CONCERNING THE TASK OF PHYSICS and the application of the principle of maximal simplicity.*

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In accordance with the way the question concerning the sources of physical knowledge has been heatedly debated over a considerable period, perhaps today it already can be said that pure empiricism has lost its dominance. Philosophy, to be sure, pronounced long ago that physics cannot be developed relying only on results of experiments but that non-empirical principles must be used as well. Yet only after exponents of the exact sciences began to investigate the methods of physics and thereby arrived at a non-empiricist position, were solutions forthcoming that could satisfy the physicist. Above all, Poincaré and Dingler achieved important results in this respect. Here we proceed from their principles but apply them more generally than has been done previously. In this way, we find an answer to the question concerning the accomplishments of physics that provides an insight into the logical relations obtaining between physical theories that seemingly contradict each other, and into the conditions under which a decision between such theories can be made.

I. The three stipulations: space postulate, time postulate, action principle.

It is the main thesis of the conventionalism propounded by Poincaré and further developed by Dingler that in the construction of physics we have to make stipulations based on our free choice. This implies that those components of the propositions of physics arising from such stipulations can neither be confirmed nor refuted by experience. However, the choice of these stipulations should not be made in an arbitrary

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manner but in accordance with certain methodological principles whereby the principle of maximal simplicity will ultimately be decisive.

Physics has to be based on three such stipulations: one for the system of space, one for the system of time, and one for the principle of (least) action.

The insight that the system of space is a matter of choice was first to be noticed. Poincaré already showed that the question whether Euclidean or non-Euclidean geometry is realized in nature is meaningless. It is up to physics to choose freely which of the infinitely many geometrical systems should be applied. According to the stipulation of the basic measurement standard, usually called a "rigid body", we get this or that geometrical system. (Strictly speaking, the measurement standard must not posit a body as rigid but may only take the distance of a pair of physical points to be unchangeable or determined by some function. Otherwise there is an over-determination. However, we will not consider this here but defer it to a later demonstration.) Hence the widely held empiricist view that rigid bodies exist in nature and could be found or produced by the physicist by means of mere experiments is incorrect. Rather, the rigid body can either be chosen freely or it results from the choice of a geometrical system. And neither can this system be found empirically; it too must be chosen freely or it has to be found by means of a freely chosen measurement standard (of a rigid body). The question as to which of the two possibilities we should adopt will be considered below.

It has been known for a long time due to Poincaré¹ and Dingler² that the geometrical system is a matter of choice. Furthermore, the limits and the feasibility of this choice as well as the functional dependence, just mentioned, between empirical facts the measurement standard and the geometrical system have been delineated elsewhere.³ So this first stipulation will not be discussed any further here.

¹ The well known books, especially: *Wissenschaft und Hypothese*, German translation by Lindemann, 1906, p. 73ff.

² Dingler, Die Grundlagen der Physik. 1919. Ein Grundproblem dermodernen Physik. Ann. d. Naturphil. XIV, 112, 1920. Physik und Hypothese. 1921.

³ Carnap, Der Raum. Erg.-H. 56 der Kantstudien, 1922. p. 32-59.

But in contrast, the necessity to stipulate a norm for the measurement of time ("temporal measurement standard") is commonly ignored. It consists in choosing one world trajectory which coincides repeatedly with another one. The sections of the first world trajectory produced by the coincidences have to be defined as certain time intervals and it also has to be determined how the division of this world trajectory can be carried over to all other trajectories only by means of coincidences. Put in more concrete terms: we choose a physical system that exhibits a "periodic process". The notion "periodic process" does not already presuppose the determination of time, because it does not imply that all periods are of equal lengths. Furthermore, this concept does not presuppose the spatial measurement standard either, for instance, in order to recognize the "same state" of the system. That is because we define "periodic process" as one in which two physical points (material points, rays of light etc.) come in contact with each other again and again. Hence the temporal measurement standard consists in assigning certain numbers to the periods of time between the mentioned coincidences of points within this chosen system (the "clock"). Additionally it has then to be determined how the processes in other systems should be measured temporally.

