in the following admission (p. 196): " ... thus the confrontation of subject and object, which we must assume with all its ramifications for the immediately given, is neither contained in the actual data of introspection, nor can it be conceptually apprehended. To impose this distinction upon the given—that is, to construe the given in analogy to thought—is to introduce a theoretical interpretation." Here, similar to §64, the strange opposition between a fact which Frischeisen-Köhler admits and that which, in his opinion, "must be assumed." The reason for this lies presumably in the fact that Frischeisen-Köhler thinks it to be impossible—as do probably many other proponents of the egocentricity of the given—to advance from a subjectless starting point to the construction of experiences which contain the self. However, construction theory will show that it can be done.

66. The *Autopsychological Basis and the Problem of Objectivity*

If the basis of the constructional system is autopsychological, then the danger of subjectivism seems to arise. Thus, we are confronted with the problem of how we can achieve objectivity of knowledge [61] with such a system form. The requirement that knowledge be objective can be understood in two senses. It could mean objectivity in contrast to arbitrariness: if a judgment is said to reflect knowledge, then this means that it does not depend on my whims. Objectivity in this sense can obviously be required and achieved even if the basis for knowledge is autopsychological.

Secondly, by objectivity is sometimes meant independence from the judging subject, validity which holds also for other subjects. It is precisely this intersubjectivity which is an essential feature of "reality"; it serves to distinguish reality from dream and deception. Thus, especially for scientific knowledge, intersubjectivity is one of the most important requirements. Our problem now is how science can arrive at intersubjectively valid assertions if all its objects are to be constructed from the standpoint of the individual subject, that is, if in the final analysis all statements of science have as their object only relations between "my"

61 Erkenntnis

experiences? Since the stream of experience is different for each person, how can there be even one statement of science which is objective in this sense (i.e., which holds for every individual, even though he starts from his own individual stream of experience)? The solution to this problem lies in the fact that, even though the *material* of the individual streams of experience is completely different, or rather altogether incomparable, since a comparison of two sensations or two feelings of different subjects, as far as their immediately given qualities are concerned, is absurd, certain *structural properties* are analogous for all streams of experience. Now, if science is to be objective, then it must restrict itself to statements about such structural properties, and, as we have seen earlier, it can restrict itself to statements about structures, since all objects of knowledge are not content, but form, and since they can be represented as structural entities (cf. § 15 f.).

A system form with an autopsychological basis is acceptable only because it is recognized that *science is essentially concerned with structure and that, therefore, there is a way to construct the objective by starting from the individual stream of experience*. Much of the resistance to an autopsychological basis (or "methodological solipsism") can probably be traced back to an ignorance of this fact, and many of the other expressions for the original subject (e.g., "transcendental subject," "epistemological subject," "superindividual consciousness," "consciousness in general") can perhaps be thought of as expedients, since from the natural starting point in the epistemic order of objects, namely, the autopsychological, no transition to the intersubjective realm seemed possible (cf. the quotations in § 64).

Only later, during the formulation of the constructional system itself, can we demonstrate the precise method for achieving objectivity in the sense of intersubjectivity (§§ 146-149). The preceding general remarks will suffice for the moment.

67. *The Choice of the Basic Elements: The "Elementary Experiences"*

After deciding to choose an autopsychological basis for our system (i.e., the acts of consciousness or experiences of the self), we still must determine which entities from this general domain are to serve as basic elements. One could perhaps think of choosing the final constituents of experience at which one arrives through psychological or phenomenological analysis (such as the most simple sensations, as in Mach [Anal.]), or, more generally, psychological elements of different types from which experiences can be formed. However, upon closer inspection, we realize that in this case we do not take the given as it is, but abstractions from it (i.e., something that is epistemically secondary) as basic elements. It must be understood that constructional systems which proceed from such basic elements are as much justified and practicable as, for example, systems with a physical basis. However, since we wish to require of our constructional system that it should agree with the epistemic order of the objects (§ 54), we have to proceed from that which is epistemically primary, that is to say, from the "given", i.e., from experiences themselves in their totality and undivided unity. The above-mentioned constituents, down to the last elements, are derived from these experiences by relating them to one another and comparing them (i.e., through abstraction). The more simple steps of this abstraction are carried out intuitively in prescientific thought already, so that we quite commonly speak, for example, of visual perceptions and simultaneous auditory perceptions, as if they were two different constituents of the same experience. The familiarity of such divisions which are carried out in daily life should not deceive us about the fact that abstraction is already involved in the procedure. This applies a fortiori to elements which are discovered only through scientific analysis. The basic elements, that is, the experiences of the self as units (which will be more precisely delineated in the sequel), we call *elementary experiences*.

REFERENCES. In opposition to the "atomizing" school of thought in psychology and epistemology, which postulates such psychological "atoms" as, e.g., simple sensations as elements, there is presently more and more emphasis on the fact that "*every state of consciousness is a unit* and is not, strictly speaking, analyzable." (Schlick [Erkenntnis].] 143 f.; italics mine). In particular, there is more and more proof that, in perception, the total impression is primary, while sensations and particular feelings, etc., are only the result of an abstracting analysis. This position has already been clearly indicated by Schuppe [Erkth.] 41, also [Imman. Phil.] 17: "The thinking of the individual begins with total impressions which only reflection analyzes into their simple elements." Similarly, Cornelius [Einleitg.] 210 f., also Gomperz [Weltansch.], with his doctrine of the "total impression" (as the feeling of unity for the impression as a whole), emphasizes this point and clarifies it with examples. He also gives a historical survey of related earlier theories. He mentions William Hamilton, Schuppe, Nietzsche [Wille] and others. Reininger [Erk.] 370, makes similar statements and refers to Kant.

