Physical Concept Formation

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Introduction

The Task of Physics

The Task of Science

Science has the task of collecting and ordering cognitions in order to achieve an ever greater degree of mastery over reality. By "mastery over reality" we can understand two different things. To man inexplicable appearances are in themselves something uncomfortable. Through explanation and understanding of their connections man overcomes the distressful influence of these appearances even if he can alter them as little as he can the weather or the movements of the stars. From this purely intellectual control a practical control can be sought and often indeed achieved. This practical control consists in the influencing of the appearances in accordance with a conscious intention. For this it is necessary to know the mutual dependence of appearances in order to know under which conditions a particular process is to be expected. Accordingly, we can distinguish between the mere *understanding* and the *prediction* of reality. Although both can usually be found in the individual sciences, we can divide the sciences (at a particular level of development, e.g. the present) according to the preponderance of one or the other aspect into *sciences of understanding* and *sciences of law.* Today the division is approximately the following. The systematic natural sciences (e.g. physics, chemistry, biology, astronomy) are predominantly sciences of law. The historical natural sciences, that is those which describe a unique period of time (e.g. geology, the theory of the origins of organisms), which are of less importance than the systematic, are developing ever more into sciences of law. The systematic cultural sciences (e.g. theory of art, sociology, economics) also set this goal for themselves, but are further from it and in the main must satisfy themselves with mere understanding. The historical cultural sciences (e.g. political history, social history, history of art, history of religion) are virtually pure sciences of understanding; whether they have any general laws to ask after at all is disputed.

The sciences discussed above, which treat the various branches and sides of reality and therefore are called *"empirical sciences",* stand opposed to the *"formal sciences".* The formal sciences do not treat a particular domain of reality, but rather the pure (i.e. empty of reality) forms which the empirical sciences require for the working up of their objects. The formal sciences (e.g., formal logic, the theory of relations, mathematics) are therefore auxiliary sciences of the empirical sciences.

The extraction of the material of cognition precedes the ordering synthesis of cognitions, regardless of whether it is directed toward mere understanding or prediction according to general laws. To this first phase of scientific activity belongs, for example, the undertaking of experiments, weather or star observations, the gathering of historical reports and documents, statistics, etc. In the second phase the forms supplied by the formal sciences are used as frames or schemata to work up this material, to assemble it into an ordered structure. In the actual practice of scientific activity the two phases are almost always combined together. Of course every individual scientific activity receives a clear imprint through the predominance of one or the other of these components. In many sciences the workers are in the habit of separating themselves according as they practice chiefly the activity of the first or the second phase (e.g. experimental and theoretical physicists).

What is Concept Formation?

A cognition (in the sense of a scientific cognition) consists in the statement of a state of affairs, in its representation by words or other symbols (mathematical, chemical or other such symbols). Every such representation --no matter how primitive the represented fact appears --already strictly speaking belongs to the ordered synthesis, and therefore to the second phase of scientific activity. A sign is introduced or, if it is already in use, subsequently legitimated, when those conditions in the representation of states of affairs in which the sign is to be used are stated. The introduction or legitimation of the word "horse", for example, occurs when those conditions which must be in hand for us to call something a horse are stated, hence through the statement of the *characteristics* of a horse (or the definition of the word "horse"). We say of a symbol introduced or legitimized in such a way, or at least that we view as legitimizable, that it designates a *concept.* A concept-symbol is therefore a lawful symbol, whether it is defined or not. Its employment is to be *lawful;* the symbol is not to be used in an arbitrary, capricious manner, but rather in a particular, constant manner; moreover the uniformity in the mode of employment can be secured either through express definition or simply through constant habit, “linguistic usage”.

We have not hereby said *what a concept is;* but only what it is for a symbol to designate a concept. That is also the only thing that strictly speaking can be done. And that is also sufficient; because whenever there is meaningful discussion of concepts, it is always about concepts designated by symbols or concepts which are in principle so designatable; and so the discussion is fundamentally about these symbols and their laws of employment.

The formation of a concept consists of the statement of a law about the employment of a symbol (e.g. a word) in the representation of states of affairs. In ordinary life and in the first stages of science, of course, there occurs a type of concept formation where no such laws about the employment of symbols (words) are expressly stated. In these cases, however, such laws are no doubt implicitly observed or at least their observance is postulated. As soon as discussion is about a concept at all, it is a question of the unified, and therefore lawful employment of a symbol. The relation of unformulated concept formation to conscious, formulated concept formation of developed science corresponds roughly to the relation of “unwritten rules” of morality to codified law.

The Task of Physics

The task of physics is the conceptual manipulation of sensibly perceivable objects, i.e. the systematic ordering of the perceptions and the drawing of inferences from perceptions in hand to expected perceptions. Other empirical sciences (whether all of them remains undecided) also in the end relate to perceptions. Physics is distinguished in that it investigates the most general attributes of the perceivable, whereas the other sciences concern only a particular selection of these inasmuch as they relate, say, only to the processes of organisms or the connections of human life.

In physics, as in most sciences, the undermost levels of the structure are already erected in the prescientific thought of daily life. Even before there is a unified physics, perceivable things and processes are compared and their spatial, qualitative, and temporal relations asserted. In this way a beginning is made to the construction of a complete ordering of perceivable events. The work of physics consists of nothing other than the furtherance of this activity in a more regulated manner; the order is carried through more strictly, certain material resources, the apparati of physics, are provided in order to widen the domain of the material to be synthesized; furthermore also certain conceptual resources are provided in order to carry through the synthesis more thoroughly, in order to be able to treat connections of higher degree. We will consider more closely precisely these conceptual resources here. In this way we will recognize the stratified edifice under consideration in the system of physics.

I. The First Stage of Physical Concept Formation

Qualitative Stage: Perceived Things and Attributes

Things and Attributes of Things

In the portion of nature that we perceive, we recognize things of various size, figure, and position and through perception determine various attributes in these things: their color, hardness, temperature, strength, elasticity, weight, etc. Not all of these attributes are equally immediately reducible to perception. We immediately perceive the color or temperature of a body as soon as the body comes into a stimulus relation to the sense organ with whose help we recognize the attribute in question, that is here, the eyes or endings of the warm or cold nerves that lie in the skin. On the other hand, in order to apprehend the body's strength under torsion, compression, or extension, we must submit it to stress through torsion, compression, or extension in increasing degree until it breaks or tears. The following examples also belong among the attributes of this second type, for whose establishment we must first bring about a particular process: viscosity and friability, plasticity, easy fusibility, solubility (in certain liquids), pitch and timbre of the tone or type of sound of a stimulated natural frequency of the thing.

If we look closer, we notice however that no sharp difference between attributes of the first and second type exists. For an attribute of the first type also expresses nothing other than the manner of reaction of the thing to certain conditions to which it is subject; it is only that in this case the conditions in question are usually fulfilled without our assistance. So, for example, the assertion that a body is red means that it reflects red light when it is illuminated with white light; the condition of being illuminated by white light is not expressly stated because it is usually fulfilled in ordinary circumstances.

Hence every assertion about an attribute of a thing says how it reacts to certain conditions or stresses; i.e. what happens to it when it is compressed or bent in such and such a way, is deprived of its support, is illuminated, heated, its fire is extinguished, is placed in water, etc. *An attribute of a thing is a manner of reaction.* The assertion of an (enduring) attribute of a thing therefore takes place through a conditional sentence ("if ..., then ..."; "Implication"); this conditional sentence for its part speaks only of singular, directly perceived attributes.

Induction

Enduring attributes of a thing ("this body is red") are inferred from the observation of momentary attributes ("the body is now reflecting red light"). This is the first step in a series of steps that lead to ever higher levels of concept formation. At each of the higher steps we find again the characteristic relation that we have just now determined for the first step: the obtaining of a conditional relation between attributes at level n gives occasion for the formation of a concept at the n+1 level, in which a new attribute is put forward that is asserted of those things for which the conditional relation in question obtains. (Those bodies for whom the following conditional relation holds: "if the body x is illuminated with white light, then it reflects red light", are conferred the (enduring) attribute Red; the recognition of this change of level is made somewhat more difficult because language does not heed the difference of level, rather the momentary attribute as well as the enduring attribute are designated with the same word "red".

All physical propositions are, consequently, conditional propositions. And therein lies a special problem already with the first step of physical concept formation that should be mentioned here where we meet conditional propositions for the first time. The peculiarity lies of course in the fact that *all physical propositions,* precisely because they are conditional propositions, *assert more than is observed,* indeed more than could be observed, *therefore assert more than one legitimately may assert.* For the conditional sentence asserts: when such and such conditions are fulfilled, where and whenever that may be, so and so occurs. What is observed is, however, only that in some cases or in many cases, in any event in all sufficiently known cases of the specific conditions the appropriate has occurred; and all cases can never lie before us as observation material since an unknown future always lies before us.

The method of inferring the general validity of a certain conditional relation from single or repeated observation is called "induction". It follows from our considerations that *induction has no logically strict justification.* It can, however, cite for its legitimation its experiential confirmation. The axiom which forms the basis of the method of induction, "Identical conditions yield identical occurrences" is, of course, protected against an experiential refutation thanks to the fact that the identical conditions never occur twice. For if ever all perceived conditions in two cases are identical but the same occurrence does not follow, it always remains possible to claim that conditions that are not directly perceived but merely inferred are unidentical. For this purpose under such circumstances new conditions which until now were not brought into account must be taken into consideration. The reverse side of the possibility of saving the axiom under all circumstances consists then in that as a result of the inexhaustability of conditions all physical propositions (as well as all the inductive propositions of other sciences) may claim only probability, not absolute validity.