This is not the place for discussing the important question concerning which conditions a time measurement standard must satisfy in order to yield a consistent system of time measurement, nor the question concerning the extension of the range of the time measurement standard satisfying these conditions, and whether one or several of these conditions have a privileged position or whether all the logically possible ones are also methodologically equivalent. Here it is only essential to emphasize in the first place that the time measurement standard can be chosen. The usual conception nowadays asserts by contrast that the equality of two time segments *[Zeitstrecken]* is not subject to an arbitrary stipulation but rather that it is given, and empirically determined, in nature on account of the fact that two processes, that continue to develop in the same manner, are also to be considered temporally identical. However, to this contention it must be objected that the stipulation of this "same course" is not possible without the conventional stipulations previously made. For, it presupposes in the first place that it is possible to decide the equality of two spatial configurations of the system. But this is

obviously only possible on the basis of a space measurement standard. The second condition for the recognition *[Erkennen]* of the "same course" leads not only, as in the case of the first, to a stipulation that must be chosen, but contains a contradiction. For in the grounding of the time measurement on the same course of the processes of a system is presupposed that the system in question is isolated; for instance, it is not meant, say, that a pendulum clock would indicate the same times on the earth and on the moon. However, whether a system is isolated, that is, whether no forces from outside act upon it, can only be decided on the basis of the third stipulation -- to be discussed later -- namely, that of the action principle. However, this principle in turn already presupposes the time measurement standard since it states which accelerations follow from determinate spatial configurations. Thus, the empiricist conception of time determination is untenable.

The third stipulation concerns the action principle. That space and time measurements are grounded on non-empirical principles does not present great conceptual difficulties. However, that the laws of nature cannot be obtained from mere empirical data even when these measurements are determined, requires a new consideration of the matter, which has been carried out by Dingler.⁴ In the laws of nature there occur quantities whose measurement is not possible directly but rather is reduced to space-time measurements (e.g. mass, gravitational force, electric charge, electrostatic field etc.). This reduction however presupposes a general action principle. If we define, for instance, mass as the quotient of force and acceleration then we cannot measure it without presupposing a force law (e.g. the Newtonian gravitational law or the basic law of elasticity). For as long as the masses are not determined acceleration is indeed measurable but not force. If one recalls that in astronomy the masses of the celestial bodies (and in fact not only of the planets but also of the invisible satellites of the fixed starts) are determined only on the basis of Newton's law, then it is easy to convince oneself that in principle any other mass action principle could be posited and carried out without contradiction with experience. To achieve this one would need on the basis of the selected law only to calculate the mass distribution necessary for the observed motions and to designate it as "real"; nothing else, exactly as in astronomy on the basis of

Newton's law. That in this, among other things, a far more complicated mass distribution had to be set up, speaks indeed in favor of retaining the usual force law but not against the possibility in principle of choosing a different one. In a way similar to what happens in the stipulation of the basic law of space determination, we can also here exploit in two different ways the available freedom of choice for the action principle: Either we choose the basic law itself (e.g. Newton's law or Maxwell's electromagnetic laws or any other) and afterwards determine empirically the distribution of the acting substance (say, of the mass or of the electron); or, the other way around, we posit a specific substance distribution in the world and from this we infer the action principle.

II. The requirement of maximal simplicity.

We now proceed one step further. According to the insight in the freedom of choice available in three moments of the construction of physics we lay down the basic principle that must direct the choice: the requirement of utmost simplicity. However, on closer consideration it becomes evident that this requirement can be applied to two different things. As a consequence, even for those who admit the described basic principles of "critical conventionalism" (that is, the fact that those stipulations can be chosen and the requirement of utmost simplicity) there are still two different ways to procede.

The parting of the two ways becomes clear on the three discussed points at which the choice of a stipulation is to be made. As we have seen, there is freedom in this matter in the choice of a fundamental law (space measurement standard or also geometrical system, time measurement standard, action principle) and upon the choice made then depends the description of the state of the world (the spatial ordering of things in the world; the temporal course of processes [events] in the world; the empirically ascertained numbers [Substanzzahlen] to be ascribed to things, for instance masses). The question then is: Does the requirement of maximal simplicity hold for the fundamental law or for the description of the state of the world made on the basis of this law?

⁴ Dingler, *passim*. Dingler chooses a space system (Euclidean) and action principle (Newtonian) but overlooks the

Let us discuss this question first of all for the simplest case, that is the space postulate. The first way here means: the basic principle of maximal simplicity is related to the geometrical system; among all the logically possible ones the simplest should be chosen. No doubt, this is the Euclidean system. This must be put at the basis of space measurement in that the measurement standard (of the rigid bodies) is determined in such a way that it satisfies the axioms of Euclidean geometry. Dingler chooses this procedure ("pure synthesis"). General relativity theory, by contrast, follows the other way. It chooses (for the four-dimensional space-time domain) a less simple geometrical system (which connects space and time stipulations) because in this way the description of the events [*des Geschehens]* becomes simpler (motion trajectories are geodesic lines, light rays are null lines).