The position just discussed has been developed especially by *Gestalt* *theory*. (Cf. Kohler [Gestaltprobl.] and Wertheimer [Gestaltth.]. It has become methodologically fruitful, especially in psychology, not only by suggesting new ways of asking questions, but also by arriving at materially new findings through a change in outlook. From this theory, new and important aspects arise for areas other than psychology.

Modern psychological research has confirmed more and more that, in the various sense modalities, the total impression is epistemically primary, and that the so-called individual sensations are derived only through abstractions, even though one says afterward that the perception is "composed" of them: the chord is more fundamental than the individual tones, the impression of the total visual field is more fundamental than the details in it, and again the individual shapes in the visual field are more fundamental than the colored visual field places, out of which they are "composed". These psychological investigations have frequently been undertaken in connection with Gestalt theory. Cf. also Wittmann [Raum] e.g., 48 ff.; note on page 19 of that work an interesting quotation from F. W. Hagen, who maintained a similar position as early as 1844.

As closely related, we must also mention the philosophical position of Driesch, with its emphasis upon "totalities" (cf. especially [Ordnungsl.] and [Ganze]).

In choosing as basic elements the elementary experiences, we do not assume that the stream of experience is composed of determinate, discrete elements. We only presuppose that statements can be made about certain places in the stream of experience, to the effect that one such place stands in a certain relation to another place, etc. But we do not assert that the stream of experience can be uniquely analyzed into such places.

68. *The Elementary Experiences are Unanalysable*

The elementary experiences are to be the basic elements of our constructional system. From this basis we wish to construct all other objects of prescientific and scientific knowledge, and hence also those objects which one generally calls the constituents of experiences or components of psychological events and which are found as the result of psychological analysis (for example, partial sensations in a compound perception, different simultaneous perceptions of different senses, quality and intensity components of a sensation, etc.). From this results a special difficulty.

We remember that class and relation extension are to be the only ascension forms of the constructional system (§ 40). Starting from any basic elements and basic relations, we can form only objects of the following kinds in the constructional system: on the first constructional level, classes of elements and relations [62] between elements; on the second level, only (1) classes of such classes, or classes of relations [62] of the first level, and (2) relations [62] between such classes, or relations [62] between relations [62] on the first level, or relations [62] between classes on the first level and elements, etc. It is obvious that construction, when carried out with the aid of these ascension forms is always synthetic, never analytic. Even if we were to suppose that the basic elements are themselves again classes of other elements, classes of "fundamental elements", we could not construct these fundamental elements with the aid of the given ascension forms. *The basic elements of the constructional system cannot be analyzed through construction*. Thus, the elementary experiences cannot be analyzed in our system since this system takes them as basic elements.

This fact agrees very well with our conception that the elementary experiences are essentially unanalyzable units, which has, after all, led us to choose them as basic elements. However, it could appear now that the previously indicated aim, namely, to construct, among other things, all objects of science and also the known psychological elements (i.e., the so-called constituents of experience), would now become unattainable. This difficulty is of fundamental importance for construction theory and requires, for its resolution, the development of a special constructional method. This is now to be discussed in more detail.

69. *The Problem of Dealing with Unanalyzable Units*

We overcome the difficulty which results from the fact that elementary experiences are unanalyzable by introducing a constructional procedure which, even though synthetic, leads from any basic elements to objects which can serve as formal substituents for the constituents of the basic elements. We call them formal substituents, because all assertions which hold for the constituents bold, in analogous form, also, for them. We call this procedure *quasi analysis*. (It is derived from the Frege-Russell "principle of abstraction": cf. the remark at the end of §73.) It is of importance wherever we are concerned with unanalyzable units of any kind, that is, with objects which, in their immediate given-ness, do not exhibit any constituents or properties or aspects. These objects are given, as it

62 Relation

were, only in point form and can therefore be treated only synthetically; nevertheless, as a result of our procedure, we can ascribe various characteristics to them. Properties and constituents are here taken to be the same thing; with psychological processes, for example, one cannot use the expression “constituent” in its original, spatial sense, but only in the sense of the equally figurative expression of “different aspects” or “characteristics”.

If unanalyzable units of any kind are given and if we are to discuss them at all, then statements about them must also be given. We have previously divided the descriptions of objects through statements into property descriptions and relation descriptions (§ 10). The statements about unanalyzable units cannot be given as property descriptions, since this would amount to saying that we ascribe characteristics to these units, which would contradict the concept we have of them. The statements can only be pure relation descriptions. Let us investigate especially the case where the relation descriptions are given in extensional form i.e., in the form of a pair list,[63] for example, through enumeration (or other characterization) of the pairs of correlated members (cf. §§ 32, 34). Notice especially the case where the unanalyzable units in question form the basic elements of the constructional system; in this case, the relation description is possible only in extensional form, since the basic relations [64] of a constructional system are given only in extension (§§43,45).

