

# FORMAL EPISTEMOLOGY:

## Some open topics, background and bibliography

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### 1 Introduction

I will divide the main open topics into four main sections: the first devoted to epistemic logic, the second to Bayesian epistemology, the third to models of belief change and the fourth to interactive epistemology and related issues in the area of rationality in general (especially its relationships with issues in rational choice).

### 2 Epistemic Logic

The issue of *logical omniscience* – should knowledge be closed under logical consequence, should agents know all tautologies, or be at least committed to know them, should knowledge be closed under conjunction, etc. – and the possibility of studying epistemic operators where logical omniscience can fail is still the object of a great deal of attention both by logicians and formal epistemologists in general.

Another foundational topic that is very interesting is related to the role played by the domain of possibilities or the domain of states considered in the models. Should they be possible worlds as traditionally understood (un-structured primitives linguistically describable via maximal and consistent sets of sentences) or should they be understood as possibilities with epistemic parts (a la Lewis or Halpern) or should they be understood minimally as in the case of Hintikka's model sets? The first topic and the second often interact in important ways.

A third, relatively open, topic is the study of first and second order epistemic logics, in particular logics of probability. Quantification over individuals, over epistemic options or even worlds (in hybrid logics) are of special conceptual

interest. Rich models where various modalities and agents interact are also of particular philosophical interest.

Finally the careful distinction between *knowledge attributions* and *knowledge claims* leads to interesting foundational issues of various degrees of depth and to interesting logical distinctions.

Models of conditional belief or conditionals in general face some of the aforementioned challenges plus some foundational conundrums of their own (like doubts about whether, say, indicative conditionals are truth carriers at all).

## 2.1 Logical Omniscience

Logical omniscience can fail due either to normative or descriptive issues. Two examples of normative reasons are (a) models of high probability which fail rule (C) – see lecture notes – and (b) models of knowledge as tracking truth a la Nozick which fail at least rule (M) – see lecture notes and a recently distributed paper contributed to the last FEW meeting.

But lack of logical omniscience can appear also due to computational limitations or, more in general, cognitive limitations. In this case there are various types of models of *bounded agents* in the literature. The very notion of bounded agent remains an open conceptual problem with no dominant model universally recognized in the literature.

A general introduction to some of the dominant models (with some exceptions) is the relevant chapter in Reasoning About Knowledge, by Ronald Fagin, et al; MIT Press, 2003. A simpler model appears in the book Modeling Bounded Rationality, by Ariel Rubinstein, MIT Press, 1997. A good complement to both series of models is the paper by my coauthor Rohit Parikh in the most recent edition of FEW: "Sentences, Propositions, and Beliefs".

<http://ist-socrates.berkeley.edu/~fitelson/few/schedule.html>

Neighborhood semantics can be used advantageously in order to deal with the so-called 'awareness models'. An example of very recent work in this direction is the conference paper a student: Giacomo Sillari posted in the page of the conference called LOFT:

<http://www.csc.liv.ac.uk/loft06/accepted>

The study of the first order and second order cases where one is able to quantify over propositions is in its infancy (see references in the paper by Giacomo).

Much inspiration about the very philosophical enterprise behind the use of

epistemic logics can be found in *Knowledge and Belief* the monograph initially written by Hintikka (see references in lecture notes). The full formalization of Hintikka's model sets was commented in passing in the lectures. You can find something more detailed in a review of a recent book by Vincent Hendricks I recently wrote for the *Notre Dame Philosophical Reviews* (it is going to appear in a couple of weeks). But much remains to be done there reflecting properly about the philosophical uses and relevance of Hintikka's notion of defensibility and the contrast between this notion and the standard notion of consistency.

## 2.2 States: fat and thin

Regarding the second aforementioned topic it is useful to contrast the possibilities used by David Lewis in his essay *Elusive Knowledge* with the global states used by Fagin et al. in *Reasoning about knowledge*. Both models of states include epistemic descriptions of agents as world-components. So, the models of logical omniscience in *Reasoning about knowledge* are influenced by this type of modeling of states; while models of awareness in terms of neighborhoods are not. The most extreme consequence of adopting models where the states have epistemic parts leads to the abandonment of the axiom of foundation in set theory. See the chapter on modal logic in: *Vicious Circles* (Center for the Study of Language and Information - Lecture Notes) – by Jon Barwise, Lawrence S. Moss, 2004. Which are the connections between these types of models and neighborhood models? There is only preliminary work in this area (mainly correspondence between Larry and myself).