Substances and Attributes of Substances

We have seen how the (enduring) attributes of things are derived by induction from individual perceptions. From these still further attributes can be inferred, e.g. the specific weight of a thing from its size and its weight, the substance of a thing, say, from its color, specific weight, elasticity, or what have you.

Substance is a particularly important concept for further concept formation. It is appears first as an attribute of certain things. For example, "Iron" initially means the attribute of being iron that certain things have, i.e. having such and such attributes, the "characteristics" of iron. This thing-attribute is then reified, i.e. conceived as a self-subsistent object, to which in turn attributes can be assigned; attributes, however, of higher level than that of the thing-attributes. That is, one assigns to this new object "Iron" all those attributes, which are common to all those things to which the attributes belong on account of that characteristic of ironhood. These attributes assigned to iron, which are originally thing attributes, become thereby *substance attributes.* Since all those bodies that on account of their specific weight, color, etc. are called "iron" are hard and easily rust, we therefore assign hardness and propensity to rust to iron as substance attributes. Most of the attributes dealt with in physics are substance attributes; especially at this first, qualitative stage.

The occasion for formation of the concept *"substance"* lies in state of affairs that bodies that agree in certain attributes also tend to agree in others. The former attributes then are considered as characteristics of the substance, the later as inferable substance attributes. The choice of the characteristics from the substance attributes of a substance is, by the way, possible in various ways. The formation of the concept substance on the basis of certain agreements among things follows from an axiom, which we will discuss in its generality only later (at the second, quantitative level: the axiom of concept formation on the basis of a transitive, symmetric relation).

Conditional Relations

We have seen that physical propositions are propositions about conditional relations, i.e. propositions of the form: if a, then b. The establishment of such conditional relations belongs to the most important goals of all other experiential domains as well, already from the most primitive cognition level on. The possibility of inferring unperceived attributes, of predicting expected perceptions, from the perceived rests on the knowledge of such conditional relations. For example, we expect a quite particular taste and smell from most of the things that belong to our diet as soon as we see them. The probability that such expectation is disappointed is smaller the more singular the total figure and constructional structure of the body are: an apple or a piece of bread won't disappoint our expectation as easily as a certain crystalline mass that we take to be sugar or a liquid that we consider to be wine.

The conditional relations that are stated in physical propositions are of various kinds. First, the simultaneous attributes of a thing can depend on one another. For example, if a body consists of table salt (i.e., roughly speaking, if it white and has the typical taste of table salt), then it has a flame with the typical yellow sodium color. Second, the processes on a body are conditioned by the processes in its neighboring environment. The conditional relations of the second kind are particularly important because they have given occasion to a peculiar concept formation; they should therefore be considered more closely.

The "Causal Relation"

The totality of the attributes of a body (or part of a body or system of bodies) at a particular time forms the *state* of the body at this time. By a *process* on a body during a period of time we understand the series of states of the body during this period.

It is evident already in prescientific experience that the processes on a body, that is, the way in which its state changes, is conditioned in a certain way by the state of its neighborhood. This is because, as we saw earlier, the most important enduring attributes of things and the substance attributes are nothing other than methods of reaction to bodies. Hence the presence of such attributes signifies that a body undergoes a certain change of state when its neighborhood changes. Since all known influence by man on the external world rests on the conditional relations of the second kind, it is understandable that he early became aware of these conditional relations and conceived them in a particular way. Thus, he noticed, for example, that a piece of ice melts if its environment warms up. He didn't satisfy himself then with establishing the conditional relation, but rather, as accords with the mythological thought of primitive man in general, through empathy imagined himself in the place of the conditioning process of the environment, e.g. fire, and believed therefore that the same relation exists between this process and the conditioned process as between an action of the will and the external process that he intentionally effects. Therefore, he called the melting of the ice the *"effect"* of the fire. Although, as regards its character, this mytho-poetic conception belongs to a time that lies more than a millennium behind us, and although it has been expressly struggled against and refuted by philosophers already centuries ago, it has still not disappeared from contemporary thought; not even among many natural scientists, who believe that physics must establish not only the conditions of processes but also the “causes” that “produce” the processes.

The discussed conditional relation of the second kind has no right to receive a special place among the conditional relations between physical states of affairs and to be conceived in the described way as the “causal relation”. If the state changes in the neighborhood of a body are known, then it's own state changes are determined by its initial state; it is just as determined however by its end state. No one will advocate the conception that the end state produces the state changes of the body; the conception that the initial state or the state changes in the neighborhood or both together produce the state changes of the body has exactly as much or as little claim to correctness.

The same conclusion follows from the comparison with the conditional relation between simultaneous thing attributes. There is no fundamental difference between a proposition of the kind “Gold is yellow”, “Ice is cold” and the proposition “bodies expand when heated”. The former propositions are, as we previously discussed, also conditional propositions. And as little as the fact that a certain body is made of ice, i.e. is made of water and is in solid state, *produces* the coldness of the body, but rather is only a (generally) sufficient condition for it, just as little can the heating of a body *produce* its expansion, but rather is also only a (generally) sufficient condition for this.

II. The Second Stage of Physical Concept Formation

Quantitative Stage: Physical. Magnitudes

Counting and Measuring

Already in everyday life it is frequently necessary to describe things or processes not through the mere mention of attributes (“qualitative” statements) but rather to append number statements (“quantitative” statements), if an exact picture is to be given through the description or if the desired result depends upon an exact observance of the conditions of a procedural rule (e.g. a cooking recipe). The use of quantitative statements, indeed ultimately the reduction of *all* qualitative statements to quantitative ones, proves itself to be necessary to a still higher degree in the scientific, that is, methodical investigation of natural processes. We will consider the grounds of this necessity later. First we must give a detailed answer to the question: How is the production of quantitative statements about reality possible and which prerequisites exist for them?

The production of quantitative statements in any field happens either through *counting* or through *measurement.* The original method is *counting.* This is, however, only usable in certain cases, namely if a collection of well separated individual things or processes whose number is to be established is in question; obviously only whole numbers can appear as the result in such cases. As opposed to this, in true *measurement* fractional numbers also appear; measurement must be used if a collection that does not consist of separated elements but rather is either continuous or is treated as continuous is in question. Stones or pulse beats can be counted; length, temperature, temporal duration must be measured.

The fractional numbers that appear as the result of direct measurement are always rational numbers; irrational numbers can also appear, not as directly measured but as inferred from directly measured numbers. For example, the diagonal of a square whose side is measured as a cm is established as $a\sqrt{2}$ cm.

All counting ultimately goes back to the counting of a temporal sequence of experiences. Originally only those processes can be counted that are experienced sequentially and that have a certain similarity with one another. The striking of a clock is counted in that the members of a known sequence, namely the words “one, two,...” are coordinated one after another with the individual strikings. Subsisting things are also”counted in this way; here the sequence of processes arises through touching each thing in turn with one's finger or glancing or merely imagining each of them. The result of the counting of this temporal sequence is then transferred to the collection of things itself.

All measurement fundamentally goes back to counting. Counting is the single original way of producing quantitative statements. The measurement of any magnitude is a type of counting that is undertaken on the basis of certain conventions. These conventions are quite different for the different types of magnitude; they follow however universal basic rules. These conventions form the foundation of quantitative concept formation in physics. We will therefore look at them in detail, not only according to their general rules but also with various physical magnitudes as examples. We will in this way see that it is a question of conventions about some type of identity or agreement and indeed chiefly a question of spatial identity ("Congruence"). *All measurement* of any magnitude *reduces in physics to measurement of spatial length* (in special cases also immediately to counting).

The Analysis of Temperature Measurement

Physical measurement signifies the coordination of numbers to any physical objects (things, attributes, phases of a process and the like) whatsoever from a certain domain. This coordination, if it is to have a sense at all, must not occur arbitrarily, but rather must conform to the qualitative behavior of the objects. In what way this adaptation of the number-coordination to the qualitative behavior has to occur, we will first investigate through the example of the concept of temperature and will thereafter form into general rules, which we will rediscover with other types of magnitude.

The formation of the concept "temperature" is occasioned through experiences of various types. First, we notice that various bodies that come into contact with our skin evoke various sensations of the warm-cold sense; as does the same body at different times. On the basis of these sensations we can determine when a body has warmed up and when it has cooled off. Imagine now that the concept of temperature were still unknown to us. We decide on the basis of the mentioned experiences to assign in an appropriate manner a particular number, which we want to call "temperature", to each body (if necessary also to the individual bodily points) at each instant such that the thermal behavior of the bodies in question is comprehended in the simplest possible way through the assigned numbers and also such that the general laws of this behavior assume the simplest possible form.

For one and the same body we want to stipulate roughly that the sequence of numbers assigned to it, its "temperature", should rise with warming and fall with cooling. (The opposite stipulation would also, of course, be possible and practicable.) Now comes the more difficult problem of the assignment of numbers to *different* bodies. First we could try to assign the same temperature to two bodies if they evoke the same sensations of warmth, and to assign to one body a higher temperature than to another body, if it evokes a stronger sensation of warmth (or a weaker sensation of cold). This method of assignment would however prove to be unsuitable in view of the fact of "thermal equalization". That is, if two bodies are brought into contact with one another and a heat alteration occurs thereby (without chemical reactions) in both, it is always in the opposite direction: in the one there is a warming up and in the other there is a cooling off; if the change is only perceptible in the one, then the other is assigned a change in the opposite direction of imperceptible magnitude (with the help of later natural laws that come forth with the concept of specific heat). And of course the cooling off occurs in the warmer body and the warming up in the cooler. This law would sometimes suffer exceptions in the attempted convention for the concept of temperature, in which the perceived warmer body would warm up and the perceived cooler body would cool off. (A piece of wood at a temperature of 50° seems cooler to sensation than a piece of iron at a temperature of 45°, it however cools off with contact with this iron.)