Generally speaking, and without restriction to the particular problem of elementary experiences, quasi analysis is to achieve the following: unanalyzable units of any kind, a pair list of which is presupposed, are to be manipulated with the constitutional ascension forms of class and relation extension (i.e., with synthetic methods) in such a way that the result is a formal substitution for proper analysis (i.e., the analysis into constituents or properties), which cannot be carried out in this case. Because of the required formal analogy between the results of quasi analysis and those of proper analysis, one can suppose that a certain formal analogy will obtain between these two procedures themselves. Thus, we investigate, to begin with, which formal characteristics we can find in the procedure of a proper analysis Which proceeds on the basis of nothing but a pair list of the objects to be analyzed. Then we shall see that the desired procedure of quasi analysis can be developed analogously.

63Since only dyadic relations are discussed in the sequel, I have translated “Relationsbeschreibung” as “pair list”, even though “list of n-tuples” would have been more precise.

64 Grundbeziehungun

70. *The Procedure of Proper Analysis on the Basis of a Pair List*

In the case of proper analysis, we are concerned neither with points that have no properties nor with unanalyzable units, but with objects which have several constituents (or characteristics). Analysis consists in inferring these constituents, which are initially unknown, from other data, e.g., from a pair list. Let us illustrate this with a simple example.

> EXAMPLE. Let our aim be the analysis of a number of things, each of which has one or more colors. Let there be altogether five different colors. Let us define the relation of "color kinship" in such a way that it is to hold for two things if these have at least one color in common. Let the things be individually designated, for example, by numbers. Now let us assume that we do not know of any of the things which colors it has. All we have is a pair list (i.e., we know only the extension of the relation of color kinship: we are told all pairs for which this relation holds, but we are not told which color these two things have in common). In other words, the relation extension of color kinship is completely given (cf. §§ 10 and 34). Now, our task consists in inferring from these data the distribution of colors. We cannot proceed by choosing one of the things at random and determining all its color kin on the basis of the pair list, for it does not follow that all these are color akin to each other.
>
> The task of analysis is attained once we succeed in determining the "color classes". Let us call the class of all things which have a certain color in common a "color class" (e.g., the class of the red [completely red or also red] things, of blue things, etc.). There are, in all, five color classes which partially overlap. What is the connection between the color classes and the relation of color kinship? Now, two properties are characteristic for the color classes. The first of these they have always; the second, most of the time, namely, when conditions are not especially unfavorable. First of all, any two elements of a color class stand in the relation of color kinship to one another (because the members of the pair both have the color which determines the color class). Secondly, the color classes are the largest possible classes all of whose members are color-akin (i.e., there is no thing outside of a color class which stands in the relation of color kinship to all the things in the class). (This second property can occasionally be absent, for example, if one of the five colors is a "companion" of a second, i.e., if none of the things has the first color without having the second also.) For example, if blue is a companion of red, then the blue color class does not have this second property, for a thing which is red but not blue does not belong to this

> color class and is nevertheless color akin to all things in this class, since all of them are also red. If there are no systematic connections between the distributions of the different colors, then this unfavorable case, namely, that the second property is missing in a color class, becomes the less likely the smaller the average number of colors of the thing and the larger the total number of things is. Let us assume that in our case the unfavorable conditions are not fulfilled (i.e., that the color classes have both of the characteristic properties). Now we have to determine, on the basis of the pair list, those classes of things which have these two properties (in the terminology of logistics: the similarity circles [65] relative to color kinship). This is possible because the two properties have been described only with reference to couples for which the indicated relation holds. The classes formed in this way will be the color classes. In this case, we will find five color classes without, of course, being able to determine which color belongs to each of them. Thus, we must assign arbitrary names to them, for example, C_1...C_5. Now if we remember that a class does not consist of its elements, but is a quasi object, whose symbol serves to express that which is common to the elements of a class (§ 37), then we can simply think of the color class C_1 as the common color of the elements of C_1. Thus C_1...C_5 designate the five colors. We do not know, of course, whether C_1 is red or green, etc. Now, if one of the things is an element of C_1 and of C_5, but not an element of any other color class, then we say of it that it has two colors (i.e., it bears the colors C_1 and C_2). In a similar way, we can make this determination for each one of the things. Thus, the analysis is complete; we have determined the constituents (or properties) of each element, even though we have not used the usual names for the qualities, but have only characterized them as common properties of certain elements, that is, as classes.

Thus, if a pair list is given whose relation extension signifies agreement in (at least) one constituent, then the procedure of proper analysis consists in establishing the similarity circles associated with the relation extension, that is to say, the classes which have the following two properties: any two elements of such a class are a pair of the given relation extension, and no element outside of such a class forms a pair of this relation extension with every element in that class. Classes which are formed in this way are then assigned to their elements as constituents (or properties).