That it is not the case that one of the two ways is correct and the other false (in the sense of a contradiction either in itself or with experience) is not, according to the previous considerations, to be doubted. It can only be a question of which of the two is more appropriate for science to follow.

Before we discuss the advantages that each of the two ways present, let me reply to the obvious objection that *only apparently are we dealing with two different cases* and that both must lead to the same goal, in that the simplest fundamental laws always yield the simplest general description. This would indeed mean the rejection of that claim of relativity theory to maximal simplicity, but this could already be part of the intent of the objection. However, that the identity of the two ways is certainly not a conceptual necessity becomes clear with the following analogy (not an example, for an example can always consist only in the state, somehow conceived, of the whole of nature). Let it be the assigned task to give an account of the spatial distribution of trees of a nursery. In case we are given no measurement rule for the arrangement of the trees, we must stipulate a system of coordinates for every single tree. For this purpose we would choose the simplest system, perhaps a right-angled, Cartesian system. Against this we note that

necessity of a time measurement standard.

if the trees stood at the points of intersection of two groups of parallel straight lines, then we would not choose that simplest right-angled system, but rather a suitably situated oblique-angled one. For with respect to this system the position of all the trees is described with a few number indications. And in case the trees lay in concentric circles, then their arrangement would be stated most easily with the help of polar coordinates. Assume now that they stand not completely without regularity, but also that neither of the two last given arrangements is precisely fulfilled; rather that the positions of the trees is approximately designated by a group of concentric circles simultaneously with two groups of crossing parallel lines. We now suppose that a definite coordinate system is chosen and also that it is justified as long as only the position of a small part of the trees is measured out and even so, only with little precision. But if the measuring out makes further progress in extent and exactness, then it can turn out that another system than the one originally chosen allows representing the total arrangement of the trees, so far as it is known, in the most simple manner.

From the above analogy it is immediately clear that the measurement system that is in itself the simplest of the applicable measurement systems does not unconditionally also yield the simplest description of the states. Later on both advantages and disadvantages of two ways will become clear, first in reference to the space-system, then also generally. The most important *advantage of the first* lies in that the entire formal system is stipulated once and for all, while with the second the possibility is never excluded that the results of future experience could require a completely new system to be constructed and form the basis of the representation. The disadvantages of such a rebuilding from the foundations may be overestimated, however. The results of earlier measurements do not become thereby worthless, but are only converted and reinterpreted, similarly to how the rebuilding of larger theories in physics proceeds (for example, in the transition from Ptolemaic, or better Tychonic, to a Copernican planetary system, or from the emission theory to the wave theory of light, and again in the transition from the theory of the elastic medium of light to that of electromagnetic fields).

The advantage of the second way lies, as the analogy already indicates, in the simpler entire representation of (above all in this case, the spatial) relations to world, more precisely: our familiar spatio-temporal surroundings. Here the question can be raised: With what right then, may it be supposed that the conditions of this yet vanishingly small surrounding in relation to the unknown region would be similar even only in some way to other space and time regions? Or how would one justify that one made the general basic laws dependent on these conditions? The reply is that during the last millennium's procession through space and time regions apparently no considerable alterations of conditions in this sense were noticed. And likewise, if the presumption itself is not in the least suggested to us, permitting calculation of the near future upon the same conditions, then we would have no possibility at all of making inferences about the future other than those upon which we proceed at present. And so, whether we adopt the first or the second way, we add to each prediction the reservation: So far as no hitherto unknown factors arise. And if the conditions of our region also only remain the same for again a further millennium, then it would be still be no doubt worthwhile in itself to show, of the physical theory thus established, that these conditions could be represented in the simplest possible manner.

Similar correspondences hold with the *choice of action principles*. Again here the first way has the advantage in being able to decide independently of experience and hence with finality ; the second way on the other hand has the advantage of the simpler representation of the states of the world, or more precisely, of our region. Otherwise expressed: According to the first way the prominence lies upon the general laws; according to the second it lies in the processes governed according to these laws. The distinction between these two object domains is of the greatest significance for the task and procedures of physics. It will now be more closely considered. Afterwards as well, the goal and meaning of the two ways will be more sharply graspable.

