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Paul Bernays
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Paul Bernays was born on October 17th 1888 in London; he died after a short illness on September 18th 1977 in Zürich. He was the son of Julius and Sara Bernays, née Brecher. His father was a businessman and—as he states in the curriculum vitae appended to his thesis—he was of jewish confession and a citizen of Switzerland. (1) Soon after the birth of Paul, the family moved to Paris and from there to Berlin. It is in Berlin that he attended school, from 1895 to 1907. He seems to have been quite happy at school, a gifted, well adapted child accepting the prevailing cultural values in literature as well as in music. It was indeed his musical talent that first attracted attention; he tried his hand at composing, but being never quite satisfied with what he achieved, he decided a scientific career. He studied engineering at the Technische Hochschule Charlottenburg for one semester, then realizing (and convincing his parents) that pure mathematics was what he wanted to do, he transferred to the University of Berlin. His main teachers were: Schur, Landau, Frobenius and Schottky in mathematics; Riehl, Stumpf and Cassirer in philosophy, Planck in physics. After four semesters, he moved to Göttingen; there he attended lectures on mathematics by Hilbert, Landau, Weyl and Klein, on physics by Born, and on philosophy by Leonard Nelson. Nelson was the center of the Neu-Friessche Schule—Bernays was quite an active member of the group and stayed in contact with it all his life. His first publication—in 1910—was "Das Moralprinzip bei Sidgwick und bei Kant", published in the Abhandlungen der Friesschen Schule. (2) There were two further publications in 1913 in the same Abhandlungen, one "Über den transzendentalen Idealismus", the other "Über die Bedenklichkeiten der neueren Relativitätstheorie". (3,4) Though we no longer have the difficulties discussed by Bernays, it is remarkable how calmly he takes part in otherwise rather heated controversies. There is no doubt that Bernays was deeply influenced by Nelson—by his liberal socialism as well as by his revised version of Kant's imperative demanding the permanent readiness to act according to duty (Nelson lived from 1882 to 1927).

In the spring of 1912 Bernays received his doctorate with a dissertation (written with Landau) on analytic number theory—the exact title being: "Über die Darstellung von positiven, ganzen Zahlen durch die primitiven, binären quadratischen Formen einer nicht-quadratischen Diskriminante." (1)

At the end of the same year he obtained his Habilitation at the University of Zurich where Zermelo was professor. His Habilitationsschrift was on function theory: “Zur elementaren Theorie der Landauschen Funktion $\phi(a)$”. (5)

From 1912 to 1917, Bernays was Privatdozent in Zürich. There are no publications (except the one's already cited of 1913) in this period. Bernays must have passed some crisis in these years. In the short biography published in the book “Sets and classes” (6), he states:
“At the beginning of the First World War, I worked on a reply to a critique by Alfred Kastil of the Fries philosophy. This reply was not published—by the time there was an opportunity to have it published I no longer agreed with all of it.” He even considered giving up mathematics at this time—but did not see anything he felt he could do better. Therefore, Hilbert's proposal to be his collaborator in Göttingen must have come as a relief to Bernays. That he found his way back to mathematics is shown by his Göttinger Habilitationsschrift, a brilliant piece of work written in a short time. Bernays left Zürich in 1917; his paper was submitted in 1918. The title is “Beiträge zur axiomatischen Behandlung des Logikkalküls”. (7) In retrospect it is hard to understand that the paper was not published at that time—parts of it appeared in 1926 in the Mathematische Zeitschrift. (8) Bernays explained this long delay once in the following way:
“To be sure, the paper was of definite mathematical character, but investigations inspired by mathematical logic were not taken quite seriously—they were thought of as amusing, half-way part of recreational mathematics. I myself had this tendency, and therefore did not take pains to publish it in time. It has appeared only much later, and strictly speaking not quite complete, only certain parts. Many things I had in the paper have therefore not

been recorded accordingly in descriptions of the development of mathematical logic". (9)

An analysis of the content of the paper will be given in some detail—the library at the ETH in Zürich is in possession of Bernays' copy.

