# On the Dependence of the Characteristics of Space upon those of Time

The "outer world" around us exhibits a two-fold order: that of succession and that of contiguity in space. Since Kant, we are given to answering the question why every object of (outer) experience fits into these orders like so: they are forms of intuition and therefore conditions that every object must conform to in order to be an object of possible experience at all.

Space and time are only connected to one another through this necessary validity for every object of experience; but neither is dependent upon the other. An event happens before another or after it or simultaneously with it: nothing is therewith determined regarding how the first is spatially related to the other, whether it occurs above or below it, whether it is near or far from it or in the same place. Admittedly, a certain relationship, which however indicates no dependency, shows itself in the conceptual, measuring, mathematical grasp of the two order-forms. In this, the temporal order appears as one dimensional, as representable by a series. The spatial possesses a higher degree of manifoldness, which is designated as three dimensional. This means that the spatial order can be represented by a three-fold series schema, without it being the case that it can be known through three series-directions determined from within. The agreement of the two orders therefore consists in their being conceivable as series, mathematically expressed: as numbers, as coordinates. The grasp of the outer world through measurement, its ordering in a system of coordinates belongs to the task of physics. When we speak here of the space and time of the outer world, we wish to think always only of their conceptually graspable, rationalizable, mathematical characteristics, that is simply: the space and time of physics.

Physics unites the one time coordinate and the three spatial coordinates in a coordinate system of four axes. In this, none of the four coordinates is dependent upon the other three, and thus, in particular, the time value is not dependent on the location or vice versa. The totality of events of the outer world, in all places, at all times, is represented in physics in a single, unchangeable, four-dimensional world. The state of the world at a moment is then represented as a three-dimensional cross-section through this four-dimensional world. If one traces the fate of an elementary particle (which, according to the physical theory at the basis might be, for example, a material particle or an elementary quantum of electric charge or an elementary quantum of energy) through all the point events that take place where it is, as one imagines them represented in the different momentary cross-sections of the position of the particle, then these "world points," each of which represents a point event of the particle, form the "world line" of the particle in the four-dimensional world; only such world lines of material particles do we designate here as "world lines." This Minkowskian representation of all processes of motion through the meshwork of world lines in the four-dimensional world has been disseminated especially in relativity theory and may be here presupposed as known.

The special theory of relativity brought time and space into a closer connection than they had before in physics or in pre-scientific consciousness. The sense of this theory can be (following Minkowski) quickly and intuitively formulated that the flat momentary cross-sections are not univocally in their positions, but can have a certain (but not every) inclination to one another without, of course, forfeiting the foundational characteristic of momentary cross-sections: merely to bind together simultaneous point events. Thus, simultaneity becomes equivocal; statements regarding space or time in themselves no longer have sense, but rather only the unification of both does. The general theory of relativity goes yet further: in it the cross-sections are no longer mandated to be flat, but rather maintain flatness and the other characteristics assigned to them by the special theory only in the limiting case of infinitely small regions. Even here, however, despite the tight connection among the coordinates, there is no discussion of mutual dependency; they are still four independent variables.

In order to achieve the intended thesis about the dependencies between time and space, we must first introduce and discuss a few further concepts.

Since the position of the momentary cross-sections and, therefore, the relation of simultaneity is not univocal, but rather contains a conventional element, in this physical space-time world "the" time order cannot be spoken of from the start. At first, only the temporal order on each individual world line in itself, the "proper time" of the corresponding world line, and, thus, nothing other than the serial sequence of world points on this line, can be taken to be determined. Temporal relations among world points on different world lines, that is, among substantially unconnected events, can be derived—"time systems" can be set up—only mediately on the ground certain rules of comparison of different proper times. The rules of comparison can, for example, be given in the form that it is stipulated when two world points are to be taken as simultaneous.

We call the class of world points that belong to a common momentary cross-section a "space class." All world points are, therefore, simultaneous to one another; one and only one point of each world line belongs to any space class. By a "space" (in the physical sense) we understand the order of the world points of a space class (thus, this designation) expressed through geometrical concepts such as distance, close, far, line, circle, and so on.