III. Manner and Result of a Completed Physics.

For the stipulation of the direction in which physics at some stage should proceed, the fiction of a *completed construction of physics*, so to speak, an infinite endpoint, can serve usefully. How must we represent to ourselves this ideal physical system; what does it achieve, and what does it contain as propositions? The achievement must obviously be that of a "Laplacean Mind" that can calculate any future or past event. For this his knowledge must be given in a three-fold way; the complete representation of physics rests, pictorially speaking, in *three volumes*.

The first proceeds in the axiomatic manner *[more geometrico]:* it sets a few *axioms* down and purely logically derives arbitrarily many theorems. It fundamentally employs in deduction only arbitrarily few or even no theorems, without which the entire system of physics would be less in a position to carry out those calculations. The derivations should only save work for the calculations in that they anticipate the most frequent elements that then need not always be repeated. The entire knowledge content of the first volume rests just in the axioms alone. These consist in the fundamental propositions of the space determination, time determination and the dependence of processes on one another, briefly: space postulate, time postulate and action principle.

The derivation of the laws of nature from these axioms appears, if also in many respects *arising* through experience, for all that without any *grounding* in experience. The first volume therefore contains *synthetic a priori propositions*, although not exactly in the Kantian transcendental-critical sense. For that would mean that they express the necessary conditions of the objects of experience, themselves conditioned through the forms of intuition and of thought. In that case, however, only a possible frame for the content of these volumes is given. In actuality, its construction is left to our repeated choice. Accordingly, we find the most varied outlines of such axiomatic systems of physics (compare the following examples). These stand to one another not as contradictory opposites, but rather are, if logically unobjectionable, fundamentally equally justified. Choice among them is to be made merely according to fundamental principles of methodology, especially according to that of simplicity. Consequently, for the identifying description of the first volume the concept "hypothetical-deductive

system", as the Peano school has employed it for (formal) geometry (which also comprises a part of the first volume) is to be preferred to that of Kantian a priori concepts.

Examples of Axiom Systems of Physics

1. Axioms of Euclidean geometry, Newtonian laws of motion, Newtonian gravitational law.

a) Without appending further axioms. This is Dingler's position. From these fundamental propositions may be derived: the theorems of Euclidean geometry, of a pure kinematics, and of dynamics: the three integral laws, the principles of d'Alembert and Hamilton, and still many others. Further, the Kepler laws and beyond them, the laws of pseudo-elastic (molecular) matter and from them the statistical-kinetic theory of large elementary aggregates: the theory of heat. The reduction of the electromagnetic laws to the above-mentioned axioms is fundamentally conceived only as a logical possibility, in any case it cannot yet be accomplished.

b) To the above axioms are appended Maxwell's equations of electromagnetism. Deduction of geometry, kinematics and dynamics as in 1a. The optical laws are located in derived propositions concerning periodic processes in the electromagnetic field. The deduction of the laws of elasticity and of molecular processes are regarded as possible but not carried out.

c) A variant of b. The elastic (collision) laws of matter for the smallest particles are set up as axioms in place of the Newtonian gravitational law and the latter is derived from them (statistic-kinetic theory of gravitation).

2. Euclidean axioms, Newtonian laws of motion, basic laws of the theory of the electron, the quantum postulate.

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Deduction of geometry, kinematics, and dynamics as in 1a. The calculated stabile forms of complexes of electrons are designated as modes of atoms *[Atomarten]*, and their behavior is deduced from the basic laws (mechanical, thermal, optical, chemical properties of atoms). Those derivations are carried out only in part. Derivation of gravitation along the lines indicated by Lenard.

3. Einstein's General Theory of Relativity; a single world law: null variation of a certain "world function". Two different forms:

a) Mie-Hilbert. Proceeding from the world law as before: the general Riemannian geometry for four dimensions. The ten fundamental equations of Mie's theory of matter.

Strictly speaking, space and time postulations fail here. These are not somehow already laid down through the Riemannian geometry (as in cases 1a,b,c and 2, the spatial measure postulates are determined through Euclidean geometry); rather they are determined through the choice of the standard measurement body and clock. This choice is not expressly acknowledged because the tacit assumption is made that all 'natural' standard measurement bodies and clocks yield the same measure determination (unspoken fundamental postulate of the metric equivalence of measuring rods and clocks).