In order to maintain the important law of thermal equalization, we must therefore hit upon the conventions for the assignment of temperature numbers in another way. We proceed in view of this in such a way that we take the behavior [of bodies] according to thermal equalization directly as the basis of the assignment. First we stipulate that two bodies that suffer no thermal change with mutual contact (we say in this case: the bodies are in "thermal equilibrium"), should receive the same number assignment. That it is possible to carry out this convention depends, however, on still another fact of experience. If two bodies, a and b, behave in the given way towards one another, and also b and c, then so do a and c always; in short: thermal equilibrium is empirically a *"transitive"* relation. If this fact of experience were not present, then we could not carry out the given method of assignment univocally, because then on the one hand we would assign bodies a and c the same number, on the other hand we would not be permitted to do so because they are not in thermal equilibrium. Moreover, the concept of thermal equilibrium has, as we can realize from its definition without having to rely on experience, the attribute of "symmetry"; i.e.:

if it obtains between a and b, then it also obtains between b and a. As one can easily see, this condition must always also obtain for a concept, which is to be used as the basis of the assignment of equal numbers (of any type of magnitude).

Likewise in connection with the behavior [of bodies] in thermal equalization we can now affect the convention for the assignment of unequal numbers: if a body in contact with another undergoes a warming up (cooling off), it is to be assigned a lower (higher) number than that body. Also here the convention can be carried out only because the relation between two bodies that it is based upon has certain formal attributes. The relation, which expresses itself when a body in contact with another warms up, is first "*asymmetric*", i.e. if it obtains between a and b, then it does not obtain between b and a. That follows from the mentioned fact of experience that the reactions of two bodies in thermal equalization never occur in the same direction. Second, the relation is empirically transitive: if it obtains between a and b and also between b and c, then it also obtains between a and c. If the conditions of asymmetry and transitivity were not fulfilled, then, as one easily recognizes, the number assigned to a body according to the affected convention would sometimes have to be larger as well as smaller than the number assigned another body; the convention would not be able to be carried out without contradiction. Third, the relation used here as a basis has a certain sequential quality: according to its definition it always obtains in one direction or the other between any two bodies that are not in the relation of thermal equilibrium. If this condition were not fulfilled, we would sometimes not be permitted to assign to two bodies the same number, but also neither of the two could be assigned a higher number than the other. In this case also the convention could not be carried out. (We would then have to assign more complicated types of magnitude than the values of a one dimensional, simple sequence forming number sequence.)

Were two physicists who were separated from one another to undertake a coordination of temperature numbers according to the affected conventions, the statements of the two would agree in whether one of two bodies receives the same, a higher or a lower number than the other. We say: they agree in the "*topological*" temperature assertions. Beyond this we want to achieve identity in the numerical assignments the two now make for the temperature of each body; we say: their temperature assertions should not agree only topologically but also "*metrically*". Only when this is the case is the concept of temperature univocally determined. Because only then does the statement of the temperature of a body at a particular time have a univocal sense. In order to reach this agreement we must affect still further conventions, the *"metrical conventions".* Let us assume roughly that one of the physicists assigns to the bodies those temperature numbers that we are accustomed to assigning them according to the usual temperature scale; the second physicist however is inclined to assign the numbers 0,1,2,3,4,5 to those bodies which according to our thermometer, and therefore also to the assignment of the first physicist, have the temperatures -20,0,10,15,18,19. In order to bring the assignment of the second physicist into agreement with that of the first, we must *first* assure that the *"scale form"* becomes the same, i.e. that two different temperature differences which are the same according to one assignment are always also the same according to the other. *Second,* we must assure the agreement of the *zero point,* and *third* the agreement of the *unit.* The first demand is fulfilled if, for example, the numbers 0,1,2,3,4,5 of the second physicist's temperature scale stand on those places on which the numbers -20,-10,0,10,20,30 stand on that of the first physicist; further the second is fulfilled if they stand in the positions 0,5,10,15,20,25; finally the third demand is fulfilled if they stand at the positions 0,1,2,3,4,5. With this the agreement is then reached. If the *conventions for equality of differences, the position of the zero point on the scale, and the magnitude of the unit* are met, then and only then is the magnitude univocally determined. Only then is it defined as a metrical magnitude; only then do the statements of the value of a magnitude have a determinate sense. We will see later how these conventions are to be met for temperature and other physical magnitudes.

In what does the definition of a physical magnitude consist?

The definition of a physical magnitude consists in the determination of the rules according to which the assignment of the values of the magnitude to objects is to occur. It has occasionally been thought that a physical magnitude (e.g. time) has in and of itself a sense, without regard to how it is be measured; the question of the method of measurement is on this view a second question. Against this it must be emphasized that the sense of every physical magnitude consists in the circumstance that certain physical objects are to be assigned certain numbers. As long as it is not determined how this assignment is to occur, the magnitude itself is not determined and statements about it are senseless. Measurement is, however, nothing more than that assignment. Only the more exact details of measurement in individual cases, the decision from among several different in principle possibilities according to their technical usefulness, etc. form secondary questions.

As we have seen in the example of temperature, five determinations belong to *the complete definition of a physical magnitude,* two topological and three metrical: la) determination of magnitude equality, 1b) determination of sequence form and positive direction, 2a) stipulation of segment identity and thereby the scale form, 2b) of the null point, 2c) of the unit magnitude. The introduction of a physical magnitude may of course occur at first quite provisionally, i.e. with the provision for later, more practical determination of certain points. So long as one of the points is, however, not determined at all (also not even tacitly determined, as is frequently the case), the magnitude in question can not be spoken of sensibly. At minimum, the assertions about the magnitude that directly concern the still missing point in any way are still without sense. That was, for example, for a long time not noticed with regard to one of the most fundamental of all physical magnitudes, time; until the statement of the theory of relativity any stipulation in accordance with la) was missing, that is, the definition of simultaneity. Many assertions in which time appears have a sense only after some filling of this gap.

Many physical magnitudes are derived from others; for such magnitudes the conventions for the five points follow from the definition and therefore do not need to be explicitly arrived at.

The five determinations of a physical magnitude

We now want to formulate into general rules, whose use will then be shown for still other physical magnitudes, that which we recognized in the example of the concept of temperature.

1. The topological definition of a physical magnitude

The prerequisite and occasion for the introduction of a type of magnitude is an empirical finding of the type that among the objects (bodies, processes) of a domain two relations obtain: one transitive and symmetric and one transitive and asymmetric. The first relation then gives the occasion for the formation of a particular concept of identity, the second for the formation of the concept of a particular type of magnitude, and indeed chiefly (that is if that relation has a certain sequentiality) a one dimensional ("scalar") magnitude. It is then determined that the assignment of numbers to the objects of the domain is to occur such that:

a) those objects between which the transitive, symmetric relation obtains are assigned the same number,

b) an object that stands in the transitive, asymmetric relation to another is assigned a smaller number than the other.

2. The metrical definition of the magnitude via three conventions

So that the magnitude is not only topologically determined, i.e. with respect to identity and the direction of difference, but also metrically, i.e. with respect to individual values, it is necessary to choose three conventions. The first is essential since it has considerable influence on the form of natural laws; the two others are inessential.

Through change of the first convention there appears, in the mathematical expression of a natural law, in the place of the magnitude in question some one-one, monotonic function of the magnitude; through change in the second or third conventions there appears only an additive constant or, respectively, a constant factor.

a) there is a *scale* form to choose, i.e. a convention to make for when two scale segments, and hence two magnitude differences of the magnitude in question, are to count as equal.

b) there is a *null point of the scale* to choose, i.e. a convention to make about when an object is to be assigned the magnitude value zero.

c) there is a *unit* to choose, i.e. a convention to make about when the magnitude value one is to be assigned to an object.

About a. The identity of scale segments or magnitude differences is, indeed, to be distinguished from the identity of the magnitude values themselves; these values and only these are already fixed through the topological determination la). The establishment of the scale form is chiefly chosen from the point of view that the laws of nature (especially the energetic laws) in which the magnitudes in question appear should take on a simplest possible form.

About b. In many cases the decision about the choice of a null point is simple and obvious inasmuch as a certain magnitude value forces itself on us as the "most natural", i.e. the simplest scale origin. In other cases a magnitude value is chosen that allows itself to be realized always and exactly in the same way via a simply produced process.

About c. For the establishment of the unit also an object (a reproducible process or even a particular individual body) must be chosen as the normal object. Here it is usually not the case that a particular object forces itself on us as the simplest for this choice. Hence a certain easily obtainable material or an individual body that is exposed to the fewest possible disturbing influences is chosen. A certain attribute or process in this material or body yields then the unit of magnitude. (Frequently a certain fraction or multiple of the normal magnitude is taken as the unit for practical reasons.)

Examples of the concept formation of physical magnitudes

We now want to look at some examples of how the necessary determinations and conventions are affected for physical magnitudes of various types.

A. Length

As was already mentioned above, length measurement is the most fundamental measurement in physics, to which the measurement of many other magnitudes is reduced. Its conceptual determination will therefore be especially exactly investigated.

The concept of length measure is much older than science. It is so familiar to us that we usually do not make ourselves aware at all of those experiences and chosen conventions on which its concept formation rests. For this reason we carried out the first investigation not with it but with the concept of temperature, with which precisely because of the greater technical difficulties in measurement the necessary prerequisites of measurement are more clearly expressed. We will see that the prescientific determination of the concept of length follows fundamentally the same rules, which determine concept formation of the types of magnitudes that are first introduced in scientific practice.

Let us assume that the concept of length measure is still not known to us and that we want to see how it is formed on the basis of certain experiences according to the established basic principles.