## 2.3 First and second order extensions

Regarding the third topic the study of the entire family of first order classical modal logics remains an interesting challenge. This includes the study of first order operators of high probability. I recently tackled this issue in three papers two of them already published:

‘First order extensions of classical systems of modal logic: The role of the Barcan schemas,’ *Studia Logica*, 71: 2002, 87-118.

Non-Adjunctive Inference and Classical Modalities, recently published in *JPL*.

and a third that is forthcoming in a special issue of *Studia Logica*:

First Order Classical Modal Logic (with Eric Pacuit) [see my web page]

But much remains to be done in this area. To have a flavor of possible open areas of research you can have a look at the slides of: "Quantified classical modal logic: The family of free classical systems" posted in:

<http://www.hss.cmu.edu/philosophy/phillogic/workshop/>

The foundational connections between this semantic program and the foundational basis of the Kripkean semantic program are interesting per se, as well as the extension of what Eric and I presented in our paper to the family of *free* quantified modal logics (normal and non normal) – Kripke only studied the subfamily of free normal modal logics.

A quick inspection some of the aforementioned papers immediately shows that the semantics in terms of neighborhoods of first order classical systems of modal logic is *very* different than the Kripkean one. Among other things it is possible to provide a full parametrical completeness result for the entire family in terms of frames with *constant domains*, which permits to re-think the philosophical dispute about possibilists and actualists from a fresh and insightful perspective.

The distinction between knowledge claims and knowledge attributions is philosophically important and we saw in the lecture notes that it can lead to important logical distinctions as well. Sometimes the distinction is framed as the contrast between the perspective of the first person and the perspective of the third person. This way of framing the problem has obvious linguistic connotations that might not be able to capture the full dimension of the contrast (as Hintikka explains in his book some paradoxes like Moore's paradox of saying and disbelieving can be formulated linguistically with proper names rather than in terms of the use of the first person singular pronoun).

## 2.4 Belief claims and belief attributions

This distinction between belief claims and belief attributions is often projected to the study of conditionals in terms of the distinction between *epistemic* and *ontic* conditionals (see the encyclopedia article of Cross and Nute on conditional logic in the second edition of the Handbook of Philosophical Logic and a forthcoming article of mine on conditional logic in the Stanford Encyclopedia). Epistemic conditionals are (for many) the ones expressed in the indicative mood in English (assuming that there is such mood, something that not all linguists agree about) and the ontic conditionals are expressed in the subjunctive mood but there are all sorts of classification problems and conceptual problems in need of clarification there. Ontic conditionals are usually considered truth carriers and there are various semantics for them, including variants of probabilistic and epistemic semantics as well.

Neighborhood models are possible in this area as well and, as usual, they are rather general and accommodating, although they presuppose that conditionals have truth values, so they exclude some types of conditionals from the general picture. The study of conditional models in terms of neighborhoods has some pedigree but in many areas remain in its infancy. Open problems are related to the study of first order models of conditionals, which are needed for most philosophical applications (study of laws and causality for example). Again the use of neighborhoods permits here a great degree of semantic flexibility. Rich models where one has in the background a Bayesian net and the language is expressive enough to capture quantification are seldom explored.

The explicit use of time and tense in epistemic models usually leads to clarity and in the case of some proposals like Nozick's notion of *method* the explicit representation of a branching time structure in the background is an unavoidable necessity (see our contribution to the last FEW conference at Berkeley). The appeal to branching time also makes easier the connection with work in learning theory (as developed by my colleague Kevin Kelly or by Daniel Osherson at Princeton).

### 3 Bayesian Epistemology

This topic is oceanic. Broadly understood the area includes much of the existing work in formal epistemology from the use of Bayesian networks to models of belief revision that are decision theoretically motivated.