---

65 Ähnlichkeitskreise

71. *The Procedure of Quasi Analysis*

The procedure of quasi analysis for elements which are unanalyzable units (that is, which have neither constituents nor characteristics) stands in exact formal analogy to the indicated procedure of proper analysis. In order to be able to use quasi analysis, it must be presupposed that a pair list is given, whose relation extension R has the same general formal property as the relation extension which forms the basis of proper analysis. The latter (in our example, color kinship) indicates agreement in a constituent and hence is symmetrical and reflexive (i.e. it is a "similarity"; cf. § 11). If R is likewise symmetrical and reflexive, then we can proceed as with proper analysis; that is, as if R also meant agreement in a constituent. Thus, we form similarity circles with respect to R (i.e., those classes *c* which have the following two properties: each pair in *c* is an R pair; no element outside of *c* forms an R pair with every element in *c*). In this case, too, we envisage the similarity circles (which correspond to the color classes of our example) as common properties of the elements and hence assign them to these elements as characteristics. But since it is presupposed that these elements are unanalyzable units, they cannot, strictly speaking, have characteristics or constituents, nor can this be a case of proper analysis. It is for this reason that we designate the procedure as quasi analysis and the entities which we find through this procedure and which we assign to the elements, as "quasi characteristics" or quasi constituents. Thus, for example, if we have found the similarity circles q_1, q_2, . . . (i.e., if we have found for each such circle the list of elements which belong to it) and if a certain element belongs, for example, to the classes q_1, q_3, q_4, then we say: this element, although as an unanalyzable unit it does not have proper constituents, has three quasi constituents, namely, q_1, q_3, q_4. Thus, the quasi analysis has been carried out and meets the requirements which we have previously laid down for it (§ 69).

EXAMPLE. Let us clarify the significance of quasi analysis through an example. As a domain of unanalyzable units, we use the so-called "compound" chords. As a phenomenon, i.e., as it is given in sensation (in contrast to the viewpoint of physics and acoustics), a chord is a uniform totality which is not composed of constituents. It may seem to us as if the chord we hear when we strike the keys *c*, *e*, *g* of the piano has three parts; however, this is due only to the fact that the character of our perception is partially determined by the tone kinship of this chord with innumerable other chords which are already known to us: the chord *c-e-g* is akin to all the chords which (acoustically speaking) contain *c* (one of these may be *c* alone). Furthermore, our original chord is akin in tone to all chords which contain *e* and likewise to all which contain *g*. Thus, it belongs to three chord classes, and this brings about the impression that it has three parts.

Let us now assume that we have not been given any qualitative characterization, but only a pair list of the chords which one can hear, for example, in a piano, that is to say, a pair list on the basis of tone kinship. Since this relation extension is reflexive and symmetrical, we can apply the procedure of quasi analysis to it. On the basis of the given pair list (i.e., on the basis of the list of pairs which are akin in tone), we determine the similarity circles. These similarity circles stand in exact formal analogy to the color classes of the earlier example of proper analysis. With the aid of this analogy, one can easily convince himself that they are identical with the above-mentioned chord classes (i.e., with the classes of such chords which [acoustically speaking] coincide in a constituent tone). Thus, for each "constituent tone" (in the language of acoustics), whether or not it occurs among the chords in isolation, we obtain such a quasi analytic similarity circle (i.e., for example, the similarity circles, *c*, *d*, *e*, etc.). Now we assign to each chord those similarity circles to which it belongs as quasi constituents. Since the chord *c-e-g* is an element of similarity circles *c*, *e*, and *g*, we assign to it these three classes, namely, *c*, *e*, *g*, as quasi constituents. (The threefold sign (*c-e-g*) of this chord refers initially only to its origin, namely, the depression of three keys of a piano, and does not refer to a tripartition of the uniform chord.) We said previously that the chord *c-e-g* does not, properly speaking, consist of three parts and that the impression of tripartition which it makes upon a trained ear is due to the fact that it belongs to three chord classes. Now we see that this impression of tripartition is the result of an intuitively performed quasi analysis. In hearing the chord, we detect—provided that we have already heard a sufficient number of other chords—three constituents, not in the sense of parts, but in the sense of three different directions in which we can proceed from it to other chords (i.e., to entire chord classes which stand to one another in the relation of tone kinship).

As we here identify what is generally called the constituent tones of a chord with chord classes (i.e., with classes of chords), it is important to recall the character of classes as quasi objects (§37). A chord class is neither the whole nor the collection of its elements. Thus, it is not the chord phenomenon which would result if the chords of this class were to be sounded in some temporal sequence or other, or even all together. A chord class, as any class, is that which its elements have in common.

> But this, again, is not to be understood in the sense of a common constituent, for the chords have none such. The "class" is not, properly speaking, an object. Its symbol merely serves to make those assertions which hold equally for all its elements. It is apparent, then, that the characteristic or, more precisely, the quasi characteristic, *c*, cannot mean anything but the mutual kinship of all the chords which (acoustically speaking) "contain" *c*. If one were to hear the chord *c*-*e*-*g*, without having previously heard any musical chords, one would hardly think of it as having three parts. Even though we say that we recognize the tone *c* as a constituent tone in the chord *c*-*e*-*g*, we should not think of it as a proper constituent of this chord, but only as a quasi constituent. Otherwise, one would come to the conclusion (which has indeed sometimes been maintained) that the chord *c*-*e*-*g* consists of the individual tones *c*, *e*, *g*, and, in addition to them, of something new which comprises the actual character of the chord. Thus we would assume four constituents, where in truth there is only an unanalyzable unit without any constituents.