The empirical fact that lies at the basis of all length measurement is that of the availability of *"rigid bodies".* Bodies that consist of certain materials, e.g. metal, stone, wood, have the peculiarity that edges that are *"congruent",* i.e. that can be brought into coincidence with one another, always remain congruent. (We speak of edges for the sake of greater intuitiveness and simplicity; for more precision we would have to speak of pairs of points on the bodies.) Bodies that exhibit such behavior we call "rigid with respect to one another". Experience shows that an iron rod and a piece of wax or rubber are not rigid with respect to one another; furthermore neither are two pieces of wax. However, all bodies that consist of, e.g. metal, stone, or wood are (approximately) rigid with respect to one another. Since these bodies form in our experience the only collection of bodies that are rigid with respect to one another, we will call them simply "the rigid bodies". From the described behavior of rigid bodies it follows that the congruence (coincidence) of the edges of rigid bodies is a transitive relation: if it obtains between a and b and also between b and c, then it obtains between a and c as well; at least this holds in so far as it is possible at all to check the edges of a and c with regard to congruence. Since transitivity is met with universally in the domain of the testable, we postulate transitivity also where it is not testable, because the edges in question cannot be brought close to one another: thus we call, e.g. two edges of the same stone as congruent (although they surely do not allow of being brought into coincidence) if each of them shows itself to be congruent with one and the same other edge (e.g. of a certain other stone). The enlargement of the extension of the basic relation through the assumption of transitivity in the untestable region represented here is, by the way, frequently undertaken in the conceptual formation of other types of magnitude; and equally so also for the second, asymmetric relation. That congruence is symmetric is shown by its definition.

The second relation upon which the conceptual formation of length rests is that of *"part coincidence"*: the edge, a, of a body can be brought into coincidence with a proper part of the edge, b, of another body, i.e. the two bodies can be placed against one another such that one endpoint of a coincides with an endpoint of b but the other endpoint of a coincides with an inner point of b. From this definition and that of rigid body it follows that the relation of part coincidence between two edges of rigid bodies continues to obtain if it once does. This relation is asymmetric and transitive.

We turn now to the above established rules for the five determinations of a type of magnitude.

la) The relation of congruence between edges of rigid bodies is transitive and symmetric. We assign therefore to such edges the same number as their "length". Momentary, not lasting, congruence is alone ascertainable for edges of other bodies; we assign to such an edge the same length as one of the edges of a rigid body with which it is momentarily congruent; because the relation of momentary congruence is also transitive and symmetric.

1b) The relation of part coincidence between edges of rigid bodies (and momentary part coincidence if one or two other bodies are in question) is transitive and asymmetric. Moreover it has the sequential quality described above. This bodily behavior occasions us in accordance with rule 1b) to introduce a one dimensional magnitude. If two edges stand (continuously or momentarily) in this relation, we assign (continuously or for the point of time in question) to the first a smaller length number than the second.

2a) We come now to the *metrical determinations.* The essential metrical convention, namely the *scale form,* is in this case not difficult to choose. A particular type of this convention has such a preponderance of simplicity and hence usefulness that it has been the basis of all length measurement from the earliest time and at the first glance indeed seems to us to be the only possible choice, so that we believe that there is no freedom of choice at all. We know, however, from the analogy with temperature measurement, the discussion of which for this reason has been given above, that also here with length measurement there is freedom of choice with regard to scale form. It is not a question of the choice of the substance of the standard measure; this choice has already been decided in favor of the substance of the "rigid bodies"; that appertains to rule la).

We should remind ourselves that a new temperature scale could be introduced in the place of the usual one through the another apportionment of numbers. We can change the scale on a normal meter in just this way, in which we put in place of the number designations arbitrary others, which however must agree in their topological order with the original (i.e. increasing numbers in the old scale must correspond to increasing numbers in the new one). We can, e.g. put the numbers 0,1,2,3…in those places where the numbers 0,20,30,35,…are originally; we can even place the numbers -2,-1,0,1,…in those places. From the standpoint of the old scale the new one is, of course, "false"; if we take the new scale as the original, then there is no "true" or "false" in judging it, but merely "useful" or "not useful". The three scales agree topologically: if any two segments are measured as equal according to the usual scale of the meter stick, they are also equal according to both of the new scales; if a segment is longer than another segment according to one scale, it is also longer according to each of the new scales. The scales give different answers only to the questions, *what* length a segment has and *how much longer* one segment is than another. The establishment of the scale form now occurs in accordance with rule 2a) through a convention for the identity of segment differences. The usual scale form of length measurement from time immemorial results if we affect this convention in the following way: two segment differences are to be regarded as equal if they have the same length when taken as segments. As we recognize from the other given scale forms, this decision is not necessary; a certain degree of triviality is not to be denied it, since it is by far the simplest. A particular objection to the freedom of choice of this convention will be discussed in the next section (on the Additive Theorem). That the decision can be made in the given form in the case of length rests on the circumstance that segment differences can be taken as segments themselves and can be measured. The corresponding does not hold for all types of magnitude; temperature differences, for example, cannot be taken as temperatures.

The well-known, age old procedure of length measurement via the repeated laying of a unit measure after itself follows from the given convention. The division of the measure stick occurs correspondingly through the circumstance that another smaller rigid edge is sought that can be placed along the unit edge a particular number of times (e.g. ten in the decimal-metrical system).

2b) Convention for the null point of the length scale: We assign the length zero to the distance between two points that touch one another or spatially coincide. This convention also at first seems self evident or necessary, but it also rests on a free decision; of course the nature of the case suggests precisely this decision to us quite plainly. We acknowledge the logical possibility of another choice with the third example of a scale that begins with -2. As previously with the question of the scale form, so here also one will object to a scale with a beginning number different from zero that with such a scale one also coordinates numbers with intervals between points, but absolutely does not measure that which we mean by the word "length" by these means. This objection is right inasmuch as the scale form and beginning number are fixed, if a quite particular meaning of the word "length" is assumed. Despite this the fact remains that at a certain point in its systematic construction physics must affect a decision about the scale form and null point for this magnitude as much as for every other magnitude type to be introduced. That in this case prescientific life has already affected a decision can of course simplify the scientific decision, or even allow it to occur unconsciously. As soon as the validity of the freedom of choice is recognized for this decision also, however, so at least is the task of consciously checking and expressly recognizing (or discarding) the prescientific decision.

2c) For the *choice of the unit of length* there is no naturally self evident decision. A particular individual body (terrestrial body) is established or expressly produced (the standard meter in Paris) on which a certain segment is the unit of length or a certain multiple thereof. In order to determine the unit of length in a way independent upon a particular individual body, one can fix it in relation to a particular length that is represented through an always repeatable process; of all the processes that in principle come into consideration for this only the optical wave lengths fulfill the practical condition of being measurable, i.e. comparable with other segments, with sufficiently high precision. For example, the unit of length is determined through a relation to an always and everywhere repeatable process through the assertion that the wave length of a particular spectral line (D1) amounts to 0.0005896156 millimeters.

As we see, the concept formation of length has its empirical foundation in the concept of the rigid body. A difficulty arises now through the fact that real bodies never precisely conform with the concept of the rigid body. The edges of two iron rods, which are once brought into coincidence, may not at another time be able to be brought exactly into coincidence. Of course, this occurs under certain conditions that we designate as thermal expansion, elastic stress, etc. We are lead in this way to not always assign the same length to the edge of an (iron, for example) body, but rather a length that stands in a particular functional dependence to its state. We proceed in the following way. We determine first with an iron measuring rod, which itself is not warmed, the law of the expansion of other iron rods with warming. We find that the dependence of the expansion on the warming is the same for the different iron rods. If we now use our iron measuring rod in cases where it itself has another temperature, we do not fix its length as equal to the original l_o, but rather that which results from the supposition that it follows the same law of expansion as the other iron rods: $l = l_o(1+ \alpha t)$, where α designates the substance dependent coefficient of thermal expansion and t the increase in temperature. To what extent no circularity lies in this reference to other magnitudes, which for their part are first definable with the help of length, in the definition of length is to be discussed later with the concept of temperature. With regard to the greatest possible simplicity of the expression of the dependency function the choice of the substance suitable for the definition of length can be narrowed: e.g., with wood a corrective term with respect to humidity must be inserted, but not with iron.

<u>The choice between Euclidean and non-Euclidean geometry</u>. If the concept of length is determined, then the question of the <u>structure of the space</u> of our reality is an empirical question. The answer on the basis of previous measurements reads that space is either Euclidean or it diverges only very little from being Euclidean.

If the propositions of the General Theory of Relativity about spatial relations in gravitational fields are correct -- and the observations of the behavior of light rays passing near the sun seem to satisfy these propositions --, there are nonetheless still two possibilities for the formation of the concept of length. Either we insert the influence of the gravitational field on the measuring rod as a corrective factor in the definition of length, as we do with the corrective factors for thermal expansion, etc.; in this case space (according to the current state of physical knowledge) has a Euclidean structure throughout, even in gravitational fields. Or we do not admit this corrective factor in the definition, rather we conceive the influence of gravitation on the measuring rod as an attribute of space, not as a physical force; then space within gravitational fields is non-Euclidean. The motivation for not conceiving the influence of the gravitational field on the measuring rod as a physical force, the thermal expansion however as such, one can show that the gravitational contraction occurs in all substances in a numerically identical fashion, thermal expansion on the other hand depends upon substance. The first procedure, which leads to a Euclidean spatial structure, has in this way the advantage of a simpler geometry of the world; with the second procedure geometry is indeed not so simple, in return for this however the laws of nature acquire a very much greater degree of simplicity. Physics has not yet decided conclusively in favor of one of the two conceptual definitions of length.

The measurement of the all other spatial magnitudes: angles, areas, volumes, is easily reducible to length measurement. This is a purely geometrical concern into which we need
not go here.