The ten fundamental equations must here be imagined in the form they would have if they included only the ten primary field magnitudes: electrical density, at every time the three components of the (electromagnetic) vector potential, and of electric and magnetic flux. For the ten other field magnitudes (scalar potential, at all times three components of electric current density, and of electric and magnetic field strengths) depending on them, we are to imagine therefore that they are determined from these primary ones by the corresponding, in large measure still unknown, functions.

b) Weyl. Here spatial and temporal measurement postulates are expressed through a four-vector field ("electromagnetic vector potential") that couples to the Einstein tensor field ("gravitational potential"). These two fields are determined through a single world law. On them depends not only the relations of spatial and temporal measurement, but also the gravitational and electromagnetic field magnitudes, and accordingly, all physical processes. Here, therefore, space, time, and action principle have been combined into a unity.

c) Kaluza⁵. Outline of a variant of Weyl's theory: in place of vector and tensor fields, only one field of a fundamental metric tensor of a five-dimensional world. Hence a yet stronger unification.

The deduction of natural laws from a world law of the theory of relativity (in the different forms of representation) is conceived in two stages: From the world law are derived the fundamental differential equations, and from these, in turn, the single laws of nature. The second derivation is realizable in great part (analogously to case 2); the first, however, is only loosely indicated since the exact form of the "world function" is still unknown.

4. The aether-theorie of Wiener.⁶ Only one physical state magnitude: the (absolute) current velocity of the aether, which remains constant for every aether particle. A single fundamental law: the (standard-) acceleration as a function of the velocity vector field. That is the action law; space and time postulations are not explicitly indicated. Perhaps the Euclidean space postulate is tacitly presupposed, for then the time postulate is univocally determined from this and from the fundamental law of constancy of velocity. Accepting this supposition, three axioms are then basic: the fundamental law, the law of constant velocity, and the Euclidean space postulate. The deduction of gravitation, of the Maxwell equations, and the other laws of nature is up to now only intimated; their possibility must be investigated nevertheless.

⁵ Th. Kaluza, "On the Unification Problem of Physics", *Session reports of the Berlin Academy of Sciences*, LIV, 966-972, 1921. The literature of the previous examples is well-known; in addition to the aforementioned names, compare Laue, *The Theory of Relativity*, v.2 (1921).

⁶ Proceedings of the Saxon Academy of Science, XXXVIII, 1921.

The second volume produces the mediation between the domain of perception and the domain portraying the object of physical theories. That these two regions are completely separate from one another can scarcely be sufficiently emphasized. The first contains the contents of sensation: colors, tones, tastes, pressures, sensations of heat, and so on, of which, strictly speaking, there is no talk in theoretical physics generally. This state of affairs is much effaced through the regular usage of the same expressions in the two regions ("pressure", "heat", also "color", "tone", and so on). This simplifying, but inexact, speech usage can hardly occasion harm within physics, although it may do so in investigations of the relations between physics and other sciences. Moreover, should the separation be indistinct in this way, then in the treatment of a physical question one tends to mix up the components that we have apportioned to the first and second volumes.

Now the second volume connects the two domains, so to speak, through a kind of dictionary that indicates which objects (elements, complexes, processes) in the second domain correspond to the particulars of the first. It does so rather like this: "Such a blue (designated, for example, according to Ostwald's color system) corresponds to a certain periodical movement of electrons (denoted by the frequency of oscillation)" (in example 2) or "... a certain region of the electromagnetic four vector field periodically ordered" (in example 3b). Obviously, the second rubric of the second volume depends on the content of the first volume. The elements of the physical domain and their relations, the physical processes, are determined by the chosen axiom system.

The epistemological (or, actually, metaphysical) question of the existential significance *[Seinsbedeutung]* of the two domains shall here be left aside entirely. Its answer does not matter at all to the solution of our problem. For in physics (contrary to a widely held opinion) it is insignificant whether one calls, in a phenomenalist-realistic sense, the contents of the first domain (e.g. the perceived color blue) "mere appearances" and those of the second domain (e.g. the corresponding electromagnetic oscillation) "reality"; or whether one, vice versa, in a positivist sense, describes the former as the "real data" and the latter as "mere conceptual complexes of those sensational contents". Therefore it is not: "Where this blue appears there really is such and such an electron

process" and also not, "Instead of this blue, we construct such and such an electron process in order to facilitate calculation". In fact, physics expresses itself neutrally with the aid of the purely formal relation of coordination, and it leaves those metaphysical interpretations to a non-physical examination.

