

Bernays Project: Text No. 7

**On Nelson's Position Concerning the
Philosophy of Mathematics
(1928)**

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Comments:

none

Subsequent to the preceding article by OTTO MEYERHOF some words on NELSON's significance for the philosophy of mathematics should be said.

NELSON belonged to those philosophers whose way of thinking resulted from a familiarity with the spirit of exact sciences. For him, mathematics and theoretical physics formed the model he tried to follow in elaborating his philosophical thoughts.

He found the demands of rigorous systematics perfectly fulfilled in mathematical axiomatics, especially in that form *Hilbert* had given to it in his "Grundlagen der Geometrie". And therefore it was his endeavor to conquer a new field for this method of axiomatics in the domain of philosophy.

In doing so NELSON stood far away from that unfruitful way of imitating mathematics which was dominant in the pre-Kantian metaphysics, based

on the belief, that knowledge can be conjured up from nothing by logical reasoning.^{143l}

As adherent of KANT he held the doctrine of the *synthetical character* of mathematical knowledge; he stressed that the amount of knowledge of mathematics is enclosed in its axioms which he considered as expression of knowledge from *pure intuition*.

In several writings, especially in the treatise “Bemerkungen über die nichteuclidische Geometrie” (1906), he criticized the sceptical and empiristic ideas that had found more and more adherents among representatives of science since the discovery of non-Euclidean geometry.

Here he showed how these ideas results from holding the old ARISTOTELIAN doctrine according to which all knowledge has its origin either in sensuality as the source of experience, or in the mind as the source of logic.

If this disjunction, which is in itself not necessary, is dropped, one keeps the possibility to accept extra-logical necessities, especially those of intuitive nature, which are expressed in synthetic sentences. Especially concerning the parallel axiom,—if this “dogmatic disjunction” is abandoned—it is by no way possible to infer from the logical possibility of a non-Euclidean geometry that the parallel axiom has no necessary validity, but only the synthetical, i.e., non-logical character of this axiom.

These ideas were further elaborated by NELSON in a lecture “Über die Grundlagen der Geometrie”, which he delivered in Paris in April 1914 (on the occasion of the foundation of the “Société internationale de philosophie mathématique”).

Here NELSON supports his claim of the *intuitive but at the same time rational character of geometrical knowledge* by a series of arguments.

He especially hints at the fact, e. g., that the difficulties of a conceptual description of the continuum (the continuity) are a clear sign for the fact that here a task is present which is posed to reasoning from the outside, i. e. through intuition.

He furthermore emphasizes that intuition cannot be charged with typical geometrical errors, like, e. g., those being due to overlooking the possibility of one-sided planes, but that they result from hasty conceptual generalizations of states of affairs seized intuitively.

In addition he objects to the demand that the non-Euclidean space could be seized intuitively. In the known presentations of non-Euclidean geometry, e. g. by the geometry of the globe with a suitable definition of congruence, is that what is shown indeed ^{143l} not a non-Euclidean space, but only that the non-Euclidean system of laws is satisfied by certain objects and relations of the Euclidean space.

If this argument is today not accepted by many, this is due to the fact that today's mathematicians have mostly lost the real meaning of the words "intuition" and "intuitive", so that intuitiveness is talked about in most cases only in a dimmed and blurred sense, where especially no distinction is drawn between real intuitive comprehension and mere intuitive analogy.

More important opposition against NELSON's standpoint starts out from the opinion that our spatial intuition has no perfect distinctiveness, that therefore geometrical laws are only approximately determined by intuition derived from the data of intuition only by a process of idealization.

NELSON argues against this demand as follows: It cannot be denied that the geometrical axioms are an idealization in their relation to the fact of observation. But this circumstance only speaks against the *empirical* character of geometrical laws. Their *intuitive* character is thereby not disputed (unless that one takes the mentioned dogmatic disjunction as a basis).

On the contrary: an idealization presupposes an ideal. Only if such an ideal in the sense of an epistemological norm is given to us, the abstraction doing the idealization has its definite distinctiveness, being free of arbitrariness, and only then, as well, the steadiness of the idealization in face of the extensions of our domain of experience is guaranteed. Hence, it is just the aspect of idealization which gives us a hint on the fact of pure intuition, on the base of which the process of idealization can simply be understood as the transition from sensual intuition to pure intuition.

For Nelson, from this doctrine of pure intuition as the norm for geometrical idealizations results the consequence that there is a foundational difference between geometrical and physical idealization: for physical idealizations the applicability to reality is always problematic in the beginning, because the assumption of a limit for the idealizing limiting process [*Grenzprozeß*] requires a justification by experience and can therefore only be shown as highly probable at the best. Compared to this, for us the geometrical idealizations of limiting objects are given in pure intuition which gives the guide for the process of geometrical idealization; for us the existence of a limit is here certain, independently of experience. ^{|¹⁴⁴}

This independence from experience is not to be understood in the sense of pure immanence, so that one should, e. g., distinguish the apriori validity

of geometry for intuition from the validity of the “real” (physical) space. On the contrary, Nelson declares explicitly—in this respect too, a true adherent of Kant: “We know only *one* space. This is the space which is dealt with by geometry and in which the physical bodies are.”