On the Addition Theorem and the Summability of a Magnitude Type

Against the previous assertion that the usual scale form of the length scale is only the simplest, but not the only possible one, one could object that with the other scale forms (e.g. the two previously mentioned, irregular forms) a segment, consisting of two segments of length a and b, would not always have the length a+b. If one desires this from the length scale, there results, of course, univocally one particular scale form, that is, the usual one. It is, however, not as necessary to claim this simple formula, c=a+b, as the *"Addition Theorem"* as it may appear at first. By an addition theorem with respect to a magnitude type we understand a convention for the dependency of the magnitude of a composite structure upon the magnitudes of the partial structures. Of course, with the magnitude type, length, we have the happy possibility of assisting that addition theorem of simplest form to obtain inasmuch as we form the definition of length, more exactly: the stipulation of the scale form for length, in the appropriate way, as we have done above. If, however, a magnitude type is already defined, i.e. is determined as to its scale form, then the question as to whether an addition theorem of the given simple form or one of more complicated form obtains for it can only be answered through experience. The question emerges at all only with those magnitude types whose magnitude differences can in some way be conceived of and measured as magnitudes themselves. With temperature, for example and as mentioned, that is not the case. On the other hand, it applies, for example, to velocity, more exactly: to the relative velocity of one body to another.

In order not to deal with vectorial but only one dimensional magnitudes, we will consider the case of a one dimensional trajectory, that is a straight line. If here a body B has the relative velocity, v_1, with respect to body A and the body C the velocity, v_2, with respect to A, we can conceive of the relative velocity, d, of C with respect to B as representative of the difference of these magnitudes. According to the earlier, usual (even in physics) conception this magnitude difference is measured through the arithmetical difference of the two magnitudes v, and v_2: $d = v_2 - v_1$. That accords with an addition theorem of the simplest form: $v_2 = v_1 + d$. The theory of relativity (indeed the so-called special theory which is scarcely debated any more) has taught us however, that this theorem is indeed valid with a degree of approximation that suffices for all practical purposes, but not with absolute precision. In its place appears rather the (Einsteinian) addition theorem:

$v_2 = \dfrac{v_1 + d}{1 + \dfrac{v_1 d}{c^2}}$ (c designates the velocity of light 300,000 km/s.).

Against the addition theorem for (relative) velocity that deviates from the simplest form it is initially often objected that it is nonsensical because it stands in a contradiction to the clear concepts of space, time, and velocity. The form of the addition theorem for any magnitude type is not however deducible from concepts, rather it must be determined through experience. That has of course only a theoretical significance for length measurement, since length, as most basic magnitude, can be defined so that the simplest addition theorem holds. For velocity, on the other hand, practical consequences result. That is due to the fact that velocity is a magnitude that is derived from length and time and whose scale form after definition of the form for length and time cannot be freely chosen anymore.

It follows from these considerations that we have to distinguish among three cases in regard to the establishment of the scale form --or, what signifies the same thing, the establishment of the identity of magnitude differences. The magnitude type can be so constituted that an addition theorem is possible, or not. The first is the case if a difference of magnitude for this magnitude can be conceived of and measured as a magnitude itself, i.e. if there is a process which one can conceive with unique sense as the composition of two magnitudes of the type in question; or (what amounts to the same thing) if there is a process which one can conceive of as the division of a magnitude. Such a magnitude we will call *"summable"* (or "composable" or "divisible"). We can compose two lengths a and b, when we lay two objects (edges of bodies) that have lengths a and b end to end; we can compose two (relative) velocities v_i and d, when we give body B the velocity v_l relative to A and the body C the velocity d relative to B. Two temperatures, on the other hand, we cannot compose; if the bodies A, B have the respective temperatures t_1, t_2, then we do not know how to determine any process on them such that a temperature appears therein that we may with unitary sense view as the composition of t_1 and t_2. Length and velocity are therefore summable magnitude types, but not so temperature.

For every magnitude, accordingly, there applies one of the three following possibilities.

Case 1. The magnitude type belongs to the summable; an addition theorem holds therefore for it; and this has indeed the simplest form: c=a+b. This can come to pass either through our choosing the definition of the magnitude type in question, more exactly: the convention concerning the scale form, with this goal in mind (as with length) or through the fact that certain other magnitudes produce this condition empirically (as with velocity according to the earlier physics). A magnitude type with such an addition theorem we will call *"additive"*. (Examples: length, area, volume, angle measure, time, velocity, [according to the previous conception], weight, mass, electrical charge, electrical potential, quantity of heat, energy.)

Case 2. The magnitude type in question is *summable,* but not *additive.* For it obtains therefore an addition theorem of another, less simple form. (Examples: velocity [according to relativity theory], the sine of an angle [and the other trigonometric functions]).

Case 3. The magnitude type in question is not *summable;* there is no composition of magnitudes and hence no addition theorem. (Examples: temperature, wave length.)

Weight is an important example that the *additivity* of a magnitude type is *not self-evident*; before this was realized, the laws of leverage were believed to be a priori, i.e. to be able to be shown to be necessities of thought, whereas they are in reality empirical theorems. This error was first discovered only very late (Ernst Mach).

B. Temperature

Since have used the concept of temperature for the derivation of the general rules of magnitude determination, a brief discussion suffices here.

la) Temperature identity is defined on the basis of the transitive, symmetric relation of thermal equilibrium.

lb) The concept "higher temperature" is defined on the basis of the transitive, asymmetric relation that exhibits itself through the processes of the thermal equilibration.

2a) The scale form can be defined such that two temperature differences are set as equal if mercury experiences the same increase in volume in both of the corresponding heatings. The scale of our ordinary mercury thermometer possesses this scale form. We see that this determination already presupposes length measurement, which is of course the most original measurement. Here, however, we seem to have argued in a vicious circle, in that we have made reference to temperature in the determinations of length measurement, namely in the correction due to the thermal expansion of the measuring rod. This appearance of a circle disappears if we take the different levels of precision into consideration. A first, rough length determination is possible without thermal correction. Only a more precise length measurement must take temperature into account. For this, however, temperature still does not have to be determined with the more precise length measure, rather only with the first, rough measure; because only a small change in the result of the length measurement depends on it. With the help of the more exact length measurement the precision of the temperature determination can then in turn be raised a level and so on. (Another possible escape from the circle consists in the fact that first only the *topological* determination of the concept of temperature is laid down: la and lb; for this the concept of length is not yet presupposed. The determination of length can then on based on measurement with a measuring rod of "unchanged temperature", since the concept of "unchanged temperature" presupposes only the topological, not the metrical, determination of the concept of temperature. Finally, the *metrical* determination of temperature can be undertaken with the help of the concept of length.)

If one chooses another standard substance in the place of mercury, one obtains another scale form; e.g. the alcohol scale, the hydrogen scale, among others. In physics one has finally introduced a scale form (which does not differ importantly from the given scales) that is not precisely described through the behavior of any actual material. For its description one must introduce corrections with regard to the actual materials much more than even with length. This "thermodynamical" temperature scale has the advantage that with the choice of it the laws of thermodynamics, which have acquired a fundamental significance in the more modern physics, take on the simplest form.

2b) Because melting ice always has the same temperature and this process is easily repeatable under identical conditions, it has been chosen as the standard process for the stipulation of the null point. For the scientific scale one takes instead of this, another temperature, the so-called "absolute zero" (-273°C). This temperature cannot of course be realized at all, its choice as null point has however the advantage that thereby the laws of thermodynamics take on a particularly simple form.

2c) The normal segment is defined through easily repeatable processes: the temperature of melting ice, the temperature of boiling water at standard air pressure. As unit (Celsius degree) one takes for practical reasons the hundredth part of this segment. The apparent circle that lies in the reference of the definition to air pressure, whose measurement is only later defined, dissolves through consideration of the levels of precision just as before.

C. Time

la. The first convention must decide when two events are be assigned with the same time value. It is a matter therefore of the definition of *simultaneity.* On the *general* concept of simultaneity of two events, which can be arbitrarily far from one another, physics has hit upon no unified and final determination. This concept determination offers particular difficulties. Relativity theory first noticed that these difficulties are present and that it is a task at all to define a general concept of simultaneity, and has resolved this problem in a certain way with reference to light signals.

Because of these difficulties we want to limit ourselves here to the consideration of processes that are spatially near to one another, say the processes in a room. Two events are then called (under this limitation) simultaneous if they are perceived conjointly. (This definition does not contain a circle; "to be conjointly perceived" does not designate physical simultaneity, but rather experiential simultaneity, which is presupposed by physics as an ascertainable basic relation.) If time measurement is introduced through the necessary conventions, then the extension of simultaneity is widened, just as we have seen with temperature and length: events that are not perceived conjointly but which stand in the same time segment as conjointly perceived events are then also called simultaneous.

1b. If an event (under the mentioned spatial limitation) is perceived before another, we assign to it the smaller time number. We can do that because the relation of being perceived earlier is transitive and asymmetric. (This relation is viewed, just as is being perceived conjointly, as a psychologically basic state of affairs that is not defined further; perhaps one could attempt such a definition: if the recollection of a perception is experienced conjointly with another perception, then the former perception precedes the latter.) Moreover, the relation has the requisite serial quality: for any two nonsimultaneous events either the one precedes the other or *vice versa.* (That originates from the fact that experiences allow themselves to be temporally ordered in one dimension; this fact is, as is often overlooked, a experiential fact.)

With this the *topology of time* is defined, that is simultaneity and temporal succession; but still not the *temporal metric,* the measure relations of time.