We must now discuss a very important difference between two types of order characteristics that is already of significance in the general theory of order and will here come into consideration as much with spatial as with temporal characteristics, namely, the difference between metrical and topological characteristics. A characteristic is called metrical if it has to do with relations of measurement, and thus in the end can only be expressed through the means of measure numbers; in many concepts the measure numbers appear unexpressed, for example, in the concept of a line (whose definition uses the concept of shortest length) and in that of congruence (which is defined via the concept of segments of equal length or vice versa). The metrical characteristics of time order are, for example: the axiom of congruence (if two points a, b lie on a world line and c lies on the same or another, then there is on the world line of c a time segment cd that is congruent to ab), the Archimedean axiom (if a, b, c are any three world points on a world line, and if we extend the time segment ab in both directions with sufficiently but finitely many segments congruent with it, then there is a time segment in this series to which c belongs). Examples of the metrical characteristics of spatial order: space is Euclidean; or: space in non-Euclidean but in every point the measure relations in vanishing volumes of the spatial part under consideration converge to the Euclidean.

The non-metrical characteristics of the time and the space order are called topological. They concern only the neighborhood and connection relations. Examples of topological propositions about the time order: the time order of points of a world line is a serial order, that is, it is homogeneous (if a+b, then a is before b or b is before a), transitive (if a is before b and b is before c, then a is before c), and irreflexive (a is not before a), and thus asymmetrical (if a is before b, then b is not before a); furthermore, it is dense (between any two points there always lies another). Examples of topological propositions about the space order: space is three dimensional; space is in itself dense (in any neighborhood of any point no matter how small the neighborhood, there are still other points); space is continuously connected (that is, every closed curve can be shrunk to a point in space).

In addition to the temporal and spatial relations between world points, we must consider another relation that plays a special role among the foundational concepts of abstract physics, that is, coincidence. If two world lines cross one another, so that the world point a of one falls together with b, then we say: a coincides with b. This relation belongs, therefore, neither to the temporal nor to the spatial order, or to a certain degree to both: it signifies the null relation or identity that is fulfilled in both orders simultaneously: coinciding points have the same position just as much spatially as temporally. Thus, nothing is determined about either the temporal or the spatial order by this relation. Furthermore, coincidence is a topological relation, since neither spatial nor temporal coincidence [Zusammenfallen] are metrical determinations.

Our thesis regarding the dependency between temporal and spatial order can now be formulated as: the topological characteristics of the spatial order can be derived from the topological characteristics of the temporal order. (The wider-ranging thesis that all the characteristics of the spatial order, and thus also the metrical characteristics, can be derived from the same determinations is here, without justification or further discussion, only mentioned.) This thesis will be justified here through a construction of time-space topology that will not be carried out but only outlined in its basic features and that proposes the derivation that is asserted to be possible.

The thesis allows of another yet more exact formulation. If a sentence x can be arrived at according to logical rules of inference through inferential consequences that begin with sentences a, b, c and the basic sentences of logic as premises without taking as help and further sentences or unexpressed presuppositions or, say, intuition, then we say simply: the sentence x is derivable from a, b, c. By an axiom system for a theory we will understand a group of sentences such that from them all the (remaining) sentences of the theory are derivable. If the axiom system a’, b’, c’ of the theory, T’, is derivable from sentences of the theory, T, that has the axiom system, a, b, c, then a’, b’, c’ and therewith all the sentences of T’ are derivable from a, b, c. The theory, T’, in this case forms only the immanent exhibition of a part of T without the addition of logically new elements; we call T' a "branch" of the theory, T. The sense of the thesis is now that spatial topology is merely a branch of temporal topology, if we introduce into the latter the coincidence relation, which belongs to neither but rather stands between them.

In order that this assertion contains a positive sense, one more limiting stipulation in relation to the content of the axioms of temporal topology must be met. For it would easily be possible to take up temporal axioms that contain a proposition about the spatial order in a more or less hidden form. For example, the sentence "space is simply connected" can be put into the form: "the time order is such that every complete cross-section the time series, which is such that any two points are mutually simultaneous, is simply connected." If similar such spatial axioms disguised as temporal axioms and associated with properly temporal axioms, it would of course be possible to derive space topology from these apparent temporal axioms. This trivial possibility must be eliminated. We will understand by a "proper temporal axiom," in contrast to a spatial axiom or a space-time axiom, an axiom that concerns only time order and the coincidence relation; the designation "proper sentence of the temporal order" will be understood correspondingly. Then our thesis maintains that spatial topology is derivable from the proper temporal axioms. For this a criterion for the propriety of a temporal axiom is necessary. We will introduce this later.