Accordingly the laws of geometry have immediate obligation for physics, they form the frame in which all natural science is bound and only by which the task of physical research gets its determination. Because—as Nelson explains—, if one makes geometry itself an object of experimental control, the option for definite conclusions from physical observations gets lost, since then, given a new observation, one can never know whether it expresses a new feature of space, previously unknown, or some other physical fact. Nelson elucidates by the following example: Given one had at the time, when the earth was thought to be a disk, found by triangulation, that the sum of angles of mundane triangles is larger than two right angles, one could have concluded with the same right the non-Euclidean nature of space or the global shape of the earth.

What is said here especially about geometrical laws covers similarly all those laws which, according to the KANTian doctrine, are taken from pure intuition, i. e. also the laws of time and of the geometric doctrine of motion (cinematics).

Due to his conviction of the apriori liability of these laws for the physic explanation of nature, Nelson had to oppose to the new physics, whose characteristic instant consists just in the fact that one had more and more released oneself from the belief in the necessity of integrating all physic facts into the framework of an apriori fixed space-time order and in the resulting in princi-

ple exceptional position of geometric-cinematic laws compared with physical laws.

However, this change in the methodological conception of physics forms only a part of the philosophical impact from the newer development of the exact sciences. Another important influence comes from research on the *foundations of arithmetic*. Nelson lively and actively shared the development of this research.

Nelson stood even ^{|144r} in close contact to the ambitions starting from the *Cantorian set theory* via several members of the NEO-FRIESEAN school founded by him, especially through GERHARD HESSENBERG, who was one of the leaders in the development of the CANTORIAN set theory.

In detail he dealt with the *paradoxes of set theory* whose first becoming known he experienced. These paradoxes had a special interest for NELSON because of their relation to certain dialectical modes of inference, he used oftentimes for disproving antagonistic views—especially showing an “introjected” contradiction, i. e. a contradiction which occurs in all such cases where accepting the validity resp. the insightfulness of a posited general claim gives already a counter-example of its validity.

The tract “Bemerkungen zu den Paradoxien von RUSSEL[L] und BURALIFORTI” (Abhandl. d. FRIESSCHEN SCHULE, vol. II, issue 3), composed by NELSON together with GRELLING, does not claim to solve the paradoxes; it served for stating more precisely and for sharpening the given range of problems—at this place, e. g. the very concise paradox was set up for the first time, that is tied up to the word “heterological”—and for rejecting unsatisfactory attempted solutions.

Nelson stayed in critical reserve concerning attempts to found mathematics by pure logic. On the other hand he felt a deep interest and a vivid sympathy for the HILBERTian enterprise of a new foundation of mathematics. In this way of founding mathematics NELSON welcomed the realization of the methodic principle of a *separation of critique and system*, i. e., the complete dissociation of the founding procedure from the deductive-systematic erection of mathematics, and the epistemological distinction connected to the former between proper mathematical matters of fact and “meta-mathematical” matters of fact which have to be demonstrated by the foundation. This unanimity of the HILBERTian approach with the basic ideas of his one methodology following Fries was a big gratification for Nelson. Even shortly before the end of his life he expounded in a paper (56th convention of German philologists and schoolmen, Göttingen, September 1927) the methodological kinship of the HILBERTian foundation with the FRIESIAN critique of reason

There is, however, still another aspect which relates the HILBERTian foundation of mathematics to NELSON’s philosophy: the “finite attitude” demanded by Hilbert has to be epistemologically characterized as some sort of *pure intuition*, because it is on the one hand intuitive and it goes anyway on the other hand beyond what can actually be experienced. |^{145l}

The prerequisite of such a foundation of knowledge is as such still independent of the special nature of the HILBERTian conception; it is true for any finite foundation of mathematics. A characteristic feature of the HILBERTian foundation is, however, that here the *finite standpoint ist related to the axiomatic foundation of theoretical science*. The conditions of the finite attitude present themselves thereby as the conditions *for the possibility of theoretical*

knowlegde of nature quite in the sense of the KANTian problem.

As soon as this connection comes to general |^{145r} awareness, the possibility will thereby be given, that the basic ideas of the KANTian critique of pure reason will be revived in a new shape, detached from the special forms of its time dependence from whose bindings theoretical science has freed itself.

Such a methodic clarification can help anyway to enforce that what is warrantable of the rational tendencies today onesidedly disregarded, for whose maintaining NELSON sticked up all the time of his life.