Further sentences are for instance: (in example 2) "Such a pungent smell (the smell of chlorine; lacking a classificatory system) corresponds to a certain mixture of peculiarly structured electron complexes (*Cl*-atoms)"; "Such a temperature sensation (lacking a classificatory system) corresponds to a certain average kinetic energy of a number of electron complexes (atoms or molecules)". -- The dictionary can be used in both directions: it serves to translate a phenomenal state of affairs to the corresponding physical one and vice versa. However, we have to note, that *only in the second case is the correlation is unambiguous*. In the first case, a determinate sensory content corresponds not just to one certain physical state of affairs but to an infinite number of these. This is for two reasons. The first reason becomes clear from the last of our example sentences. In that sentence, an average kinetic energy of a number of molecules (and, consequently, the same temperature sensation) corresponds to those infinitely many velocity distributions, differing in amount and direction, which result in the same average value. The second reason lies in the psycho-physical fact of the limit of sensitivity.

The *third volume* contains the description of the physical state of the world at any two points in time. In order to calculate what happens at any time at a certain place it is not enough to indicate the state at merely one point in time. At least not if only the state magnitudes themselves are indicated but not their differential quotients. As logical analysis shows (Russell, Mongré-Hausdorff⁷), the latter do not belong to the momentary properties, although they may be treated as such mathematically. The two states to be described do not have to be neighboring in time, as is sometimes surmised. For based on any two states two neighboring ones can be calculated, and consequently their differential quotients. For that it is however required that the identity of the corpuscular elements *(Substanzelemente)* is indicated; e.g. in the case of electron theory not only the spatial

⁷ "Paul Mongré" was a pseudonym of the mathematician Felix Hausdorff [Editor's note].

distribution (besides the field distribution) of the electrons for the two points in time must be indicated; it also has to be made clear which electron in one distribution is identical with which of the other. If it is possible to calculate, from the coordinate values of the n elements at the time t_0 and the 3n components of their velocities, the 3n coordinates at time t_1 , then we only need to imagine that we have solved the 3n equations, referred to the latter coordinates (i.e., for t_1), for the 3n velocity components in order to understand that these components are then determined by the coordinates for the time points t_0 and t_1 .

Incidentally, it seems possible to assume that the description does not have to be limited to two sections of so special a kind (t = const.) through the four-dimensional space-time world, but that perhaps any two three-dimensional sections can be chosen. However, apparently this has not yet been investigated.

Sometimes the opinion been expressed that only future but not past events could be calculated definitely based on the two state descriptions alone. However, this holds only if we do not indicate the spatial distribution of the single elements (e.g., electrons) but only the average values of certain state magnitudes for areas containing a very high number of elements (e.g., temperature, gas pressure). Practically, only this latter case can occur (because of the above-mentioned ambiguity of correlations of the second volume). But here we begin by presupposing ideal physics, and we take a state description to be the indication of the distribution of elements. In this case there is no difference between the calculation of the future and of the past.

Obviously, the character of the third volume depends on the question which axiomatic system we have assumed for the first volume. For according to the choice of this system, the state of the world at one moment will be described as a momentary distribution of matter particles, or of streaming aether particles, or of electrons and the electromagnetic field, or as a three-dimensional section through the four-dimensional vector field and tensor field, and so on. The relation of the three volumes in achieving that task of fictive, completed physics is as follows. We assume that we want to determine what happens at a certain time at a certain place, i.e. what is perceivable, using the state descriptions provided by the third volume and the aid of the sentences provided by the first. To achieve this, the state in the environment of that space-time point is calculated and expressed in sentences about sensation qualities through the inverse translation provided by the second volume (which in this direction is unambiguous).

In order to achieve a better approximation to the actual state of affairs, let us for a moment forget about the fiction of completeness for our first and second volume, though not for the third. Thus we assume we have complete knowledge of the laws of nature and the relations between physical states of affairs and sensation contents, but not about the state of the entire world. Then the task is: To calculate from the observed state of a restricted domain, namely our own spatio-temporal region, the state of another spacetime region. For this, however, the second spatio-temporal region must initially be temporally near the first and spatially smaller and completely enclosed by it (and indeed such that the smallest distance between a limit point of the first and one of the second region is constantly greater than the temporal interval multiplied by the velocity of light). In this way, for example, the state of such a small spatial portion could be predicted only for a second, near the beginning of the second the state could be known inside of a cone of more than 300,000 km radius. Yet fundamentally this is taken only to be a technical difficulty. More difficult, however, is the circumstance that the state of a region should be determined through observations. However, such a determination is fundamentally impossible due to the ambiguity of the relations of the second volume. Thus even the solution of that restricted task is not possible.