Because of the exclusivity assertion it contains, the demonstration of the thesis comes to nothing, if the following two conditions are not strictly fulfilled: 1: the basis axioms must be proper temporal axioms, 2) the derivation of spatial topology from these axioms must be "logically pure," that is, free from unacknowledged premises. In order to be in accord with these two conditions, the somewhat greater formality of special methodical tools must not be avoided.

The use of symbolic logic offers the greatest certainty for the logical purity of the derivation. Logic, in its various systems (Schroeder, Frege, Peano, Russell, among others) has certified itself in many ways in the handling of the foundational questions of mathematics, indeed just as much of arithmetic and analysis as of geometry, where a similar demand for purity was also to be fulfilled. We attain the necessary criterion for the propriety of the temporal axioms through the theory of relations, which forms a branch of symbolic logic. The system of symbolic logic that Whitehead and Russell constructed in "Principia Mathematica" has the advantage over the others in that in it the theory of relations is so well developed that it is available as a complete method for the handling of extralogical regions, as temporal and spatial topology here. The temporal axioms must be comprehended in the forms of the theory of relations and expressed in the symbols of symbolic logic; the sentences about time and, if possible, also those about space must then be derived from them according to the logical rules of inference. Here, the entire construction will not be represented (see the works cited at the end), rather we will provide only the basic thoughts from which it proceeds, and, further, its chief steps in gross outline and, finally, its result.

We understand by a "relation extension" the extension of a relation, analogous to the "class" which forms the extension of a characteristic. Many propositions (extensional logic erroneously presumes: all) about a particular characteristic are representable through corresponding propositions about the class that belongs to it. This is the case when the proposition does not depend upon the actual sense of the characteristic but only upon which objects possess the characteristic and, therefore, are elements of the class and which do not. In an exactly analogous way, it is often the case that the handling of a specific relation does not depend on its actual sense but only on the question of which pairs of objects stand in the relation; these pairs are then the member pairs of the relation extension that belongs to it. This abstraction from content and limitation to extensional handling is in the case of relations still more frequent than in the case of characteristics, hence, investigation may be directed to the relation extension instead of to the relation itself. This is especially the case in all investigations of order systems, hence, the foundational significance of the theory of relations for the theory of order and, thus, for all formal disciplines, for example, mathematics. In accordance with what we have said, we will call a relation extension, say Q, "completely given" if for each pair of objects, x, y, it is determined whether it falls under that relation extension (in symbols: xQy) or not; the corresponding relation is, however, not fully, but only extensionally, described.

We begin our construction of the time and space topology with two basic relation extensions among world points, which we will designate with K and Z. The relation of the relation extension K is that of coincidence: aKb means, therefore, that the world point a spatio-temporally coincides with the world point b. The relation of the second relation extension Z is the basic relation of temporal topology, the earlier-occurrence on the same world line: cZd means that the world points c and d lie on the same world line, and thus represent point events of the same physical particle ("genidentical" world points), and indeed that c is (temporally) before d. We now assume that the two relations (coincidence; serial following of points of each world line) are completely extensionally known; thus, that both relation extensions, K and Z, are "completely given." That means that for every pair of world points, x, y, it is known whether xKy holds or not and whether xZy holds or not. The epistemological and methodological questions that pertain to this assumption we leave to the side. It is important that other than what is here demanded nothing about the physical space-time world is given. In particular, this is not given: if two world points, a, b that do not coincide and do not lie on the same world line, the point a earlier or later than or simultaneous with b is; still less, then, which spatial distance or other spatial determinations of position hold between a and b.