Unfortunately there is a tendency of conflating all work in this area with the adoption (conscious or not) of a probabilist epistemology admitting only probability as the main epistemological primitive notion. So, to some extent the discussion about Bayesian epistemology ramifies as a discussion about the tenability of such forms of probabilism.

One book that assumes this strong form of probabilism tacitly is the recent book by Hartmann and Bovens called *Bayesian Epistemology*. The book's first part deals with probabilistic models of the notion of coherence as developed in the early work of C.I. Lewis and the most recent work of L. Bonjour. A good counterpoint to this account is the book of Eric Olsson called *Against Coherence: Truth, Probability and Justification*, Oxford, 2005; especially Part III section 8. For the remaining chapters David's introduction to Bayesian networks and related issues can be very useful.

A very recent book which tackles the problem of whether probabilism is tenable is the book by Christensen: *Putting Logic in its Place: Formal Constraints*

on *Rational Belief*. You can use advantageously parts of the lecture notes to assess on your own whether this defense of probabilism is tenable. I wrote a review of this book that is about to come out in a new issue of the *Journal of Philosophy*. The main idea there is to show that there is a form of probabilism that neither Christensen nor Stephan or Luc are taking seriously, namely the form of *unified* probabilism as presented in my:

The main foundational problem here is related to the use of conditional probability as the main epistemological primitive (either the sole primitive or one among others) and therefore the abandoning of Kolmogorov's axiomatization of probability as the one one should use in philosophical applications. At least two articles are salient there:

Seidenfeld, T. 2001 Remarks on the Theory of Conditional Probability: Some Issues of Finite vs. Countable Additivity, in *Probability Theory*, V. Hendircks et al., Kluwer, 167-78,

as well as Alan Hajek's What Conditional Probability Could not Be, recently published in *Synthese*. There are a family of open problems in this area of varied depth and nature (from the purely philosophical to the mathematically involved). Parikh and I contributed to this discussion in:

Conditional Probability and Defeasible Inference (with Rohit Parikh), *Journal of Philosophical Logic*, 34, 97-119, 2005. The paper contains a proof that Countable Additivity – and therefore Kolmogorovian probability – cannot be used for some philosophical and computational applications, like developing a probabilistic model of conditionals. Van McGee argued along these lines in his contribution to a volume in honor of E. Adams and R. Thomason and myself elaborated further on his line of argument in:

'Iterative probability kinematics', *Journal of Philosophical Logic*, 46, 479-524, 2001 (with Richmond Thomason) [this one is very technical – it requires some basic knowledge of probability defined over non-standard real numbers).

The discussion about lotteries remains a central topic in Bayesian Epistemology. The challenges they present for logical closure as a rationality requirement are presented in a series of articles starting with van Fraassen's:

ottery scenarios (rather than abandoning classical logic or adopting a paraconsistent formalism):

'First order extensions of classical systems of modal logic: The role of the Barcan schemas, *Studia Logica*, 71: 2002, 87-118.

Non-Adjunctive Inference and Classical Modalities, recently published in JPL.

and

First Order Classical Modal Logic, forthcoming in an special issue of *Studia Logica* (see my web page) – this one is also technical in nature but it contains a lot of examples related to probability defined over rich languages.

The line of argument in these articles is reminiscent of the type of argument used by Gödel to argue that a deviant logic (intuitionistic logic) is mappable to and *extension* of classical logic (the system S4). In this case non-Adjunctive and paraconsistent logics are mapped to modal extensions of the weakest classical system  $E$  – although these systems are non-normal systems weaker than the weakest normal system  $K$ .

## 4 Belief Change

This is another very broad topic with plenty of unexplored areas and open topics. The selection of topics here will be limited by my own theoretical biases to some extent.

Much of what we saw about belief change is based on models of belief change that articulate shifts in view as *cognitive decisions*. The philosophical issue of to what extent we can change our mind at will, and in which areas it makes sense to adopt this point of view, is, nevertheless, a live issue. There is an influential tradition in epistemology (including David Hume and W.V. Quine among other salient proponents) where belief is seen either in terms of responses to stimuli or dispositions to such responses. And as such they cannot be under the control of the believer. Inquirers cannot choose what to believe.