The importance of the procedure of quasi analysis becomes evident when we recall that, in the position here maintained, the elementary experiences (i.e., the basic elements of the constructional system) are unanalyzable units and that many psychological, especially phenomenal,[66] objects, which traditional psychology thought of as being compounds, are likewise unanalyzable. In the case of such entities, one can apply the language of analysis (i.e., one can speak of their constituents or components, etc.), but one should never forget that he is, strictly speaking, concerned with quasi constituents, since these entities—as they were originally given—have no proper constituents. (Cf. the references to recent psychological positions, especially Gestalt theory, and to holistic notions in philosophy in §67.) An example is the notion of chords as indivisible units, which we have just discussed at some length. In summary, analysis or, more precisely, quasi analysis of an essentially unanalyzable entity into several quasi constituents means placing the entity in several kinship contexts on the basis of a kinship relation, where the unit remains undivided.

72. *Quasi Analysis on the Basis of a Part Similarity Relation* [67]

The indicated procedure of quasi analysis treats the relation extension of a given pair list as if it meant agreement in a constituent part. Conse-

---

66 sinnesphänomenal
67 Teilähnlichkeitsrelation

quently, the results are called quasi constituents. There is still another form of relation description, which we can consider analogous to quasi analysis. It is not the relation of having constituents that are identical, but the relation of having constituents that are alike. This kind of relation description gives rise to a second type of quasi analysis which does not have the same general importance as the first, but which must be explained because it is later applied in the constructional system.

EXAMPLE. Let us again begin with a comprehensible example. Let a large number of things be of such a nature that each of them has one or several colors. Here a much larger number of things is required than in the case of the first type of relation description (§ 70). However, in this case, the number of different colors is not to be restricted to five, but a very large number of colors from all parts of the color solid are to occur. We call two things *color similar* if, among other colors, they each have a color which is similar to that of the other (i.e., which, on the color solid, has a distance from the other which is smaller than a certain arbitrarily chosen magnitude). As in the earlier example, no information about these things is to be given, except enumeration of the pairs of this relation (i.e., pair list). It is impossible in this case to determine directly the *color classes* (i.e., the classes of all and only those things which, among other colors, bear a certain color); this can be done only through a complicated procedure which we shall develop later. On the other hand, we can easily determine a different type of class, namely, the "color similarity circles". All else develops from them.

The largest possible parts of the color solid, which contain nothing but colors that are similar to one another, are spheres which partially overlap each other, and whose diameter is the arbitrarily fixed maximal distance of similarity (which may be different in different parts of the color solid). Thus, to these color spheres belong not things, but colors. The class of things which have one of the colors of a certain color sphere is called a color similarity circle. We can now easily see that the characterizing properties of the color similarity circles, as based on color similarity, are the same as those of the color classes as based on color kinship in the earlier example: any two things belonging to a color similarity circle are color similar; no thing which does not belong to a certain color similarity circle is color similar to all things belonging to this circle. Hence, the color similarity circles are the similarity circles based on color similarity. (As in the earlier case, it is again required that certain unfavorable conditions are not present if we are to arrive at a correct determination of these classes. For example, it must not be the case that a thing *a*, even though it does not bear any of the blue colors, on the basis of which other things form the color similarity circle *c*, is nevertheless "accidentally" color similar to all these things in *c* by being similar in a color other than blue to each thing that belongs to *c*. We shall later return to this point.)

So far, we have derived only the color similarity circles and not yet the color classes, but, as we have indicated in the earlier example, only the color classes can be envisaged as the representatives of the colors themselves and can, as such, be assigned to the things. Now, the color classes are to the individual places of the color solid what the color similarity circles are to the color spheres. Since the individual places of the color solid are the largest parts of the color solid, which remain always undivided in the mutual overlapping of the color spheres, we can determine the color classes correspondingly as the largest subclasses of the color similarity circles which remain undivided through the mutual overlapping of these circles.

As we can see from the example, quasi analysis on the basis of a part similarity relation P consists first of all in establishing similarity circles relative to P, just as in the previous case. In this case, the quasi constituents are derived from the similarity circles only indirectly, namely, as the largest subclasses which remain undivided by the mutual overlapping of the similarity circles. (This explanation is not altogether precise; we shall give a more precise one later on when we explain the application of this procedure [§§ 81, 112].)

In view of the formal analogy of the first step in this second type of procedure with the first type of procedure, we can always carry out this step without having to decide antecedently whether the relation extension of a given pair list, to which we want to apply quasi analysis, is to be construed as part identity (i.e., agreement in a quasi constituent) or as part similarity (i.e., approximate agreement in a quasi constituent). After the first step has been carried out, the decision can easily be made, for similarity circles behave toward one another in an entirely different way in the first case than in the second. In the second case, there is a multiple mutual overlapping of similarity circles. Thus they can be put into one or more systems, such that those similarity circles which are close to one another in the system have a large number of elements in common. In the first case, on the other hand, similarity circles are either mutually exclusive (namely, if their elements each have only one quasi constituent) or else they have only insignificant parts in common with one another, and, even then, an order does not generally result from this fact. Thus, if we do not know whether a given similarity relation Q is to be envisaged as part identity or as part similarity, then we must investigate the similarity circles on the basis of Q as to whether they show mutual overlappings characteristic of the first or of the second case. In the first case, the similarity circles themselves must be taken as quasi constituents. In the second case, the quasi constituents must be derived from the similarity circles, namely, as the largest subclasses which are not divided through the overlapping of the similarity circles.