The construction of time topology begins with setting up axioms about K and Z, which express those formal characteristics that we have already mentioned: transitivity, irreflexivity, and asymmetry of Z; in addition, for example, the symmetry of K, the incompatibility of Z and K, and so on. The content of the axioms must, of course, remain within the bounds of what can in be in principle physically experientiable. We will not go into that now. Another limitation is important for the point of view followed here: every axiom should be a formal proposition about K or Z or K and Z. By a formal concept is to be understood a concept of logic, more exactly: a concept that either belongs to the (few, countable) basic concepts of logic or can be defined from these alone. A proposition is called a "formal proposition about one or more determinate concepts" if it uses beyond those concepts only formal concepts (accordingly, a formal proposition about K and Z, for example, can be called a proposition about K but not a formal proposition about K). Now it is in the main not easy to know whether a sentence is a formal proposition about one or more of the concepts occurring in it. In accord with the experiences in the axiomatic handling of geometry and also arithmetic after Euclid, this difficulty is considerable if the sentence to be judged is given in textual words. If we are dealing with an extralogical and extramathematical domain, say, a proposition that contains physical concepts, then the difficulty is not eliminated if the proposition is given in the form of a mathematical equation, say, a differential equation between determinate state magnitudes and time. For this equation has in itself alone no meaning; the assertion must be added that the occurring variables, time, spatial coordinates, temperature, and so on, refer, and in addition to what the whole proposition is meant to relate: say, to a closed system of material particles, to an incompressible fluid, or whatever. Thus, the physical proposition even in the form of an equation contains more concepts than it seems at first sight. These difficulties fall away if the time axioms are expressed in the symbols of symbolic logic, and that is possible in the handling of temporal topology in the theory of relations without further ado. If, other than K, only logical symbols occur in an axiom expressed symbolically, then this is a formal proposition about K. Whether this axiom contains all that is necessary for its complete meaning must be shown late in the deduction of the sentences: if a sentence that the system is supposed to contain can be proven from certain of the axioms through the mechanical-computational use of logical rules of inference; if this is confirmed for all sentences of the theory, then the axiom system is established as a self-sufficiently meaningful system.

Through the demand that all axioms be formal propositions about K or Z or K and Z and through the symbolic-calculational method of deduction it is now guaranteed that the axioms and theorems contain, other than logical concepts, no concepts other than K and Z. Now, Z, as the series-froming relation extension for the time-series on the world lines (physically expressed: the topological determination of "proper time"), is certainly not a relation extension of spatial order, but rather one of time order, indeed, from our points of view, the foundational relation extension. Since we wanted to call the time axioms and the theorems of time order "proper" if they concerned only relations of time order or coincidence, the criterion we sought for "proper time axioms" and for "proper theorems of time order" consists in this: the theorem in question must be a formal proposition about K or Z or K and Z.

If we were to use no more than the tools already discussed, then a theory of time order could be constructed and it would also consist only of proper time theorems; but in this nothing would be achieved for our thesis. Since every theorem would contain only K, Z, and logical symbols, no theorem could appear at all that would say something about space.

The introduction of definitions makes it possible to handle further concepts without damaging the demand for conceptual purity. Because of this demand, however, at first only explicit (nominal) definitions may be used, that is, such definitions as simply declare a new symbol to be synonymous with an expression that contains only old symbols. These definitions introduce only an abbreviating mode of expression. Of course, their methodical value, as we will see, greatly surpasses this merely economical value. Secondly, a new symbol may only be defined via an expression that contains only K, Z, still earlier defined symbols, and logical symbols.  The strictness in the fulfillment of these demands to be applied in the construction cannot, naturally, appear in the representation under discussion of the bases of the system.

We will now give the main steps of the construction.

One of the most important definition is that of the relation extension, W.  aWb means that there is a chain of time segments between the world points, a and b.  By this we understand a series of segments of world lines such that a is the starting point of the first and b is the endpoint of the last and also such that the endpoint of each segment coincides with the starting point of the next.  In order to make intuitive the physical sense of a chain of time segments and, thus, the relation extension W, it should be remembered that the world lines represent not only the lines of material but also of energetic elements—for example, light rays.  A simply consideration shows that the chains of time segments form precisely those lines along which physical causation propagates.  The meaning of aWb is thus: physical causation goes from a to b.  Such interpretations of the relation extensions and classes that appear in the system stand however to a certain extent outside the system itself.  For in the derivation of the theorems these interpretations are never taken into account.  They appear only for the purpose of making the process of derivation intuitive and for understanding the physical meaning of the derived sentences.

The simultaneity between two world points on different world lines is defined as: a and b are called simultaneous if neither aWb or bWa holds.  Also here a discussion of the physical concept of simultaneity has to show that it agrees with this definition.  In the newer physics the dependence of the definition of simultaneity upon the concept of causality (or signal) appears clearly.  We will take the physical time theory as it appears in the general theory of relativity as our basis, since it is currently the only one that is noncontradictory and methodically satisfactory and that stands in agreement with empirical findings; in doing this, we are in no way bound that presuppose the entirety of relativity theory, since its controversial points do not immediately touch time theory.  Our definition of simultaneity is congruent, then, with that in physics.  Moreover, there is a result of this one axiom which precludes absolute time: there is for each world point not only one world point on another world line that is simultaneous with it, but rather many, that is, the world points of an entire time segment.  Of course, the presumption of absolute time could just as well have been taken as the basis of the further derivation; the system of space-time topology would then have a somewhat different form, but this would make no difference to our thesis.