As to his activity in Göttingen in the period 1917–1934, Bernays describes it as follows: My work with Hilbert consisted an the one hand in helping him to prepare his lectures and making notes of some of them, and on the other hand in talking over his research, which gave rise to a lot of discussions. (6)

Bernays also gave lectures on various subjects of mathematics and became an extraordinary professor of mathematics in 1922. Besides a series of papers written as R collaborator of Hilbert (a typical title being: Die Bedeutung Hilbert's für die Philosophie der Mathematik (10)), there are two papers where he is on his own: One with Schönfinkel (Zum Entscheidungsproblem der mathematischen Logik (11)), where a case of the decision problem is treated, the other: Zur mathematischen Grundlegung der kinetischen Gastheorie (12) (where a special case the ergodic theorem is proved). That his activity is only partially reflected by his publications in shown by references in the literature of students of Hilbert's. We learn e.g. from Ackermann in 1928 that the axiomatization of predicate calculus based on the rules

$$\frac{\varphi \to \psi}{(\exists x)\ \varphi \to \psi} \qquad \frac{\psi \to \varphi}{\psi \to (\forall x)\ \varphi}$$

(with the well known condition on ψ is due to Bernays. (20, p. 54)

In 1933 Bernays, as a "non-aryan", lost his position at the University of Göttingen. In 1934 be moved to Zürich—to call it a return would give a wrong impression; still in the fifties he could say: "bei uns in Göttingen" (we in Göttingen). In the same year 1934, the first volume of the Grundlagen der

Mathematik appeared. (13) It was hailed from the beginning as a masterpiece on a par with the works of Frege, Peano, Russell-Whitehead.

From 1934 to his death, the home town of Bernays was Zürich. Being single he first lived with his mother and two sisters, in the last years with his sister Martha, who survives him.

Twice he spent a year at the Institute for Advanced Study in Princeton (1935/36 and 1959/60), and three times he was visiting-professor at the University of Pennsylvania in Philadelphia. In Zürich, he was first a Privatdozent, then a professor till his retirement in 1958.

During his first stay in Princeton, he lectured on mathematical logic and on axiomatic set theory. His lectures on logic have been published as notes under the title: "Logical calculus" (14)—much of the material is taken up in the second volume of the Grundlagenbuch (1939). (15) The consistency theorem (a central theorem of the second volume) is contained in the Princeton notes.

In set theory he lectured on his own axiomatization. He had presented this system already in Göttingen in a lecture of 1929/30, but hesitated to publish it because he felt that the axiomatization was, to a certain extent, artificial. As Bernays records, he expressed this feeling to Alonzo Church, who replied with a consoling smile: That cannot be otherwise. (6) This persuaded him to publish—the work appeared in seven parts in the years 1937/1954 (16) and (almost unchanged) in the volume "Sets and classes". (6) (The same volume contains an English translation of a paper published in the anniversary volume of Fraenkel: "Schemata of infinity in axiomatic set theory". (17)

From the first volume of the Journal of Symbolic Logic up to volume 40, he published about a hundred reviews—it is he who reviewed there the fundamental papers of Gödel, Church, Gentzen, and many others. His last review is on the first volume of Schröder's algebra of logic—an essay of 6 pages in small print.

But it was not only through reviews that he reacted to the development—his correspondence was immense. The list of his correspondents counts up to a thousand persons, there are preserved up to 6000 copies of letters, many of them rather an essay than a letter.

In the second part, the work of Bernays shall be discussed in more detail. His main works may perhaps be grouped under three headings:

(1) Logic; (2) Set theory; (3) Philosophy.

There are, of course, papers which do not fall within these groups—papers on theoretical physics, calculus of variation, and especially on elementary geometry.