So, according to this influential tradition epistemology should engage in explaining responses to certain stimuli rather than prescribing *norms* for *choosing* beliefs, which in turn are seen (also normatively) as *commitments to the truth* or undertakings. Philosophically the work of Isaac Levi offers the most sophisticated account of both traditions. See chapter 3 on expansion of *The Fixation of Belief and its Undoing: Changing Beliefs Through Inquiry* for an account of the dispositional view in terms of *expansion routines*. And the two last lectures reflected aspects of Levi's work and Database Updating (Applied Logic Series) by Sven Ove Hansson, Springer, 1999.

And a companion book for a different way of thinking in terms of choice functions (used in our last lecture):

*Change, Choice and Inference : A Study of Belief Revision and Nonmonotonic Reasoning* (Oxford Logic Guides), by Hans Rot, 2001.

Another interesting paper that can be read as a companion piece for the first lecture on belief change is: H. Rott and M. Pagnucco (1999) Severe Withdrawal (and Recovery), *Journal of Philosophical Logic* 28, 501-547. This paper converges on the same axiom system than the one determined by our decision-theoretical model without appealing to the idea that changes in view are decisions. So, if you see this decision-theoretic model of choice as implemented a misguided voluntaristic epistemology, you might still have reasons for utilizing the same axiom system than the one determined by our semantics.

Among the early philosophical books devoted to this topic (aside from the seminal essays by Quine like *Two Dogmas*) one should count:

*Change in View : Principles of Reasoning* (Bradford Books), by Gilbert Harman, 1988 as well as Peter Gardenfors (recent) *The Dynamics of Thought* (Synthese Library) by Peter Gärdenfors August 22, 2005 and the earlier *Knowledge in Flux: Modeling the Dynamics of Epistemic States*, by Peter Gardenfors, 1990. For a pretty recent result that might be used to motivate our use of partitions see:

‘On the Degeneracy of the Full AGM-Theory of Theory-Revision, in *Journal of Symbolic Logic*, vol. 71, no. 2, pp. 661-676

Downloadable in:

<http://people.cohums.ohio-state.edu/tennant9/>

as well as a reformulation (Note on an observation by Neil Tennant) downloadable at:

<http://www.princeton.edu/~osherson/>

See also the bibliography of the lecture notes for work done by Wolfgang Spohn and by Judea Pearl in terms of *ranking functions*. One of the most comprehensive articles in this latter area remains:

M. Goldszmidt and J. Pearl (1992) Rank-based systems: A simple approach to belief revision, belief update, and reasoning about evidence and actions, in B. Nebel, C. Rich, and W. Swartout (Eds.), *Proceedings of the Third International Conference on Knowledge Representation and Reasoning*, Morgan Kaufmann, San

Mateo, CA, 661-672, October 1992.

downloadable in Pearl's home page containing an explicit manner of linking work on belief revision to work on Bayesian networks, especially the Markov condition in DAGs. The conjecture of completeness formulated in this article remains unproved and the connections between belief revision and causal reasoning incorporating DAGs remains also unexplored (in spite of its obvious interest).

## 5 Rationality and Interactive Epistemology

A very good introduction to issues related to rationality and their connection with issues of rational choice in general is offered in Amartya Sen's *Rationality and Freedom*, by Isaac Levi, *Journal of Philosophy* VOLUME CI, no 5, 2004.

Connections with topics in logic and with issues in social epistemology and social intentionality see the web page of Christian List at LSE:

<http://personal.lse.ac.uk/LIST/research.htm>

The so-called paradoxes of Allais and Ellsberg remain one of the main conundrums utilized to test the adequacy of theories of choice under uncertainty or risk. A very good introduction can be found in the corresponding chapter of *Decision, Probability and Utility: Selected Readings* by Peter Gärdenfors on consensus is also relevant. The best Bayesian account of consensus results available can be found in the excellent:

*Rethinking the Foundations of Statistics* (Cambridge Studies in Probability, Induction and Decision Theory, 1999) by Joseph B. Kadane, et al.