2a. For the convention of the *scale form* we must determine when two differences of time value, i.e., two time segments, are to be fixed as identical. Toward this end we must chose a periodic process that is either always observable or can be realized at all times through an easily produced device. In principle, any periodic process can be used for this, i.e. any process in which a certain state always appears in a similar and in a easily and sufficiently precisely recognizable manner. (The expression "periodic" does not, therefore, include in itself the meaning that the "periods" are of the same length; what "the same length" means is to be defined first now.) Periodic processes are e.g. the daily motion of the sun, my pulse, the rotation of a wheel driven by weight or water, swings of a pendulum, and so on. Now, the choice of a standard process for time occurs in a similar fashion as the choice of a standard of measure for length. If I choose, say, my pulse and measure time according to it, I will find that the sun, wheel, and pendulum are irregular processes; if I choose the wheel, I will establish just as before the other three as irregular. If I choose the sun, however, I will find that the pendulum estimates equal time segments to a fair approximation; similarly for the sun, if I choose the pendulum. Thus, I affect a choice of processes according to which time is not "more correctly" but more usefully measured than according to others; by their choice there turn out to be many different regular natural processes. As with the standard length still greater precision can then be introduced: the motion of the stars agrees more precisely with the pendulum than does the motion of the sun; a pendulum, for which certain conditions are held constant and "perturbations" are avoided, agrees with the motion of the stars more precisely than does another. Thus a time scale is ever more precisely determined, which is constituted such that by the choice of it every periodic physical process is measured ever more precisely as regular, however more similar the state of its environment is held.

The more precise time measurement allows of reduction to space measurement, e.g. as when marks are registered on a moving strip of paper not only of the points of time to be measured but also the standard points of time, whose spatial intervals are measured thereafter.

2b. We know of *no beginning point* for the time sequence. Hence, an arbitrarily chosen time point must be stipulated to be the null point of the time scale, as is done in the various calendar systems (first year; first day of the year).

2c. The duration of any particular, continuously observable or repeatable process is stipulated to be the *unit* of time. One uses for practical purposes the year, i.e. the time of the revolution of the earth around the sun; the day, i.e. the mean temporal duration between two zeniths of the sun (southern passages); the second, a certain fraction of the day. Just as one can make the unit of length free of reference to concrete bodies by relating it to optical processes, one could in principle dissociate the unit of time from reference to astronomical processes and relate it to the unit of length with the aid of the velocity of light. Practically, however, this possibility finds no employment (at least in the current state of physics), since the velocity of light --in contrast to those optical magnitudes --cannot be measured with sufficient precision.

Analogy between the concepts of length and time. We have carried out the consideration of both magnitude types such that the concepts of the length of a segment, the identity and difference of length of two segments stand in an analogy to the concepts of the time of an event, and the simultaneity or temporal interval between two events. From the point of view of the mathematician and the physicist another analogy is more natural, that is the analogy between the concepts of the length of a segment, the identity and difference of length of a segment on the one hand and the concepts of the length of a temporal segment (i.e. the temporal interval of two events), the identity and difference of length of a temporal segment on the other. This analogy is of a formal-mathematical type; it appears, if temporal and spatial position are expressed with coordinates. The first analogy is of a epistemological type. For the discussion of concept formation, we must consider the first analogy, while the second has the greater significance in the final measurement system, the physical space-time system.

D. *Derived Magnitudes: Velocity and Acceleration*

If time and length measures are defined, then the magnitude types velocity and acceleration can be derived from them. By the *"velocity"* of a motion is understood at first the quotient (relation) of the length of the traversed path and the duration of the time spent on the path; if the velocity changes during the course of the motion, then by the velocity at a particular time is understood the limit of those velocities that are obtained if one calculates, according the first concept determination above, the velocity of the motion in various temporal segments, each beginning with the time point in question and getting ever shorter. By the *"acceleration"* of a motion is understood the quotient of the increase of the velocity during a certain time and the length of that time; here also in the case of changing acceleration a limit must be formed in similar fashion to the above.

Through these concept determinations the scale form, null point, and unit for velocity and acceleration are defined, for they follow necessarily from the respective parts of the determination of length and time measurement. Further conventions, therefore, are not met with here. The result that follows from this for the addition theorem for velocity has already been discussed above.

Area and *volume* are derived in a simple fashion, which does not need to be discussed further, from length measurement.

Angular measure appears to be independent of the definition of length measure. This does not prove to be entirely correct; it is independent only with respect to the unit of length. However, the first convention, that of the identity of angles, makes reference to the rigidity of bodies and, hence, to the identity of length.

E. Mass

The introduction of the concept of mass occurs on the basis of certain phenomena that take place if two bodies stand in mutual mechanical interaction to one another, i.e. if they push or pull one another. It is a matter of indifference for this whether the mutual interaction is occasioned by gravitation, or a existent, stretched or compressed feather between the bodies, or through electrical charges, magnetism, or some other force.

In order to deal only with one coordinate in the measurement of velocity and acceleration we will consider only motion in a straight line. Experience teaches that the accelerations that two bodies in mutual interaction acquire are always in opposite directions. Therefore, in the following we need not take into account the direction of the acceleration but only to attend to its numerical magnitude.

la. Experience teaches that the relation of the accelerations that two bodies in mutual interaction experience depends only on the two bodies, but not the nature of the interaction. The relation between two bodies, which then occurs when they acquire the same acceleration by mutual attraction, is clearly symmetric; in experience it proves to be transitive. To any two such bodies we assign the same number as their "mass".

1b. If, given two bodies, A and B, A acquires a smaller acceleration than B, when they stand in any type of mutual interaction, then that is also the case if these bodies stand in mutual interaction to another body. The relation determined thereby is by definition asymmetric; experience teaches that it is also transitive and has the sequential character required by rule 1b. From this we determine that with the obtaining of this relation body A will be assigned a greater mass than body B.

2a. We will determine the scale form --as we did with length --through the convention that the addition theorem have the simplest form: if two bodies have the masses, a and b, then the body that arises from their composition should have the mass, c=a+b. It emerges empirically that with this convention for the scale form (and the immediate and obvious convention for the null point) that the masses of two bodies stand in inverse relation to the accelerations that they acquire in mutual interaction. If we have, for example, three bodies, A_1, A_2, A_3, each with mass, m, (determined according to la), the convention under consideration says the body, B, which is the composition of A_1 and A_2, should have the mass 2m. And experience shows then that body B, whose mass is twice as great as body A_3, obtains an acceleration half as great A_3 when they are in mutual interaction. It follows from this state of affairs that with the chosen scale form the laws of mechanics take on a particularly simple form.

2b. The choice of the *null point* of the mass scale is yielded up very simply. That is, for bodies of identical material mass depends only on volume; it becomes proportional to volume if we assign the mass zero to zero volumes.

2c. For the convention for the *unit* we do not need to stipulate a particular, individual body as with the convention for unit length. For, since for a single substance (and temperature) the mass depends only on the volume, the stipulation of a substance and a volume suffices. Indeed, it is stipulated: the mass of 1 cc of water (at 15°C) is the unit mass (1 gr).

Derived Magnitudes. Density is the relation between mass and volume. Density is a *"substance constant",* i.e. it depends (for a certain standard temperature) only on the substance of the body, not its volume or figure; the last follows from the empirical independence of the mass of a body with respect to changes of figure.

If a body of mass, m, in mutual interaction with another body obtains the acceleration b, we say: the *"force"* mb operates on it. This concept determination is useful because the laws for motion produced by mutual interaction acquire a more simple form if we express them not as laws about the acquired acceleration but rather as laws about the product of the mass and acceleration that obtains for every body. Thus, many of the laws formulated in physics are laws about "forces". Forces are, therefore, for physics certain usefully defined calculation magnitudes; whether "reality" belongs to them, and indeed in a fuller sense than to the remaining physical magnitudes (e.g. accelerations, potentials, self-inductions), is a metaphysical question, which has no meaning for physics.

F. Electrical Charge

In order to consider a magnitude of a quite different type as well, we shall choose, as an example, electrical charge from the large number of physical magnitude types. Certain bodies exhibit under certain known conditions (e.g. ebonite or glass that has been rubbed) a mutual interaction that we designate as electrostatic attraction (or, repulsion).

la. The relation between two bodies that is determined when both affect a third body electrostatically identically, i.e. impart to it the same acceleration in the same direction at the same distance, is by definition symmetric and empirically transitive. We assign to both such bodies to same number as *"electric charge"*; in case of identical acceleration in the opposite direction we assign the same number with opposite sign.

1b. If one body acts electrostatically more strongly than another, i.e. imparts to a third body a greater acceleration at the same distance, we assign to it a greater absolute value of electric charge; in the case of acceleration in the same direction with the same sign, in the case of opposite direction with a different sign. The given relation is asymmetric and transitive. The determination of the sign must occur by reference to a particular material: the charge of a glass body rubbed with silk is defined as positive.

2a. Convention for the scale form, as with length and mass, through the choice of the simplest theorem: the electric charge after unification of two bodies with charges, a and b, is to be a+b.

2b. The null point of the scale is thereby already determined: two bodies that have (according to la) charges +a and -a have together zero charge.

2c. Convention for the *unit* through specification of a standard process. Since the force of mutual electrostatic interaction depends empirically only on distance and electric charge, but not on substance, we can stipulate: if two (smallest possible) bodies have the same electric charge and mutually affect one another at the distance of the unit of length with the unit of force, then both of the charges have unit value.

Derived Magnitudes. Electric *current* = the relation of flowing electric charge and time. (This is the "electrostatic definition"; there is also another, the "electrodynamic" definition; it differs from the first only on account of a different unit).

Review of the examples

We have seen in some chosen examples of physical magnitudes that the definition of a particular magnitude concept rests chiefly on certain empirical matters of fact. These empirical matters of fact must be such that they exhibit certain relations with particular formal attributes on the basis on which it is possible to assign numbers to various objects (things, processes, states of things, phases of processes, etc.)

according to certain rules; the rules of assignment then form the definition of the magnitude type in question.