73. *Quasi Analysis on the Basis of a Transitive Relation*

For a relation extension R, on whose basis a quasi analysis is carried out, we have so far presupposed only that it is symmetrical and reflexive. The indicated procedure is independent of the property of transitivity (about this concept, cf. §11). In the examples which we have discussed so far, we were concerned with relation extensions which were neither transitive nor intransitive. However, the case of a quasi analysis on the basis of a transitive relation extension deserves special treatment, for precisely this case obtains frequently in the formation of concepts in various different fields, and, moreover, it is of a special formal simplicity. The classes which are to be formed as quasi constituents fulfill, in this case too, the previously indicated conditions, but they can be defined also in another, simpler way. Since in this case R is transitive, symmetrical, and reflexive (i.e., an "equivalence", §11), it follows that no element outside of a similarity circle can be akin to any element within the similarity circle, for then it would have to be akin to all other elements of the similarity circle, and thus, contrary to our assumption, would have to belong to it. From this it follows, first, that if R is transitive, then the similarity circles do not have any elements in common. Of the two conceptions of a relation extension which were discussed in § 72—part identity and part similarity—only the first can obtain in this case: the similarity circles of R must here- themselves be considered the quasi constituents; in this case, we shall call them *abstraction classes* of R. It follows, moreover, that the class of elements which stand to any given element in the relation (extension) R forms an abstraction class. Hence, the abstraction classes and thus the quasi constituents can here be defined as the (non-empty) classes of elements which are akin to a given element.

REFERENCES. The procedure of quasi analysis in this simplest case of a transitive relation extension corresponds to the "principle of abstraction", which was first explicitly mentioned by Russell ([Principles] 166: cf. also Frege [Grundig.] 73 ff.). It had been used previously by Frege and then by Whitehead and Russell for the construction of the cardinal numbers (cf. § 40). Cf. Couturat [Prinz.] 51 ff.; Weyl [Handb.]

9 f., there also a reference to Leibniz; Carnap [Logistik] § 20. Whitehead and Russell have also referred to the extramathematical applicability of the principle and have used it for their constructions; cf. Russell [External W.] 124 ff.

74. *About Analysis and Synthesis*

Later on, in the formulation of the lower levels of our outline of the constructional system, we shall illustrate the application of the procedure of quasi analysis to the elementary experiences as basic elements. Then we shall see how this procedure puts us in a position, for example, to construct the different sense modalities and, within the sense modalities, the various sensory qualities, without disclaiming the unanalyzable character of elementary experiences.

Many epistemological systems (especially the positivistic ones) which are otherwise closely related to our constructional system have used, not experiences themselves, but sensory elements or other constituents of experiences as basic elements, without paying heed to their character as abstractions. The reason for this was perhaps that it seemed impossible to construct all objects of psychology and, among them also, those "constituents of experiences" if experiences themselves were chosen as basic elements. After we have shown, through the procedure of quasi analysis, that this impossibility is only apparent, there seems to be no reason for any epistemological position (and this holds especially for a positivistic one) why elementary experiences should not be acknowledged to have the character of unanalyzable units and why they should not be taken as basic elements.

In order to avoid any misunderstanding, let me emphasize again that, with the conception of elementary experiences as unanalyzable units) we do not brand a psychological statement, such as "this experience (or this act of consciousness) consists of a visual perception with such and such constituents, of an auditory sensation, of a feeling with such and such components, etc." as false or even meaningless. All we assert is that in such a statement the expression "constituents" refers only to quasi constituents. In other words, we say that every so-called constituent relates to the experience as the chord class *c* in the above example (§71) to the chord *c*-*e*-*g*, namely, as an entity which is constructed through kinship relations, that is, a "quasi constituent".

REFERENCES. Our position is closely related to that of Cornelius: "The value of such an analysis does not consist in the recognition of

> every single state of consciousness—no analysis of these can be possible —but in the recognition of regular connections between various such states." [Einleitg.] 314. Cf. also the quotations in § 67.

If class and relation extension are acknowledged as the only constructional steps (§68), then *the methodological unanalyzability* of the basic elements follows for any constructional system, and, from the choice of the essentially unanalyzable elementary experiences (§ 67) follows a *materially determined* [68] *unanalyzability*. From this arise the following consequences with respect to the general relationship between analysis and synthesis of scientific objects, which we assume to be constructed according to our constructional system. Since every object of science is constructed from the basic elements, to analyze it means to trace back the procedure of construction from the object itself to those elements which are required for its construction. Any analyzing beyond this point will have to take on the form of quasi analysis, since proper analysis is no longer possible. The same holds when the object to be analyzed is not a constructed entity, but a basic element. Now, quasi analysis leads to entities which we have called quasi constituents (in order to stay close to established usage, which calls them constituents). But this is done by forming classes of elements and, furthermore, relation extensions of these classes; hence, by way of synthesis, not analysis. We can therefore say: *Quasi analysis is a synthesis which wears the linguistic garb of an analysis*.