The first (and most detailed) analysis is on his Habilitationsschrift of 1918. The paper is on propositional calculus—following Russell and Whitehead, it is (except in studies of independence) based on the connectives of negation and disjunctions; implication is considered an abbreviation. The starting point of Bernays are the following axioms (slight variations of the axioms of PM):

$$(p \vee p) \to p$$

$$p \to (p \vee q)$$

$$(p \vee q) \to (q \vee p)$$

$$p \vee (q \vee r) \to (p \vee q) \vee r$$

$$(q \to r) \to ((p \lor q) \to (p \lor r))$$

Rules of inference are substitution and modus ponens. Bernays then introduces the following two notions: Derivable formula (ableitbare Formel) and identity (allgemeingültige Formel), the latter being defined by the truth table method. He shows that a formula is derivable if and only if it is an identity. The non trivial part is based on the following lemma: If a non derivable formula is added to the axioms, then every formula is derivable. The lemma is proved via the normal form theorem: Every formula is equivalent to a conjunction of "simple disjunctions". Bernays goes on to remark: This consideration gives furthermore a uniform procedure to decide whether a formula is derivable or not.

In the next paragraph is stated (though not proved) what is known as "Post-completeness":
"With respect to the logical interpretation of our calculus (which was at the origin of this study), we obtain the result that the totality of provable formulas coincides with the totality of identical formulas. And this means that our calculus contains a formal systematization of those laws of logic which concern relations of truth and falsity of propositions subsisting independently of their structure and content. Indeed, all relations between truth and falsity of propositions may be expressed with the help of conjunction, disjunction and negation, and therefore also with the help of the symbols of our calculus, and insofar the relations hold for arbitrary propositions, the corresponding formal expressions must be identical formulas in the sense defined."

The next question considered is the problem which connectives form a I basis. Besides giving complete answers (for the classical cases), partial systems are introduced and the following theorem in stated: If α is any formula (say, in $\neg, \vee$), either α or $\neg\alpha$ (and not both) is equivalent to a formula in $\vee, \wedge, \rightarrow$.) (It is added that there exists a finite system of axioms for the identical formulas in $\vee, \wedge, \rightarrow$.)

In the next paragraph, the independence of the five formulas is studied. First of all, it is shown that the formula expressing associativity is derivable from the 4 others.

Next, it is shown that none of the four others are derivable from the rest.—It is here (as far as I know), that "many-valued" logic occurs for the first time.

Bernays describes the method as follows: "In each one of the following proofs of independence, the calculus is reduced to a finite system for the elements of which a composition ("symbolic product") and a "negation" is defined, and this reduction is carried out in such a way that the variables of the calculus are related to the elements of the system as their values. The "correct" formulas shall be characterized by the fact that they assume only values of a certain given subsystem."

The most complicated system,introduced for these independence proofs, is on with 4 elements and a subsystem of 2 elements.

The last paragraph contains a detailed study on the possibility of replacing axioms by rules. There is given e.g. a system containing the only axiom $p \rightarrow p$ and six rules.

Bernays' best known contribution to mathematical logic is the work "Grundlagen der Mathematik", published under Hilbert's and his name, but written by Bernays alone. It is unique because of the wealth of material it contains—

published there for the first time (as much of what had been achieved by the Hilbert school in proof theory) or published in a more detailed form than in the original papers. It was—for a long time—a standard reference on mathematical logic, proof theory, arithmetization of metamathematics, recursion theory. But it is unique also in another respect—"foundation" is taken quite literally, it does not reduce mathematics to logic, or logic to mathematics—both are developed at the same time and (to some extent) the philosophy of mathematics along with it. Bernays once was asked why he had preferred, mathematics to music as a career. One of the reasons he gave was that he had difficulty in following three tunes simultaneously—the Grundlagenbuch shows that he hadn't this difficulty in foundation.

The work contains, of course, much material original to Bernays, but as he chose not to divide his work in definitions, theorems, proofs, it is perhaps best not to single out results which might be given the name "theorem of Bernays".