If magnitude types are introduced in this way, then many qualitative statements about things, processes, and their laws can be replaced by statements of number. We will now examine whether all statements are quantitatively expressible.

Are all perceivable attributes measurable?

Now and then one encounters the opinion that the mathematically worked physics *replaces* the qualities of perceivable nature with quantities and thereby loses an essential side of events. We will show later that this conception is incorrect, since in quantitative treatment qualities are not left unnoticed, but rather are only named in a particular way, namely by numbers. Here the question is raised whether all perceivable attributes are then measurable. If that is not the case, then there be would attributes that would not be comprehended by the quantitative method of physics. With help from an example we want to make clear to ourselves that there cannot be such attributes, if we assume that they behave lawfully. If there were certain attributes that didn't behave lawfully, there would no possible conceptual and, hence, no scientific treatment of them at all, regardless of method.

The various qualities of any sensory domain (e.g. colors or tones) allow themselves always, to the extent that they behave lawfully at all, i.e. have regular, constant relations to one another and are bound in their appearance to certain conditions, to be brought into an order. We can order them either according to their qualitative, sensory similarities or indirectly with the help of an ordering of the processes through which they occur. Every ordering, however, allows of being made into the foundation of a measurement, inasmuch as its elements are coordinated with numbers (or, according to its dimension number, pairs, triples, etc. of numbers).

Let us consider as an example auditory qualities and for the sake of simplicity not noises in general, but only *tones.* It can easily be established that every tone (musical chord) can be produced through the composition of individual tones; hence the consideration of (musical) *tones* suffices. In these we distinguish pitch, timbre, and intensity. Pitch is measured in physics by frequency. On the basis of the empirical matter of fact that in every tonal process a mechanical oscillation process occurs whose frequency stands in a univocal relation to the pitch, one can name the pitch by the frequency of the correlated oscillation process and measure it thereby. The opinion could arise that the possibility of our measuring pitch at all is owed only to the presence of the mentioned empirical matter of fact and, hence, so to say, to a happy accident. That is, however, not the case. If that fact did not hold or (in order to avoid such not unquestionable fiction) if it were as yet not known to us, the measurement of pitch would still not be impossible. Through the fact of the various relations of concord, especially the octave, the sequence of tones is ordered into a scale, namely the musical tone scale. If then a particular level is defined by a standard pitch fork (as analog to the standard meter in Paris), then the pitches can be signified according to level and, by numbering the levels, also measured. If this fact also failed to obtain or was not known, we would not need to renounce the measurement of pitch. For example, we could introduce levels into the tone sequence so that we fix the just perceivable tone differences in the various places in the sequence of tones as equal differences. (An analogous method was employed by Ostwald in his definition of the order of colors.) The employment of this method would still, however, not be necessary. It would suffice, to order a sequence of tone producers (e.g. of pitch forks, pipes, or rods) according to pitch, to number them, and to interpret it as the standard sequence. Perhaps one would in this already hit upon some useful form of the scale, in which one graded the pipe or rod lengths according to some simple rule; such a scale would then have a simple, lawful relation to the contemporary physical tone scale. Whether the discovery of such a *useful* scale would succeed or not, in any case *some* scale would follow from the sequence of tone producers and, hence also, would the possibility of measuring pitch. We will not go into intensity and timbre; it is known that the former stands in lawful relation to the energy of oscillation and the latter to the form of oscillation (or, otherwise expressed, to the composition of harmonic part tones); also for these tonal qualities there would follow in every case a possibility for ordering and thereby also measurement.

Laws of nature, hypotheses, and theories

If all perceivable attributes are measurable, then the same holds for the not immediately perceivable, but rather derived attributes. For the latter rest in a lawful way on the former, as we saw in the consideration of the first, qualitative level of concept formation. According to the undertaken considerations, all physical attributes are, therefore, measurable, and hence, expressible by numbers as values of certain magnitude types. Thus, the lawful relations of such attributes that occur in nature can be formulated as relations between numbers. Now all *laws of nature* are assertions with general content about the reciprocal dependencies of physical attributes; they can, therefore, be comprehended in the form of *mathematical identities.* The special form of these identities, in which physical causality is expressed, will be discussed later.

The assertion of natural laws goes through various levels of certainty in the course of progressing research. A law that at first is only conjectured is asserted as a *"hypothesis"*, until it acquires the certainty of acknowledged scientific theses through sufficient confirmation. That this certainty can never be absolute, but has only a greater or lesser probability, has already been mentioned in the discussion of induction.

Physics attempts to unite the natural laws of every domain and finally of the whole of nature into a coherent order: *"theories"* are asserted and elaborated. First through its ordering into such a comprehensive structure of a theory does the assertion of a natural law, which at first is founded on certain individual appearances, obtain a sufficient certainty. It is a remarkable fact, but not one explainable within science, that natural events not only proceed in accordance with law (otherwise physics would be impossible), but that the individual laws always allow of being combined into ever more comprehensive and united theories. Thus, acoustics has in the course of its development been merged into mechanics, mechanics and thermodynamics have been united, optics and magnetism have become parts of the theory of electricity; finally, in the new atomic theory, all physical and chemical appearances except for those of gravitation have been reduced to electromagnetism, or at least their reducibility in principle has become known. Here we cannot go any farther, however, into the assertion, establishment, and unification of physical theories, since we are dealing here not with theory formation but with concept formation (to which it is, admittedly, in many places closely connected).

The superiority of the quantitative method in relation to the qualitative

The difference between the quantitative and qualitative methods is basically a difference in method of designation: the quantitative method "dispenses with" the various forms of appearance (degrees, phases) of an attribute, i.e. it designates them with *numbers,* the qualitative method designates them with other symbols, mainly *words.* Now, a designation with numbers has the following advantages with respect to a designation with words: 1. In the numbers an inexhaustible collection of designations stands at our disposal, while an inventory of always new word names for thousands of individual phases is scarcely feasible. 2. The designation with numbers can be adapted to the qualitative order of the elements (phases) such that the name of every element gives at the same time its place in the ordering (advantage of the numbering of houses on a street to the designation with individual names). 3. The designation with numbers makes it possible to collect general laws into one expression (that is, *through* the mathematical relations between the numbers, the "functions"); with word designation thousands of individual sentences would have to appear in the place of a single mathematical identity; practically said: with a word designation one would not express at all what the identity expresses, but rather would satisfy oneself with the assertion of vague relations. The last mentioned circumstance also operates on research activity: only if a means is available to express more exact laws in concise form (and thereby usable in practice for the first time), does research acquire a sufficient stimulus to seek such laws. Here, as in many other places in the development of science also, the essential improvement of a means of representation works as an inducement to more exact question formulation and more penetrating investigation.

The struggle between the qualitative and quantitative methods has found its, so to speak, classic form in a certain part of physics, namely the optics of the visible, the *"theory of colors"* in the opposition of *Goethe* and *Newton.* Newton, although he did not yet know light to be a wave process, had already ascertained through experiments the numerical relations among the various spectral lines, and hence, among the various colors and which were later interpreted as relations of frequency. Goethe fought against him here and there in his *Theory of Colors* with the most vehement words. To him synthetic resources and mathematical calculations were repugnant, he wanted to tie knowledge always to immediate intuition, and based his investigations, therefore, on the immediately seen colors or, at most, colors transformed by prism, mirrors, and the like. If physics had not, as it actually happened, followed Newton but rather Goethe, it would have been able to make many steps beyond Goethe's correct cognitions, of course, but would soon have reached a boundary and would have had to give up the largest and most important parts of today's physical knowledge. Above all, in this way the most important result in the development of physics in the last hundred years would not have been reached: the ever greater unification of the parts of physics into a unified theory. In the Goethean, i.e. sensory-qualitative method physics dissociates into a sequence of partial domains, which correspond to the various sensory domains. These can, of course, exhibit some relations to each other, but only with the Newtonian, quantitative method can the partial domains fuse with each other. This result coheres with the large development that the theory of electricity has made in the last century. With the qualitative method it had barely been able to go beyond the primitive initial levels; for it is not based on the perceptions of a particular sense, but rather rested essentially on inferences, reasoned calculations from observations of various other sensory domains. And yet the theory of electricity has achieved a revolutionary significance in physics. From the theory of some peripheral, infrequent appearances has developed the theory of the fundamental force that determines the construction of all matter and upon which all perceivable occurrences (except for gravitational phenomena) rest, whether they be mechanical, acoustic, thermal, optical, chemical processes or what have you.

It cannot, therefore, be doubted that the passage from the qualitative to the quantitative level signified a decisive and necessary step for physics. That is true not only for the form of contemporary physics, since perhaps another form is thinkable that could achieve the same accomplishments with a purely qualitative method; rather it is true for any physics that has for its goal a causal natural system, i.e. a system in which every event is univocally determined by earlier events.

The passage of physics to the quantitative level took place at the beginning of modern times. The decisive step occurred when *Galileo* prepared himself to ascertain the laws of free fall through measurement experiments with drainpipes.

III. The third level of physical concept formation

Abstract level: The four dimensional world process

The four dimensional world

Since space is three dimensional every spatial point can be designated by giving three numbers. How the designation is undertaken depends on the choice of the reference system ("coordinate system"); e.g. geographical latitude and longitude and the height above sea level can be chosen. If we add a fourth number as the time coordinate, we have designated through the four numbers a "space-time point" or *"world-point".* In order to express, for example, that a particular state magnitude has the value z at the space point {a,b,c} and a time d, we can say: its value at the world-point {a,b,c,d} is z. The whole state of the world is given if the values of the physical magnitudes are given for every space point. The whole process of the world through all time is given, if the values of the physical magnitudes are given for every world-point. Since many physical magnitudes depend on other magnitudes for their distribution of values, and can, hence, be calculated, if the distribution of the others is known, for representation of the world process it is not necessary to take all physical magnitudes into account, but only some of these. Which magnitudes suffice for the description of the entire process is still not entirely known; according to the current position of physics it seems to be ten specific magnitudes.