Since the basic elements are not accessible to proper analysis, but only either to quasi analysis or to other constructional procedures, all of which are synthetic, it follows, if we concentrate not upon the linguistic expression but upon the actual nature of the procedure, that these elements are accessible exclusively to synthesis and not to analysis. All other objects are synthetic entities constructed from the basic elements and analyzable only to the point where these basic elements are reached again. *Analysis is possible only if, and to the extent in which, synthesis has preceded; it is nothing but a retracing of the path of synthesis from the final structure to intermediate entities and finally—if the analysis is "complete" in the sense of construction theory—to the basic elements*. To be sure, an analysis is then not yet "complete" in the scientific sense, but its continuation is then a quasi analysis (i.e., a new synthesis).

68 inhaltlich bestimmt

2. THE BASIC RELATIONS[69]

75. *The Basic Relations as Basic Concepts of the System*

We have realized earlier (§61) that, to lay down the basis of a constructional system, we need not only the basic elements, but also certain initial ordering concepts,[70] since otherwise it is not possible to produce any constructions starting from the basic elements. The question whether these first ordering concepts are to be given the form of classes ("basic classes") or of relation extensions ("basic relations") remained at first open. But after the basic elements were chosen (§67) and the elementary experiences which were chosen turned out to be units unanalyzable in principle, it appeared that any assertion about them would have to have the form of a pair list (§69). From this it follows that (one or more) *basic relations* must be chosen as the first ordering concepts. These basic relations, and not the basic elements, form the undefined basic concepts of the system. The basic elements are constructed from the basic relations (as their field).

> REFERENCES. Cassirer ([Substanzbegr.] 292 ff.) has shown that a science which has the aim of characterizing unique entities through contexts of laws without loss of individuality must utilize, not class ("generic") concepts, but *relational concepts*, since these can lead to the formation of series and thus to the establishment of systems of ordering. Since one can easily make the transition from relations to classes, and since the opposite is possible only very rarely, it follows that it is relations extensions which must initially be posited.
>
> Thus, two entirely different and frequently hostile philosophical positions have the merit of both having discovered the necessary basis of the constructional system. Positivism has emphasized that the only material of cognition consists in the undigested experientially given. It is here that we have to look for the basic elements of the constructional system. Transcendental idealism, especially the Neo-Kantian school (Rickert, Cassirer, Bauch), has justly emphasized that these elements do not suffice. Order concepts, our basic relations, must be added. We want to determine the basic relations in such a way that they are isogenous (§29) to one another (i.e., that they are all of the same level, §41). In fact, the terms of each of the basic relations are never to be

---
69 Grundrelationen
70 Ordnungssetzungen

anything but elementary experiences. In order to form the basic relations, we must now consider which relations between the elementary experiences are to be considered fundamental. We are not here concerned with the quest for psychologically fundamental relations or relations which are of especial importance for the processes of consciousness. Since the basic relations are to serve as the basis for the construction of all objects (of cognition), they must be chosen in such a way that, through them, all recognizable states of affairs can be expressed. According to our detailed discussion (§§50, 51), expressibility is to be understood only in the sense of a definite description; here we pay attention only to logical and not to epistemic value, nor are we concerned with the question whether, in the actual occurrence of a process of cognition, a state of affairs which can be expressed through certain basic relations [71] is actually derived from these basic relations. It happens occasionally that a certain state of affairs is fundamental and cannot be reduced to simpler ones, as far as the psychology of cognition is concerned, while it is logically dependent upon others in such a way that it can be constructed from them and that it therefore does not have to be postulated as a basic relation. We shall later on find examples for this.

In searching for the basic relations, we have to pay especial attention initially to the requirements of the construction of physical objects (i.e., we shall test our findings by applying them to facts of perception). Whether we need other basic relations for the construction of objects of higher levels (heteropsychological or cultural), we shall consider afterward. The present investigations whether certain relations are required as basic relations and especially whether they are sufficient for the demands that we put upon them can only be provisional. The correctness and appropriateness in the choice of the basic relations can be confirmed only through the fact that, in the formulation of the constructional system, the most important constructions, upon which all else rests, can be carried out with the aid of the chosen basic relations. This logical performance is the essential criterion for the basic relations. On the other hand, an investigation of whether a certain relation is fundamental as far as the psychology of cognition is concerned has mostly heuristic value.

In order to discuss which relations are meant as basic relations and what entities can be constructed from them, we have to speak of the experiences in the customary *factual language*,[72] in this case, the *lan-*

71 From here to the end of §75, "basic relation" is a translation of "*Grundbeziehung*".

72 Sachverhaltssprache

guage of psychological analysis: we have to speak of their constituents, of sensations, of the various senses, of quality, intensity, etc. In using these expressions, we do not mean to suggest that these constituents, etc., are presupposed for the construction. This would lead to a vicious circle. The only purpose of these expressions is to indicate certain known states of affairs, especially fundamental relations between the elementary experiences. This can only be done in a mode of expression as it is customary in the discussion of experiences and their relations, hence, in the language of psychology. In Chapters C and D we shall enclose, for clarity's sake, expressions which are to be understood in this way in p-symbols (e.g., ^{p}qualities^{p}). If an expression does not belong to the factual language, that is, if it is not meant in the sense of customary usage, but relates to the constructional system (hence, to a constructional definition, which either has already been given or the formulation of which is intended) or else, if it relates to an undefined basic concept of the system, then it will be enclosed in c-symbols (e.g., ^{c}qualities^{c}). (We shall not use this symbolism in headings and in reference remarks.)