A class of world points that has one world point in common with each world line and is such that any two world points are simultaneous with each other we call a “space class.” It can be shown that any such class has only one world point in common with any world line.  Thus, it corresponds to a cross section through the physical space-time world that contains no time-like but only space-like line elements, thus, to one of the momentary spaces that form, in continuous sequence on top of one another, the four dimensional space-time-world

A space class comprehends, of course, all the world points of a space, but for the task of deriving the space order, nothing has yet been done. The relation extensions, K and Z, induce in the space class no division: since from every world line only one point belongs to the class, there are in it no world points at all for which the relation extension, Z, holds; K-pairs, that is, coincident points, there indeed are, but through them we make no progress. In order to be able to introduce a space topology, we need as basic concept either that of spatial neighbor or that of neighborhoods. The task consists in deriving one of these concepts from K and Z or from other concepts already defined. The sense of this task and the question of its solvability shall now be discussed through the aid of making intuitive the manifold of world lines. In this we represent, in the usual way, space in two rather than three dimensions and picture the time dimension via a space dimension. In a closed space fibers are stretched, which all run in different inclinations and curvatures from bottom to top and that cross one another often. About this manifold of fibers, nothing more is known than this: that for any two points, x, y, it is determined whether x and y are coincident points on different fibers or not (this corresponds to the relation extension, K) and whether x lies on the same fiber as y and beneath it or not (this corresponds to the relation extension Z). To a chain of time segments corresponds an upwards-running route through the manifold of fibers that always runs along fibers but at intersection points can move over to any fibers to any other interesting with it. To a space class corresponds a cross section through the fiber manifold, which cuts each fiber once and only once and which either proceeds horizontally or (in accord with certain conditions) can have a (changing) inclination to the horizontal. Here a question is raised: can inferences about the neighborhood and connection relations of the points in such a cross section be made from such limited information about the fiber manifold. An affirmative answer to this question cannot be expressed with certainty but can be conjectured on the basis of mere intuition. That is, consider a deformation of the fiber manifold such that the K and Z information remains valid, then the fibers may be extended and within certain limits also bent but not ripped; furthermore, existing fiber intersections cannot be disconnected not can new ones arise. (The allowable deformations are precisely the “topological deformations” of the space-time world, whose invariants form the objects of the space-time topology.) On the basis of intuition one will conjecture that with such a deformation a given cross section remains such and that the position relations of the points of a cross section certainly change but only like the position relation of the points of a rubber surface with lawlike bending and extension of the surface, if no ripping or touching occur. The position changes would thus, so intuition seems to teach, allow all neighborhood and connection relations and, thus, all topological characteristics to remain.

Making the manifold of world lines intuitive brings us, then, to the conjecture that the mere information about K and Z do indeed suffice to determine also the space topology. Motivated by this conjecture, we seek for a way of strict derivation and find, ultimately, the conjecture to be fulfilled. We succeed, that is, in defining the concept of space point neighborhood in the following way. Consider any space class and any world point, d, in it. We follow the world line of d from d backwards, that is, in the direction of temporally earlier points, and arrive thereby at the points, c, b, a, one after the other, so that the following hold: aZb, bZc, cZd, and, furthermore, aZd and bZd. Then we determine through use of the relation extension W defined earlier the class of those world points to which c stands in the relation W; we will call this class the W-region of c. Similarly, we determine the W-region of b and of a. Then we can easily show that d belongs to each of these three W-regions (since a world-line segment is the simplest W-segment), that the W-regions of c is a subclass of the W-regions of b, and this is in turn a subclass of the W-region of a. All three W-regions are subclasses of our space class. Thus, we have formed within our space class a series of concentric neighborhoods around the world-point, d. If we consider in addition to the arbitrarily chosen points a, b, c, also arbitrarily many of those in between them and before them, then we achieve arbitrarily many such neighborhoods of d. In order to recognize the physical significance of these neighborhoods, let us presume as a simple case that a-d is the world line of a light source in a homogenous medium. Then the W-region of a as neighborhood of d corresponds to a spherical space simultaneous with d that is filled with the rays which have gone out since the time point a. The W-regions of b and c correspond to smaller concentric spheres.