The collection of world-points is four dimensional. If a four dimensional order is difficult or perhaps impossible to conceive of intuitively as a spatial ordering, still mathematicians are in the habit of conceiving and describing the order of collections that have more than three dimensions as if they were spatial orderings. Thus we say: just as a three dimensional spatial domain can be conceived as consisting of a stacking on top of one another of two dimensional domains, so we conceive of the four dimensional domain of physical events during a certain duration of time as a stacking on top of one another of "momentary spaces", which represent the individual states of the space points at the individual time points.

If we imagine the position of a certain material particle is marked in every momentary space, we produce the life line of this particle in the four dimensional space-time domain as the sequence of these marked world-points; we designate this line as the *"world line"* of the particle. , Although we said above that the physical process can be described in principle completely by giving the *values of the state magnitudes* of the world-points, we must however add that physics still does not at present know for certain whether in fact this would suffice. The conception of "physics as pure field theory" assumes this. Another conception holds against this that the mere giving of the state magnitudes does not suffice even to describe the position and motion of the smallest building blocks of material (say, the electrons): *in addition to the state magnitudes the world lines* of these building blocks must be given. A third conception is even more distant from that of pure field theory, since it forgoes the state magnitudes entirely: it holds that giving the *world lines alone* suffices for the representation of the entire physical process. However this may be --whether state magnitudes alone, world lines alone, or both must be given --,in any case, *the process of a domain or the entire world can be conceived as a certain, four dimensional structure.* This conception has the advantage that in place of the fleeting, ever changing picture that the process offers to perception and even to the usual, quantitative point of view, there appears a single, steady, unchanging structure, whose rigid state is investigated. Laws of nature here become statements of the reciprocal dependency of the conditions of parts of the four dimensional structure.

The representation of the physical process in the form of a four dimensional, rigid structure originates with *Minkowski.* It has been stimulated by the theory of relativity and has led to a particularly simple formulation of this theory, but is in itself independent of it and can be used for the representation of every theory about the physical process.

Physical causality

The state of affairs that the physical process is subject to regularities reads as an assertion about the four dimensional world thus: if the value of certain state magnitudes are fixed for certain world-points, then these state magnitudes cannot any more take on for the remaining world-points any value that is possible for them, rather for certain world-points the values are either univocally determined or at least contained within certain boundaries. This is the general concept of a regularity or "determination". The form of regularity met with in nature, "*physical causality*", forms a special case of this concept. It is constituted such that the change of a state magnitude at a particular point is determined through the spatial distribution of certain state magnitudes in the spatial neighborhood of the point in question. (Hence, differential equations as the form of natural laws.) With respect to the whole world process: the process in the future and the past is determined, if the state is fixed for one point of time.

In actuality, of course, the state of the entire world can never be taken into consideration, because it is never determined. For the process in a finite region the regularity is to be expressed in the following form: the physical process in a certain spatial region during a certain period of time is univocally determined by laws of nature, if some state of the region (it need not be the initial state) and the "boundary process" are known, i.e. first the distribution of values of the state magnitude for the entire region at some point of time in the period of time and second for the boundary points of the region during the entire time segment. In the language of the four dimensional world: the condition (more exactly: the attribution of values of the state magnitudes) of a four dimensional temporally cylindrical region is univocally determined through the condition of some three dimensional cross section of the cylinder and the condition of the (three dimensional) cylinder boundary. In certain simple cases, which admittedly are never quite exactly realized, the boundary process does not need to be given, rather the description of a state suffices; this is the case with the so-called "isolated systems", i.e. those regions in which no energy either enters or leaves during the time period considered.

The world process represented as a number system

Since the individual world-points can be designated by giving four numbers, i.e. a "number quadruple", we can ascribe the values of the physical state magnitudes directly to the number quadruples instead of assigning them to the world-points of the aforementioned four dimensional world. Let us assume that ten state magnitudes are necessary and sufficient for the description of the physical state. Then the physical description of a world-point consists of a sequence of fourteen numbers; the first four characterize the world-point uniquely, the last ten give its physical state. The physical process in a certain spatial region during a certain period of time or indeed in the entire world for all time is, then, to be represented by a collection of such sequences of 14 numbers, in which such a sequence belongs to each world-point of the region or the world. Natural laws are to be expressed in this representation as relations of dependency among the numbers of the various 14-tuples. The earlier described form of physical causality is so constituted that (roughly speaking) the process at a point during a short time period depends on the process at that point and at its neighboring points at the start of the time period. That is expressed as an assertion about 14-tuples thus: with reference to a particular 14-tuple, R, the numbers in the fifth through fourteenth positions in those 14-tuples, which agree with R exactly in the first through third positions and approximately in the fourth position, are determined univocally, if the numbers in the fifth through fourteenth positions of those sequences, which agree with R exactly or approximately in the first through third positions and exactly in the fourth position, are known.

The consideration of the physical process as a four dimensional space-time world is to a certain extent the transformation of the physical consideration of a process into the *geometrical consideration* of a rigid structure. The currently --indicated consideration of the physical process as a collection of 14-tuples of numbers goes still a step further in the formalization: here the physical consideration is transformed into the *arithmetical consideration* of a certain number system.

Retranslation into statements about qualities

Is the indicated, most abstract form of physics, then, still to be called physics? Does it still say something about nature and does it teach us to calculate later perceptions from ones already had? That is certainly the case, although space and time, or perceivable qualities or any other *qualities are no longer discussed at all.* Only numbers of a particular number system and their mathematical relations are spoken of. The essential attribute of this number system for the goal of physics is now not that it, precisely as a number system, can be subject to a completely mathematical treatment, a calculation of a part from the others; this is only a methodological advantage. What is essential lies much more in that *the statements of number of the system always allow of univocal translation into statements of quality* (which is occasionally not sufficiently heeded). From this it follows that the reproach raised against mathematical physics as to its one-sidedness, that is, the sole consideration of the quantitative through the neglect of the qualitative, is not correct; this will be discussed more fully below. There isn't a qualitative and a quantitative side to nature. Rather the quantitative is a certain conceptual form; the quantitative view is a particular method (resting on measurement and formalization) for the purpose of being able to cognitively comprehend and predict nature, i.e. the totality of perceivable reality with all its qualities.

The retranslation of the pure number statements of abstract physics into qualitative statements is possible, because to a particular distribution of values of particular state magnitudes is always univocally correlated particular physical qualities, ultimately sensory qualities. A particular constitution of a collection of the 14-tuples of numbers is, for example, to be interpreted as a particular movement of electrons in a particular spatial arrangement, and this again as a chlorine atom or a sodium atom or a sodium chloride crystal, i.e. table salt; to a collection of 14-tuples so constituted is to be coordinated, then, the qualities white and salty. Another such collection is to be interpreted, say, also as the movement of a collection of electrons, and these then as a certain periodic distribution of air molecules of a certain frequency; to a so constituted collection of 14-tuples of numbers is, then, coordinated with a tone determined as to pitch, timbre, and intensity or an exactly described sound.

If this possibility of retranslation did not exist, physics at this abstract, i.e. completely formalized, level would be, of course, nothing more than a harmless calculating game. Only because that possibility exists may physics say that it speaks of reality and only because the retranslation is univocal can physics make *particular* assertions about perceptions, which no one has yet had, but rather will only have at a certain future point in time, or general assertions about what perceptions are always to be expected under certain conditions.

Does physics abstract from qualities?

We have designated the third level of physical concept formation as "abstract". This is, however, based only on the fact that the concepts employed at this level are abstract, formal; this designation is not, however, meant to indicate that physics in a real sense abstracts from the qualities, that it leaves them out of account. Occasionally this view about mathematical physics is represented; one believes that it in some sense sifts through nature, retaining only the quantitative in its hand, while the qualitative --wherein, of course, the essential lies --runs through its fingers.

Is physics, as the representation of the colorful process of nature through purely formal concepts, to be compared with the representation of a colorful object by a black and white picture? No, there is an essential difference between the two representations. A photograph of a colorful thing or a description of the thing made on the basis of the photograph in fact neglects the colors; it gives, of course, various attributes of the thing; but neither do the colors appear among these nor can they inferred again subsequently from them. In the physical description of the natural process through number statements, the colors, of course, also do not appear; but they are obtainable again subsequently from these statements, hence not lost as with the photograph.

In the consideration of the quantitative level we saw that *measurement* of qualities is not some sort of violent treatment of the qualities, but rather nothing other than a *particular type of designation,* namely a designation with *numbers* instead of with word names. As little as one objects that the geographer replaces, on account of his designation of cities and mountains, actual objects with words and therefore suppresses essential attributes, just as little may one object that physicist, because we wants to measure everything, replaces actual objects and qualities with numbers and thereby loses an essential character of them. The chemist speaks not of “water” but rather of “H_2O”; despite this no one will maintain that he abstracts from substance. Similarly, the quantitative method of physics is only systematic designation, not abstraction from qualities.

At the abstract level of physical concept formation, however, qualities rarely appear at all, not even under other names. For, those structures of partial domains of the number system to which the sensory qualities are correlated, are not especially emphasized. In spite of this, the qualities are not lost even at this level, but rather are tacitly fixed in the mathematical description of the number system. That shows itself in the discussed possibility of reclaiming the statements of quality univocally from the mathematical condition of the number system. As a comparison for the representation of the natural process in physics, the representation of a melody by notes can serve better than the black and white picture of a colorful flower: the score itself does not, of course, make any sound, despite this the tones are not lost in it, since the score can be translated back into the audible melody at any time (assuming that the coordination between notes and tones is known).

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