> EXAMPLES. When we speak of ^{p}constituents of experiences^{p}, then this does not contradict the notion of ^{c}elementary experiences^{c} as unanalyzable units, for by the expression "^{p}constituents^{p}", we mean the commonly understood entities. We express through the p-symbols that we have adopted this nomenclature without wanting to express the contention that we are here concerned with actual constituents. It is after all one of the problems to be dealt with, to find out what these entities actually are, namely, how they can be constructed and how they are then to be described in constructional language.
>
> The expression "^{c}sensation qualities^{c}" or "^{c}qualities^{c}" will be used to refer to the ^{c}quality classes^{c} as soon as these classes are constructed or at least when the type of their construction has been indicated (§ 81). On the other hand, the expression "^{p}sensation qualities^{p}" or "^{p}qualities^{p}" shall mean what is commonly intended by this word. This distinction is necessary in order to be able to deal with the question of whether the constructed "qualities" are really of such a nature that they can represent the known ^{p}qualities^{p}, e.g., the ^{p}sensation qualities^{p}. We must likewise make a distinction between ^{c}time order^{c} and ^{p}time order^{p}, etc.
>
> The ^{p}elementary experiences^{p} are the known ^{p}total objects of psychology^{p}, the ^{p}processes of consciousness^{p}. The ^{c}elementary experiences^{c} are propertyless, pointlike arguments of relations. The ^{p}elementary experiences^{p} have ^{p}constituents^{p}, e.g., ^{p}sensation qualities^{p}. The ^{c}elementary experiences^{c} have ^{c}quasi constituents^{c}, e.g., ^{c}sensation qualities^{c} or ^{c}quality classes^{c} to which they belong as elements to classes.

76. *Part Identity*

In order to be able to construct the physical world, we need certain [p]constituents of elementary experiences, especially sensations with their determinations of quality and intensity, later on also spatial and temporal order which must refer back to certain characteristics of sensations which themselves do not have to be of a spatial or temporal nature in the proper sense[p].

The [p]constituents of elementary experiences[p] will have to be quasi constituents, since in our system the [c]elementary experiences[c] are indivisible units. [p]Every sensation quality, whether it is a color, a tone, a fragrance, etc.[p], will have to be a [p]common property of those elementary experiences[p] in which it occurs as a [p]constituent[p] (i.e., as a quasi constituent). This [p]common property[p] is constructionally represented as the class of the appropriate [c]elementary experiences[c] ("[c]quality class[c]"). Above, we have discussed in some detail the fact that a class is not the whole or the collection of its elements, but a property which they have in common (§ 37). This class could be constructed, for example, for every [p]sensation quality[p] through the procedure of quasi analysis on the basis of the relation of [p]agreement of two elementary experiences in such a quality[p]. Thus we consider that relation which [p]holds between two elementary experiences, *x* and *y*, if and only if in *x* there occurs an experience constituent *a* and in *y* an experience constituent *b* such that *a* and *b* agree in all characteristics,[73] namely, in quality in the narrower sense, in intensity, and in the location sign [74] which corresponds to the place in the sensory field, provided that the sense modality in question has these characteristics. Thus, two color sensations agree with one another if they agree in hue, saturation, brightness, and in location sign (i.e., in the place in the visual field); likewise, two (simple) tones, if they agree in pitch and loudness[p]. The just-discussed relation of [p]agreement of two elementary experiences in an experience constituent[p] is a kind of part identity; we call it, in brief, "[p]part identity[p]". For the logistic formulation of the constructional system, we assign to this relation the symbol "Pi", so that "*x* Pi *y*" means: [c]the elementary experiences (i.e., the elements of the constructional system) *x* and *y* are part identical[c]; and this means [p]the elementary experiences *x* and *y* are part identical[p] (in the previously indicated sense). Since one can envisage the relation of [p]part identity[p]

---

73 in allen Bestimmungsstücken
74 Lokalzeichen

as a fundamental fact of cognition, it seems reasonable to introduce the relation "Pi" as a basic relation. But we shall see later on that this is not very useful, since it can be derived from another relation which is likewise required for construction, but which itself cannot be derived from ppart identityp.

We have already seen that, from ^{p}part identity^{p}, we can either derive, through quasi analysis, the ^{p}sensation qualities^{p}, or, if these can be obtained from another basic relation, one can conversely derive ^{p}part identity^{p} from the ^{p}sensation qualities^{p}. In our construction we shall employ this second method.

Among the ^{p}sense modalities^{p} we always wish to include the ^{p}domain of the emotions^{p}.[75] This holds not only for the above explanation of ^{p}part identity^{p}, but also for the subsequent investigations. We do not wish to assert thereby (but we also do not wish to deny) that ^{p}emotions are sensations^{p}. However, we need a short expression for ^{p}the domains of constituents of experiences which are either the sense modalities or the domain of the emotions^{p}. In this context we always mean by ^{p}*sensation qualities*^{p} also the ^{p}qualities of emotions^{p} (cf. § 85).