The problem is solved with these spatial point neighborhoods. It follows from what has been said that they can be defined on the basis of the relation extension, W, and thus in the final analysis on the basis of K and Z. It can be shown that they fulfill the (Hausdorff) neighborhood axioms of space topology; thus, space topology can be derived from them via the processes familiar from point set theory. In our system a somewhat deviant process turns out to be the most expedient. After the point neighborhoods are defined, we define the "continuous space curves" on the basis of the characteristic that in every interval of each of its points lies an interval of the curve, that is, a double interval (on both sides). Then "connected space parts" are defined as those subclasses of a space class such that any two arbitrary points of the subclass can be connected via a continuous space curve completely contained within the subclass. We define "separation" as a helpful concept for dimension number: two points of a space part are called "separated" an that space part by a second space part if there is no continuous space curve that connects them and is wholly contained within the first space part that has no point in common with the second space part. A point class is called zero dimensional if it consists only of one point or of several isolated points. A point class is called n+1 dimensional if for any two arbitrary points in it there is always an n dimensional but not an n-1 dimensional subclass that separates them in the point class. Accordingly, a continuous space curve is one dimensional since any two of its points are separated by any point between them, and thus by a zero dimensional class; a plane is two dimensional and a body is three dimensional. Thus, all these definition fulfill the demand for conceptual purity given earlier: they can be transformed into (admittedly, in the end quite complicated) expressions that contain, other than K and Z, only logical symbols. Furthermore, all the propositions in the system about the defined concepts are formal propositions about K and Z, and thus proper sentences of time topology in the sense given earlier.

In the end, the proposition "space is three dimensional" will appear in space topology. With this as example we want to represent once more the meaning of our assertion that every sentence of the system is a "formal proposition about K and Z" and thus "a proper time proposition." If we view the sentence about three dimensionality as expressed in symbols, we will find in it admittedly not merely K and Z and logical symbols, but rather K and Z not at all and in addition to the logical symbols those for the concepts of space class and three dimensionality. But we can transform the sentence step-by-step with the help of explicit definitions without altering its content. The definition of three dimensionality says that the symbol of three dimensionality is synonymous with a compound expression that consists of earlier symbols, that is, the symbols for two dimensionality, one dimensionality, separation, continuous space curve, and logical symbols. We introduce this compound expression into the place of the symbol for three dimensionality in the sentence about the three dimensionality of space. The second step consists in introducing in the place of the symbol for two dimensionality a compound expression on the basis of its definition in which the symbols for one dimensionality, zero dimensionality, separation, continuous space curve, and logical symbols appear. The third step removes the symbol for one dimensionality from our sentence, the next removes the symbol for zero dimensionality, the one after that the symbol for separation. Thus, the symbols for continuous space curve, space class, simultaneity, the relation extension W (if we may give here only the previously mentioned concepts without the intermediate steps necessary for the actual execution) eliminated. And now the sentence contains only K, Z, and logical symbols. It is to be noted that the transformation through the introduction of the defining expressions brings with it no diminution of content. In most logical or mathematical operations, the inferred sentence is indeed poorer in content than the premises; they cannot be retrieved from it through reverse inference. Here, on the other hand, the sentence finally attained is equivalent with the original, that is, when one holds, so does the other; the transformation is here reversible. The original sentence, in the example the sentence regarding three dimensionality, is thus not richer in content than the one attained from it via the transformation, it is logically equivalent with it and thus also as is this one a formal proposition about K and Z.

The most important concepts of space topology, from which all the others can be derived, are thus derived from K and Z. The entirety of space topology up to dimension number consists, hence, of formal propositions about K and Z and is derivable from proper "time axioms." Space topology is a mere branch of the K-Z system, that is, time topology with the additional relation of coincidence; that was the claim of our thesis.

The proof of the thesis cannot be seen however as carried through in the foregoing sequence of thoughts. Only the complete execution of the axiom system of space-time topology can accomplish that. This execution is to be more generally taken on than the thought sequence on our thesis, whose particular engagement with the question of the dependence between time and space order makes a self-sufficient consideration both necessary and possible.

