Part I – Syntax and Symbolization:

1. Given the translation key stated below translate the following sentences in English to the appropriate symbolization:

<table>
<thead>
<tr>
<th>B</th>
<th>Bob is gathering eggs.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Bob has fed the chickens.</td>
</tr>
<tr>
<td>D</td>
<td>Bob has fed the ducks.</td>
</tr>
<tr>
<td>S</td>
<td>The chickens are playing in the pond.</td>
</tr>
<tr>
<td>Q</td>
<td>The ducks are quacking loudly</td>
</tr>
</tbody>
</table>

(i) If Bob has fed the chickens and ducks then Bob is gathering eggs.

\[(C \land D) \rightarrow B\]

(ii) Whenever Bob is gathering eggs the chickens are playing in the pond or the ducks are quacking loudly.

\[B \rightarrow (S \lor Q)\]

2. Using the translation table form problem 1 translate the following formula into English sentences.

(i) \[(B \lor Q) \rightarrow S\]

If Bob is gathering eggs or the ducks are quacking loudly then the chickens are playing in the pond.

(ii) \[S \rightarrow (C \land D)\]

If the chickens are playing in the pond then Bob has fed the chickens and the ducks.
3. For each of the following expressions of sentential logic, determine whether or not it is a well-formed formula, and if not, explain why not.

(i) \((P \lor Q) \rightarrow R\)

This is obviously not an atomic formula, it also is not a negation. Furthermore it does not match \((P \rightarrow Q)\), \((P \lor Q)\) or \((P \& Q)\) so it cannot be a well formed formula.

(ii) \((P \& Q) \rightarrow (P \lor \neg Q)\)

This expression is a well-formed formula.
Part 2 – Semantics:

1. State what it means for an argument to be valid, then give an example of a valid argument.

   An argument is valid just in case whenever all the premises are true the conclusion is likewise true.

   An example of a valid argument would be:

   $\begin{align*}
P &\implies Q \\
Q &\implies R \\
\hline
P &\implies R
\end{align*}$

2. State whether the following claims are true or false and give a brief explanation of your answer.

   (i) An argument with a contingent formula as a conclusion and tautologies as premises is always valid.

   This is false. Since the conclusion is contingent it must be false on at least one truth value-assignment. But the premises are true under every assignment so this would count as a counterexample.

   (ii) An argument with a tautology as a premise and a contingent formula as a premise is always valid.

   This should've read: An argument with a tautology as a CONCLUSION and a contingent formula as a premise is always valid.

   This is true, since the conclusion is always true there couldn't possibly be a counterexample.
3. For the following formula give a truth table and state whether it is a tautology, contingent or contradictory.

(i) \((P \lor Q) \implies (\neg P \implies Q)\)

<table>
<thead>
<tr>
<th>P</th>
<th>Q</th>
<th>((P \lor Q))</th>
<th>(\neg P)</th>
<th>(\neg P \implies Q)</th>
<th>((P \lor Q) \implies (\neg P \implies Q))</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>T</td>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
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<td>T</td>
<td>F</td>
<td>T</td>
</tr>
</tbody>
</table>

The formula is clearly a tautology since it is always true.

4. Using truth tables determine whether the following argument is valid or invalid.

\[
\neg P \implies Q \\
\therefore \neg P
\]

This argument is not valid take the following counterexample: 
\(P = T\) and \(Q = T\). For this truth value-assignment the two premises are true whereas the conclusion is false.
5. Answer the questions below.

(i) Suppose we introduced a new connective * in our language that represents the “exclusive or” (that is, either A or B, but not both). First list the truth table for * and then explain why v-introduction is no longer a valid rule of inference when “v” is read as “*”.

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>A * B</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>T</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>F</td>
<td>F</td>
<td>F</td>
</tr>
</tbody>
</table>
Part III - Proofs

Note: For all the following proofs you may ONLY use the primitive rules given in module 4 and 5 (don't use DNI or DNE from 5).

1. Prove the following arguments using only the primitive rules.

(i) \[ P \rightarrow \neg Q \]
\[ \therefore Q \rightarrow \neg P \]

1. \( P \rightarrow \neg Q \) Prem
2. \( Q \) Assum
3. \( P \) Assum
4. \( \neg Q \rightarrow E \ 1, 3 \)
5. \( \_ \) \_ I 2, 4
6. \( \neg P \rightarrow I 5 \)
7. \( Q \rightarrow \neg P \rightarrow I 6 \)

(ii) \[ P \rightarrow R \]
\[ Q \rightarrow R \]
\[ \therefore (P \lor Q) \rightarrow R \]

1. \( P \rightarrow R \) Prem
2. \( Q \rightarrow R \) Prem
3. \( P \lor Q \) Assum
4. \( P \) Assum
5. \( R \rightarrow E 1, 4 \)
6. \( Q \) Assum
7. \( R \rightarrow 2, 6 \)
8. \( R \lor E 3, 5, 7 \)
9. \( (P \lor Q) \rightarrow R \rightarrow I 8 \)
(iii) 

\[ P \land Q \]

\[
\therefore \neg(P \rightarrow \neg Q)
\]

1. \( P \land Q \) Prem
2. \( P \rightarrow \neg Q \) Assum
3. \( P \) \& E 1
4. \( \neg Q \) \rightarrow E 2, 3
5. \( Q \) \& E 1
6. \( \_{\_} \) \_{\_} I 5, 4
7. \( \neg(P \rightarrow \neg Q) \) \neg I 5

(iv)

\[
\therefore ((P \rightarrow Q) \rightarrow P) \rightarrow P
\]

1. \( (P \rightarrow Q) \rightarrow P \) Assum
2. \( \neg P \) Assum
3. \( P \) Assum
4. \( \neg Q \) Assum
5. \( \_{\_} \) \_{\_} I 3, 2
6. \( Q \) \neg E 5
7. \( P \rightarrow Q \) \rightarrow I 6
8. \( P \) \rightarrow E 1, 7
9. \( \_{\_} \) \_{\_} 8, 2
10. \( P \) \neg E 9
11. \((P \rightarrow Q) \rightarrow P \rightarrow P \) \rightarrow I