Nonetheless that a physics even still far removed from this more circumspect fiction can generally assign predictions on the ground of observations, has the following basis. To a certain store of observations, to be sure, there corresponds an infinite set of physical states of the region, and along with it also an equally large set of such states for the future moment that is to be ascertained, and even a still larger set, if one, as always in practical application, does not exactly fulfill the presuppositions of proximity in time and enclosure in space. But with the inverse translation of this infinite set of physical states back into sensory contents, there arises in many cases a relatively small set of sensory contents, that, in favorable cases, forms a continuous domain of qualities (e.g. a domain of similar shades of color). The effort is then first directed to line the observations up in such a way that further qualitative domains that do not hang together do not arise for that future instant, and then to narrow the limits of each qualitative domain as much as possible. The two defects of the prediction, ambiguity and inexactitude, can thus be reduced more and more in the progress of science. In special cases, for time intervals that are not too long, they can be entirely gotten rid of, and univocalness of prediction achieved. Even so, it remains in principle impossible to get rid of them for arbitrary time intervals. This much is true for what is practically the only thing we seek, prediction of the contents of perception. From the unambiguous prediction of physical states, in contrast, science always remains infinitely far removed, even for spans of time that are just as small.

IV. The Two Possibilities for the Application of the Basic Principle of Greatest Simplicity

After we have thus seen how the ideal physics consists of three parts, the axiomatic, the part concerning phenomenal-physical coordination ("dictionary"), and the descriptive, we can more exactly address the question of Section II: With which part of physics is the requirement of greatest simplicity concerned? Two different paths to a unified science suggest themselves, according to the answer one gives to this question.

The first part certainly counts more often than not as the most important part of physics. It is even commonly seen as the sole one. That is the case, e.g., if physics is defined as the science of natural laws, or when it is stipulated that it must eliminate "subjective qualities", by which are meant sensory contents. This interpretation can be sustained; what we then designate as the second and third part of physics would have to be assigned to the fields of physiological psychology of the senses and of a (not extant)

descriptive total science, to which *inter alia* astronomy and geography belong. In this delineation of the scientific field (the evaluation of which is at bottom only a question of linguistic usage), Haas is correct to understand the ideal as "physics as geometrical necessity", i.e. as an axiomatic system of pure deduction. In this case it may very well even seem reasonable for physics to relate the requirement of greatest simplicity to the first part, without taking into account the other parts that then do not belong to it. If it sets out upon this first path, then Euclidean geometry and Newton's laws are in themselves the simplest to establish axiomatically; in this case, there is no reason available to choose other laws. Preference is then to be given, among the axiom systems discussed, to Example 1a. Since Dingler proceeds from the above-mentioned presupposition (physics as axiomatic science), he thus accordingly has the right to place this system at the foundations of his procedure of "pure synthesis", rejecting the other systems. If an action-at-a-distance law is held to be inadmissible (the grounds for this cannot be discussed here), preference then may be given to Wiener's system.

The matter stands entirely otherwise when one also takes into consideration not only the first, but also the second and third parts of physics (according to our division). Earlier considerations have already shown that, according to the choice of axioms for the first division, there result different forms for the second and third. And in fact, the third will be just as much simpler as the second is. For since the succession of our perceptions is given and is subject to nothing else that determines our choice, the state of the world that arises from it through the translation will be just as much simpler as is the second column of the dictionary that provides the basis of the translation. For this reason the result comes out the same, whether the measure of greatest simplicity is applied to the second or the third division. Since, however, testing for simplicity the physical processes corresponding to individual sensory contents is easier than testing for simplicity the state of the entire world (which is practically never really known), the criterion is more usefully applied to the second division, and the requirement expressed as follows. Physical axioms are to be chosen so that the physical processes correlated with individual sensory contents and complexes of such are as simple as possible; and among the axiom systems that satisfy this requirement in an equivalent manner, the one that is simplest in itself is to be chosen. That is the second way.

A definite verdict cannot yet be delivered as to which of the axiom systems, discussed or otherwise, the choice should fall upon. For neither has any one of the theories been constructed with an eye to this path, nor have investigations been conducted of how the individual theories behave if they are tested for greatest simplicity from this viewpoint. In addition, more exact criteria for the determination of the degree of simplicity of a physical process must first be drawn up. Even so, there are regardless already a few preliminary conjectures that can be indicated. The systems of Dingler and Wiener, though axiomatically the simplest, do not seem to have the advantage here. Regarding optical processes in particular, Dingler, who has not yet attempted the undertaking, must assume on top of the customary material atoms also atoms of second and presumably a few higher levels, which will result in a exceedingly complicated structure. For the relatively simple construction of atoms out of a structure corresponding to electrons is not possible in his system, since only attractive forces are assumed. For Wiener, the construction of the theory is indicated in its basic outlines. On this account, the structure seems to become very complicated; e.g., to the electron there probably corresponds a spiral vortex ring [schraubiger Wirbelring] of third order made up of aether particles.