To this point we have done nothing to make our thesis more intelligible. Through the highly abstract handling of the highly abstract relation extension we might have allowed ourselves to be made more puzzled than convinced, so that we have perhaps a similar impression as Schopenhauer did of the "Euclidean mousetrap proof." Is it a matter now in the present case of an insight that is to be first won through a tedious formalistic investigation and that opposes an immediate approach? This is not the case. The basic thought of the thesis has arisen in intuition and is intuitively graspable; the circumstantial, abstract method is only requisite for the scientifically necessary conceptual formulation and justification. Here is a few remarks in the direction of making it intuitive must suffice. If we have first made fully clear the idea (which basically goes back to Minkowski) that the lines of physical causality are chains of time segments (see above, p. ?), then it will not be difficult for us to make ourselves intuitively familiar with the idea upon which our thesis rests, that is, that the proposition "the spatial distance of two physical elementary particles is small or it is large" means nothing other than: on the world point of the first we meet the lines of causation of the other in an early or a late time point (world point). Not: if two bodies are spatially near to one another then it follows that they are connected by temporally short lines of causation; but: [spatial] nearness means nothing other than temporally short connectedness. Thus, the whole spatial order rests on the time order of causal connections.

The choice of the relation extensions K and Z as the basic relation extensions for the derivation of the system of the space-time topology is in agreement with the presupposition that physics is given to make, that is, that the coincidence between two points and the serial succession of processes at a point are in principle empirically ascertainable and that the observation of all other facts is reducible to these two basic facts. It can now be shown it is logically possible to construct the system from other basic relation extensions. In particular, there are two other system forms that do not contain the relation extension K at all but rather conceive of coinciding world points as identical. Whether these two forms must from a physical point of view appear unsatisfactory or perhaps have certain advantages will not be decided here. The first of the variants has as its one basic concept the class of time relation extensions on a single world line; Z is not here the basic concept, but is to be defined as the union relation extension of that relation extension class. All axioms here are propositions about the topological characteristics of this system of proper times. The characteristics of space order lie in the manner of the intertwining among the proper-time series through identical coincidence of individual points. From the point of view of the present essay this system form would merit preference since it fulfills the presented thesis even more sharply than does the K-Z system; for here the topological characteristics of space order are derivable from the topological characteristics of time order alone. The second variant has as its one basic concept the relation extension, W. It differs more sharply from the K-Z system since not only K but also Z and the concept of genidentity and of world line do not appear at all. Here all the axioms are propositions about the topological characteristics of the causal relation. This form of the system is especially suitable to allow the previously mentioned point of view to become clear: space order is order of causal connections.

Still more thoughts, which can only be mentioned here, can be established regarding the relation between time order, causal order, and space order, as it comes to be represented in the different system forms. From the point of view of the methodology of physics we may recall the question on account of which physics strives to eliminate action at a distance from its theories and gives itself the task of expressing natural laws as differential equations. From the point of view of a more general epistemology prospects are yielded regarding the constitution of objects of experience and the relations of dependence between the categories.

Works Cited

The execution of the system of space-time topology on the foundation of the relation extensions K and Z is to be given in a later essay under the title "Topology of the space-time world, axiomatically presented through the means of the symbolic theory of relations." For the reader who is not conversant with symbolic logic and the theory of relations the basic concepts and principal theorems will be presented in an introductory section.

On symbolic logic and the theory of relations: The foundational work is Whitehead and Russell, Principia Mathematica. Cambridge, I, 1910 (2nd edition 1924); II, 1912; III, 1913. An introduction to the basic concepts without using symbols: Russell, Introduction to Mathematical Philosophy. Munich, 1923. I plan to give an elucidating overview of symbolic logic and the most important theorems in a summary of symbolic logic and of the theory of relations.

On time topology, see: Lewin, The time-like genetic order. Zeitschrift f. Physik, 13, 1923, pp. 62-81. Reichenbach, Axiomatization of the Theory of Relativity. Die Wissenschaften, volume 72, Braunschweig, 1924. Reichenbach constructs, before the introduction of metrical concepts, a time topology that is close to ours. From our point of view, he provides very valuable discussions about the co-ordination of certain system concepts to the synonymous physical concepts (for example, coincidence, simultaneity, and so on).

On the derivation of topological concepts in point set theory, see: Hausdorff, Foundations of set theory, Leipzig, 1914. (Axiom of neighborhood, p. 213).