A further major contribution of Bernays is his system of set theory, first presented in the year 1929/30 in Göttingen, and then elaborated in a series of papers in the years 1937/54. The basic result is, of course, known to everybody—how the fact that predicate logic can be based on, say, disjunction, negation and existential quantification is transformed into an axiomatization of the concept of class in such a way that a finite number of class axioms suffice to prove a general scheme of class existence. What is perhaps less well known is the careful study of basing parts of mathematics on subsystems of the full system of axioms—there are discussed e.g. three systems, sufficient to develop analysis. It is here that Bernays introduces his weakened forms of the axiom of choice (as the axiom of dependent choice) and shows that they are sufficient for analysis in a wide sense—including, say, Lebesgue measure theory and the theory of function spaces. If a general tendency of these studies is to be mentioned, the most distinctive feature is that Bernays tries to get along without sum axiom and power set axiom as long as possible. This is shown to be feasible in number theory, analysis, as well as in "general set theory".

In the book "Axiomatic set theory" (18) of 1958, a somewhat different version of the system is given—in stating the axioms, the existential form is replaced by the use of primitive symbols. Furthermore, the succession of steps in the development of the theory is different.

About half of the papers of Bernays' may be classified as philosophical. A collection of fourteen, referring to mathematics, has been edited by the Wissenschaftliche Buchgesellschaft under the title "Abhandlungen zur Philosophie der Mathematik". (19) The first article published there dates from 1927, the last from 1971.

The thinking of Bernays is characterized by his constant effort to do justice to all aspects of the problems he considered. He believed in their inherent complexity and always resisted the temptation of explaining away. A typical example is geometry—for him it had not vanished in an abstract structure and/or a part of physics, nor did he adhere to reductionism so common in foundational studies It is clear that such an attitude excludes short answers to almost all problems. Nevertheless, in order to give some impression of Bernays' way of philosophical thinking, a short text is presented. (The German contains a word—"Sachhaltigkeit" which no dictionary lists. It has been translated by "reality".)(The text from an essay on philosophy of mathematics, presented at the International Congress of Philosophy in 1969. (19, p. 174,175)

"It seems appropriate to attribute to mathematics a reality which, however, is different from that of material world. That there exist other types of objectivity than that of material world is shown by objectivity in the domain of phenomena. Mathematics is insofar phenomenological as it is concerned predominantly with the study of idealized structures and is furthermore governed by the method of deduction. In the process of idealizing,the phenomenological and the conceptual come into contact. (It is therefore inappropriate to oppose these two to such an extent as is done in Kantian philosophy.) The specific character of mathematics as opposed to empirical science does not mean that we have in mathematics knowledge a priori. It seems necessary to concede that we have to learn also in this domain of mathematics and that we have there a kind of experience sui generis. (We may call it mental experience.) This is not prejudical to the rationality mathematics. Rather it seems a prejudice that rationality is necessarily linked to certainty. Certain knowledge in the simple and full sense is given us almost nowhere. This is the old insight of Socrates."

For those who have not known Bernays personally, a few words on his personality may be added.

As his immense correspondence, his friendliness to visitors, his acceptance of invitations to congresses until the last years of his life, clearly show, he liked the contact with other human beings. He was extremely benevolent, helping many an author with his papers—from.Hilbert to a high-school teacher having made some small discovery. On the other hand, he lived in an aura of detachment. He was unique in his refusal to judge other people; he never spoke badly of anybody there is every reason to assume that he did not even think badly of others. When, once, reference was made to a statesman almost universally recognized as one of the villains of this century, in order to induce him to a negative judgment, he replied: "My situation is so different from his, that it is not for me to pass judgment". There is no doubt that his gift of seeing everywhere the best and refraining from judgment where he could not see anything good, helped a great deal to free foundational studies from the situation where different schools are expected to fight one another.

In the name of all those who have known Bernays personally, it certainly may be said: We are grateful for the privilege to have been in contact with Bernays.

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