According to general relativity theory, in contrast, there arises, presumably, a simpler structure. There what is involved, e.g. for electrons, are determinate configurations of the four-vector fields on the basis of certain singular solutions of the equations (which, however, have not yet been carried out). These configurations may show spherical symmetry, ignoring the minimal, in general negligible perturbations through reciprocal interaction. Above all, however, it is to be emphasized that both for vector and for tensor fields, what is involved are only determinations of the world-metric. "Physical" state magnitudes in the genuine, old sense, namely such as do not determine merely the geometrical mass relationships of the world, do not exist at all in this theory (in the form Weyl gives it, 3b). On the other hand, though, the structure of physical

processes according to this theory is, preliminarily, not yet secure against further complications, which perhaps will yet arise from the phenomena connected with quantum theory; while, so it seems, here the structure according to Wiener's theory (in itself quite complicated) has less to fear from this.

V. The Result. Conditions for evaluating physical theories.

Our considerations have shown that three stipulations, which can be chosen freely, have to be made for the construction of physics: space postulate, time postulate, and action principle. The choice should follow the principle of maximum simplicity. There are still, however, two different possibilities for how this principle is applied: it can either be taken as referring to the axiomatic part of physics, or to the phenomenalphysical correlation and therewith also to the descriptive part. No decision between these two possibilities is made here. The purpose of the considerations is rather to exhibit the relevant questions for deciding between different, logically equally appropriate physical theories.

The following antecedent questions have to be answered, before a decision between a number of given physical axiom systems can be made. First of all, science has to come to a consensus as to which kind of principle of maximum simplicity should be applied. If we are to take the first path, then criteria must be given according to which we can determine the degree of simplicity of an axiom system in itself, i.e., without regard to its application. These criteria then have to be applied to the given axiom systems, or a new axiom system has to be developed according to these criteria. Dingler has already made important progress in this direction. If, on the other hand, we conclude that the second of the proposed paths is to be followed, then the task becomes more difficult. For in that case, it is not just the axiom systems themselves which have to be examined. It is not, as a popular realistic conception would have it, that we have to pick out, among the axiom systems, those which stand "in correspondence with the facts of reality". This is so since the meaning of the axioms is not given by observation contents -- they are rather purely formal determinations that are coordinated with items of perceptual content-- and thus "correspondence with reality" can be achieved for any arbitrary axiom system. To do this, one only needs to put the coordination relations [Zuordnungsbeziehungen] in the proper form (the "valid coordinations"). When this is done, we may obtain "valid coordinations" for the different axiom systems greatly differing in simplicity. This is the correct core of the logically untenable distinction between "correct" and "false" systems. We say: A theory T_1 accords with certain observations, but another T_2 does not; or also, and this means the same thing: In contrast to T_1 , T_2 requires new and ad hoc hypotheses in order to explain these observations. The correct sense of this sentence is that we have to assign structurally much more complex physical processes to the perceptual content of these observations on the basis of T_2 than on the basis of T_1 .

If we consider the second path to be the right one, we will first have to produce the "valid coordinations" (the "phenomenal-physical dictionary") for each of the axiom systems among we want to choose. The coordinations are "valid" if they assign to each actually given, temporal sequence of items of perceptual content (at least) one physical course of events *[Ablaufreihe]* possible on the basis of the axioms. In order to determine the simplicity of these valid coordinations, we have to first establish guidelines as to which perceptual contents and complexes are to be taken as *essential* and which should serve as test points *[Prüfpunkte]* (since we cannot take into account the infinite set of all of them). Secondly, it is necessary to provide standards of measurement to be able to determine the degree of simplicity of the structure of physical processes. Incidentally, it is not as difficult to choose such standards in a non-arbitrary manner as it might seem at first glance, since we understand "physical processes" as purely formal complexes ("order complexes" of the theory of relations). Hence, when evaluating the simplicity of their structure we need not consider properties other than those exhibited by, e.g., the figures of (formal) geometry.

Thus we have shown which decisions have to be made and which criteria have to be established in order to evaluate a physical theory and to decide between several competing theories, without appeal to scientific instincts that have so far reigned supreme in this area, and within the scope of conscious principles of the theory of science *[Wissenschaftslehre]*.